Dark energy and radiation in novel gauge gravity theories

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Part 1: Novel quadratic gauge theories

Gauge Theories of Gravity

General Relativity

- Gauge **diffeomorphisms** $\mathbb{R}^{1,3}$:

$$g_{\mu\nu} \to R^{\mu}{}_{\nu\alpha\beta}$$

- **Einstein-Hilbert** action:

$$L_{\mathrm{T}} = -\frac{1}{2\kappa} R + L_{\mathrm{m}}$$

Einstein-Cartan Theory

- Gauge **diffeomorphisms** and **rotations** $\mathbb{R}^{1,3} \rtimes \mathrm{SO}^{+}(1,3)$:

$$h_{a}{}^{\mu}, A^{ab}{}_{\mu} \to \mathcal{R}^{a}{}_{bcd}, \mathcal{T}^{a}{}_{bc}$$

- **Einstein-Hilbert** action:

$$L_{\mathrm{T}} = -\frac{1}{2\kappa} \mathcal{R} + L_{\mathrm{m}}$$

- **Torsion** algebraically bound to **spin**.
- Still very popular as a 'minimalist' GR extension e.g. 1801.08076 [physics.pop-ph], 1911.08232 [astro-ph.CO].

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Full ten-parameter PGT$^{\text{q},+}$

- This gauge theory formalism is known as **Poincaré gauge theory** (PGT) and has a long history from Ryoyu Utiyama (1956), D. W. Sciama (1964), T. W. B. Kibble (1961).
- We consider the most **general** case:

$$
\begin{aligned}
L_{\mathrm{T}} = & -\tfrac{1}{2\kappa}\alpha_0\mathcal{R} + \alpha_1\mathcal{R}^2 + \alpha_2\mathcal{R}_{ab}\mathcal{R}^{ab} + \alpha_3\mathcal{R}_{ab}\mathcal{R}^{ba} \\
& + \alpha_4\mathcal{R}_{abcd}\mathcal{R}^{abcd} + \alpha_5\mathcal{R}_{abcd}\mathcal{R}^{acbd} + \alpha_6\mathcal{R}_{abcd}\mathcal{R}^{cdab} \\
& + \tfrac{1}{\kappa}\beta_1\mathcal{T}_{abc}\mathcal{T}^{abc} + \tfrac{1}{\kappa}\beta_2\mathcal{T}_{abc}\mathcal{T}^{bac} + \tfrac{1}{\kappa}\beta_3\mathcal{T}_a\mathcal{T}^a + L_{\mathrm{m}}
\end{aligned}
$$

- **Parity-preserving**.
- Plausible **Yang-Mills** structure (quadratic in ${\mathcal{R}^a}_{bcd}$ and ${\mathcal{T}^a}_{bc}$):

$$
{\mathcal{R}^{ab}}_{cd} \equiv 2{h_c}^{\mu}{h_d}^{\nu}\left(\partial_{[\mu}{A^{ab}}_{\nu]} + {A^a}_{e[\mu}{A^{eb}}_{\nu]}\right), \quad {\mathcal{T}^a}_{bc} \equiv 2{h_b}^{\mu}{h_c}^{\nu}\left(\partial_{[\mu}{b^a}_{\nu]} + {A^a}_{d[\mu}{b^d}_{\nu]}\right)
$$

- Torsion and curvature both propagate, spin and energy source currents.

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- Extra Lagrangian symmetries emerge if the $\{\alpha_i, \beta_i\}$ obey **criticality equalities**: these **critical cases** must be considered separately
- Lin *et al* 1812.02675 [gr-qc], 1910.14197 [gr-qc] performed an exhaustive $\mathrm{PGT}^{\mathrm{q},+}$ survey, building on earlier work Donald E. Neville (1978), Donald E. Neville (1980), E. Sezgin et al. (1980):
  - 1918 **critical cases** in **total**
  - 450 of which are free of **ghosts** and **tachyons** under further **unitarity inequalities** on the $\{\alpha_i, \beta_i\}$
  - 58 of which are **power-counting renormalisable** (**PCR**)
  - 33 of which we consider...

Particle content of the 33 critical cases

#	criticality equalities	ghost-tachyon exorcism inequalities	0^-	0^+	1^-	1^+	2^-	2^+	d.o.f
1	$l=r_1=t_1=t_3=r_3-2r_4=0$	$0<t_2, r_2<0, r_3(2r_3+r_5)(r_3+2r_5)<0$	●		○	○○		○	● ○ ○
2	$l=r_1=t_1=r_3-2r_4=0$	$0<t_2, r_2<0, r_3(2r_3+r_5)(r_3+2r_5)<0$	●	○○	○○○	○○		○	
8	$l=r_2=r_4=t_1=t_2=r_1-r_3=0$	$r_1(r_1+r_5)(2r_1+r_5)<0$		○○	○○○	○	○		
*1 9	$l=r_2=r_4=t_1=t_2=t_3=r_1-r_3=0$	$r_1(r_1+r_5)(2r_1+r_5)<0$			○	○	○		
*3 10	$l=r_1=r_2=t_1=t_2=t_3=r_3-2r_4=0$	$r_3(2r_3+r_5)(r_3+2r_5)<0$				○		○	
*4 11	$l=r_1=t_1=t_2=t_3=r_3-2r_4=0$	$r_3(2r_3+r_5)(r_3+2r_5)<0$	○		○	○		○	○ ○
12	$l=r_1=r_2=t_1=t_3=r_3-2r_4=0$	$r_3(2r_3+r_5)(r_3+2r_5)<0$	○		○	○○		○	
*2 13	$l=r_2=t_1=t_2=t_3=2r_1-2r_3+r_4=0$	$0<r_1(r_1-2r_3-r_5)(2r_3+r_5)$		○	○	○	○		
14	$l=r_1=r_2=t_1=t_2=r_3-2r_4=0$	$r_3(2r_3+r_5)(r_3+2r_5)<0$		○○	○○○	○		○	
15	$l=r_1=r_2=t_1=r_3-2r_4=0$	$r_3(2r_3+r_5)(r_3+2r_5)<0$	○	○○	○○○	○○		○	
16	$l=r_1=t_1=t_2=r_3-2r_4=0$	$r_3(2r_3+r_5)(r_3+2r_5)<0$	○	○○	○○○	○		○	
20	$l=r_1=r_3=r_4=r_5=0$	$0<t_2, r_2<0$	●	○○	○○○	○○	○	○○	
21	$l=r_1=r_3=r_4=r_5=t_1+t_2=0$	$r_2<0, t_1<0$	●	○○	○○○	○○	○	○○	
22	$l=r_1=r_3=r_4=r_5=t_1+t_3=0$	$0<t_2, r_2<0$	●	○○	○○○	○○	○	○○	
23	$l=r_1=r_3=r_4=r_5=t_1+t_2=t_1+t_3=0$	$r_2<0, t_1<0$	●	○○	○○○	○○	○	○○	
24	$l=r_1=r_3=r_4=t_1=0$	$0<t_2, r_2<0$	●	○○	○○○	○○			
*5 25	$l=r_1=r_3=r_4=r_5=t_1=0$	$0<t_2, r_2<0$	●	○○	○○○	○○			
*6 26	$l=r_1=r_3=r_4=r_5=t_1=t_3=0$	$0<t_2, r_2<0$	●			○○			
27	$l=r_1=t_1=t_3=r_3-2r_4=r_3+2r_5=0$	$0<t_2, r_2<0$	●			○○		○	
28	$l=r_1=r_3=r_4=t_1=t_3=0$	$0<t_2, r_2<0$	●		○	○○			●
29	$l=r_4=t_1=r_1-r_3=2r_1+r_5=0$	$0<t_2, r_2<0$	●	○○	○○○	○○	○		
*7 30	$l=r_4=t_1=t_3=r_1-r_3=2r_1+r_5=0$	$0<t_2, r_2<0$	●		○	○○	○		
*8 31	$l=r_1=t_1=t_3=2r_3-r_4=2r_3+r_5=0$	$0<t_2, r_2<0$	●	○		○○			
32	$l=r_1=r_3=r_4=r_5=t_3=0$	$0<t_2, r_2<0$	●		○○○	○○	○	○○	
33	$l=r_1=r_3=r_4=r_5=t_3=t_1+t_2=0$	$r_2<0, t_1<0$	●		○○○	○○	○	○○	
34	$l=r_1=t_1=t_3=2r_3-r_4=0$	$0<t_2, r_2<0$	●	○	○	○○			
*9 35	$l=r_1=t_1=t_3=r_3-2r_4=2r_3+r_5=0$	$0<t_2, r_2<0$	●		○	○○		○	
*10 36	$l=t_1=t_3=2r_3+r_5=2r_1-2r_3+r_4=0$	$0<t_2, r_2<0$	●	○	○	○○	○		
37	$l=r_1=t_1=r_3-2r_4=2r_3+r_5=0$	$0<t_2, r_2<0$	●	○○	○○○	○○		○	
38	$l=r_1=t_3=2r_3-r_4=2r_3+r_5=0$	$0<t_2, r_2<0$	●	○	○○○	○○	○	○○	
39	$l=r_1=t_3=2r_3-r_4=2r_3+r_5=t_1+t_2=0$	$r_2<0, t_1<0$	●	○	○○○	○○	○	○○	
40	$l=r_1=t_1=t_3=r_4+r_5=0$	$0<t_2, r_2<0$	●	○		○○		○	
41	$l=r_1=t_1=r_3-2r_4=r_3+2r_5=0$	$0<t_2, r_2<0$	●	○○	○○○	○○		○	

- We tend to work in modified coordinates (not in this talk!):

$$\{\alpha_i, \beta_i\} \to \{\breve{\alpha}_i, \breve{\beta}_i\}$$

$$\breve{\alpha}_0 = \alpha_0, \quad \breve{\alpha}_1 = \alpha_1, \quad \breve{\alpha}_2 = \alpha_2, \quad \breve{\alpha}_3 = \alpha_3,$$
$$\breve{\alpha}_4 = 2\alpha_4 + \alpha_5, \quad \breve{\alpha}_5 = \alpha_5, \quad \breve{\alpha}_6 = 2\alpha_6,$$
$$\breve{\beta}_1 = -2\beta_1 - \beta_2, \quad \breve{\beta}_2 = \beta_2, \quad \breve{\beta}_3 = \beta_3$$

- Then Lin *et al* scramble these again:

$$\{\breve{\alpha}_i, \breve{\beta}_i\} \to \{r_i, t_i, l\}$$

$$r_1 = \breve{\alpha}_4 - \tfrac{1}{2}\breve{\alpha}_5, \quad r_2 = \breve{\alpha}_4 - 2\breve{\alpha}_5,$$
$$r_3 = \tfrac{1}{2}\breve{\alpha}_4 - \tfrac{1}{2}\breve{\alpha}_5 - \tfrac{1}{2}\breve{\alpha}_6,$$
$$r_4 = \tfrac{1}{2}\breve{\alpha}_2 + \tfrac{1}{2}\breve{\alpha}_3, \quad r_5 = \tfrac{1}{2}\breve{\alpha}_2 - \tfrac{1}{2}\breve{\alpha}_3,$$
$$r_6 = \breve{\alpha}_1, \quad \kappa t_1 = -\breve{\beta}_1 - \tfrac{1}{2}\breve{\alpha}_0,$$
$$\kappa t_2 = -2\breve{\beta}_1 - 6\breve{\beta}_2 + \tfrac{1}{2}\breve{\alpha}_0,$$
$$\kappa t_3 = -\tfrac{1}{2}\breve{\beta}_1 + \tfrac{3}{2}\breve{\beta}_3 + \tfrac{1}{2}\breve{\alpha}_0, \quad \kappa l = \tfrac{1}{2}\breve{\alpha}_0$$

Particle content of the 33 critical cases

#	criticality equalities	ghost-tachyon exorcism inequalities	0^-	0^+	1^-	1^+	2^-	2^+	d.o.f
1	$l = r_1 = t_1 = t_3 = r_3 - 2r_4 = 0$	$0 < t_2, r_2 < 0, r_3(2r_3 + r_5)(r_3 + 2r_5) < 0$	●		○	○○		○	● ○ ○
2	$l = r_1 = t_1 = r_3 - 2r_4 = 0$	$0 < t_2, r_2 < 0, r_3(2r_3 + r_5)(r_3 + 2r_5) < 0$	●	○○	○○○	○○		○	
8	$l = r_2 = r_4 = t_1 = t_2 = r_1 - r_3 = 0$	$r_1(r_1 + r_5)(2r_1 + r_5) < 0$		○○	○○○	○	○		
[*1] 9	$l = r_2 = r_4 = t_1 = t_2 = t_3 = r_1 - r_3 = 0$	$r_1(r_1 + r_5)(2r_1 + r_5) < 0$			○	○	○		
[*3] 10	$l = r_1 = r_2 = t_1 = t_2 = t_3 = r_3 - 2r_4 = 0$	$r_3(2r_3 + r_5)(r_3 + 2r_5) < 0$				○		○	
[*4] 11	$l = r_1 = t_1 = t_2 = t_3 = r_3 - 2r_4 = 0$	$r_3(2r_3 + r_5)(r_3 + 2r_5) < 0$	○		○	○		○	○ ○
12	$l = r_1 = r_2 = t_1 = t_3 = r_3 - 2r_4 = 0$	$r_3(2r_3 + r_5)(r_3 + 2r_5) < 0$	○		○	○○		○	
[*2] 13	$l = r_2 = t_1 = t_2 = t_3 = 2r_1 - 2r_3 + r_4 = 0$	$0 < r_1(r_1 - 2r_3 - r_5)(2r_3 + r_5)$		○	○	○	○		
14	$l = r_1 = r_2 = t_1 = t_2 = r_3 - 2r_4 = 0$	$r_3(2r_3 + r_5)(r_3 + 2r_5) < 0$		○○	○○○	○		○	
15	$l = r_1 = r_2 = t_1 = r_3 - 2r_4 = 0$	$r_3(2r_3 + r_5)(r_3 + 2r_5) < 0$	○	○○	○○○	○○		○	
16	$l = r_1 = t_1 = t_2 = r_3 - 2r_4 = 0$	$r_3(2r_3 + r_5)(r_3 + 2r_5) < 0$	○	○○	○○○	○		○	
20	$l = r_1 = r_3 = r_4 = r_5 = 0$	$0 < t_2, r_2 < 0$	●	○○	○○○	○○	○	○○	
21	$l = r_1 = r_3 = r_4 = r_5 = t_1 + t_2 = 0$	$r_2 < 0, t_1 < 0$	●	○○	○○○	○○	○	○○	
22	$l = r_1 = r_3 = r_4 = r_5 = t_1 + t_3 = 0$	$0 < t_2, r_2 < 0$	●	○○	○○○	○○	○	○○	
23	$l = r_1 = r_3 = r_4 = r_5 = t_1 + t_2 = t_1 + t_3 = 0$	$r_2 < 0, t_1 < 0$	●	○○	○○○	○○	○	○○	
24	$l = r_1 = r_3 = r_4 = t_1 = 0$	$0 < t_2, r_2 < 0$	●	○○	○○○	○○			
[*5] 25	$l = r_1 = r_3 = r_4 = r_5 = t_1 = 0$	$0 < t_2, r_2 < 0$	●	○○	○○○	○○			
[*6] 26	$l = r_1 = r_3 = r_4 = r_5 = t_1 = t_3 = 0$	$0 < t_2, r_2 < 0$	●			○○			
27	$l = r_1 = t_1 = t_3 = r_3 - 2r_4 = r_3 + 2r_5 = 0$	$0 < t_2, r_2 < 0$	●			○○		○	
28	$l = r_1 = r_3 = r_4 = t_1 = t_3 = 0$	$0 < t_2, r_2 < 0$	●		○	○○			●
29	$l = r_4 = t_1 = r_1 - r_3 = 2r_1 + r_5 = 0$	$0 < t_2, r_2 < 0$	●	○○	○○○	○○	○		
[*7] 30	$l = r_4 = t_1 = t_3 = r_1 - r_3 = 2r_1 + r_5 = 0$	$0 < t_2, r_2 < 0$	●		○	○○	○		
[*8] 31	$l = r_1 = t_1 = t_3 = 2r_3 - r_4 = 2r_3 + r_5 = 0$	$0 < t_2, r_2 < 0$	●	○		○○			
32	$l = r_1 = r_3 = r_4 = r_5 = t_3 = 0$	$0 < t_2, r_2 < 0$	●		○○○	○○	○	○○	
33	$l = r_1 = r_3 = r_4 = r_5 = t_3 = t_1 + t_2 = 0$	$r_2 < 0, t_1 < 0$	●		○○○	○○	○	○○	
34	$l = r_1 = t_1 = t_3 = 2r_3 - r_4 = 0$	$0 < t_2, r_2 < 0$	●	○	○	○○			
[*9] 35	$l = r_1 = t_1 = t_3 = r_3 - 2r_4 = 2r_3 + r_5 = 0$	$0 < t_2, r_2 < 0$	●		○	○○		○	
[*10] 36	$l = t_1 = t_3 = 2r_3 + r_5 = 2r_1 - 2r_3 + r_4 = 0$	$0 < t_2, r_2 < 0$	●	○	○	○○	○		
37	$l = r_1 = t_1 = r_3 - 2r_4 = 2r_3 + r_5 = 0$	$0 < t_2, r_2 < 0$	●	○○	○○○	○○		○	
38	$l = r_1 = t_3 = 2r_3 - r_4 = 2r_3 + r_5 = 0$	$0 < t_2, r_2 < 0$	●	○	○○○	○○	○	○○	
39	$l = r_1 = t_3 = 2r_3 - r_4 = 2r_3 + r_5 = t_1 + t_2 = 0$	$r_2 < 0, t_1 < 0$	●	○	○○○	○○	○	○○	
40	$l = r_1 = t_1 = t_3 = r_4 + r_5 = 0$	$0 < t_2, r_2 < 0$	●	○		○○		○	
41	$l = r_1 = t_1 = r_3 - 2r_4 = r_3 + 2r_5 = 0$	$0 < t_2, r_2 < 0$	●	○○	○○○	○○		○	

- All **critical cases** switch off **Einstein-Hilbert**:

$$l = \alpha_0 = 0$$

- Particle content by **spin-parity** J^P sector.
- **Definite** J^P of **massive** gravitons (filled circles).
- **Possible** J^P of **massless** gravitons (circles) but definite number of D.o.F.
- Colors represent gauge fields (symmetric/antisymmetric tetrad and spin-connection excitations).
- Sometimes we have coupled excitations (mixed color).
- Sometimes field character changes via a gauge transformation (extra circles).

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$\mathrm{PGT}^{\mathrm{Q},+}$ COSMOLOGY

- QFT gives a tractable number of well-motivated gravity theories: we now want to use **cosmological IR** to further constrain them.
- **Cosmological curvature** from **FRW metric** with **spatial curvature** $k \in \{\pm 1, 0\}$ and **scale factor** R:

$$\mathrm{d}s^2 = \mathrm{d}t^2 - \frac{R^2 \mathrm{d}r^2}{1 - kr^2} - R^2 r^2 (\mathrm{d}\vartheta^2 + \sin^2 \vartheta \mathrm{d}\varphi^2)$$

- **Cosmological torsion** has **scalar** U and **pseudoscalar** Q freedoms:

$$\mathcal{T}^{a}{}_{bc} = (\hat{\boldsymbol{e}}_t)^d \left(\tfrac{2}{3} U \delta^a_{[c} \eta_{db]} - Q \epsilon^{a}{}_{dbc} \right)$$

- **Cosmological fluids** are spinless and defined by **equation-of-state parameter** $P_i = w_i \rho_i$:
  - **Radiation** including relativistic species: $w_{\mathrm{r}} = 1/3$.
  - **Matter** including baryons and CDM: $w_{\mathrm{m}} = 0$.
  - **Dark energy** added by hand: $w_{\Lambda} = -1$.

Towards a systematic approach

- Define $\{\sigma_i, \upsilon_i\}$ from **quadratic** $\mathcal{R}^a{}_{bcd}$ and $\mathcal{T}^a{}_{bc}$ sectors $\{\alpha_i, \beta_i\}$ which **uniquely affect cosmological field equations**:

$$\begin{aligned}
\sigma_1 &= \tfrac{3}{2}\alpha_1 + \tfrac{1}{4}\alpha_2 + \tfrac{1}{4}\alpha_3 + \tfrac{1}{4}\alpha_5 - \tfrac{1}{2}\alpha_6 \\
\sigma_2 &= \tfrac{3}{2}\alpha_1 + \tfrac{1}{2}\alpha_2 + \tfrac{1}{2}\alpha_3 + \tfrac{3}{2}\alpha_4 - \tfrac{1}{4}\alpha_5 + \tfrac{1}{4}\alpha_6 \\
\sigma_3 &= \tfrac{3}{2}\alpha_1 + \tfrac{1}{2}\alpha_2 + \tfrac{1}{2}\alpha_3 + \tfrac{1}{2}\alpha_4 - \tfrac{1}{4}\alpha_5 + \tfrac{1}{2}\alpha_6 \\
\upsilon_1 &= \beta_2 - 2\beta_1 \\
\upsilon_2 &= 2\beta_1 + \beta_2 + 3\beta_3 \\
(\alpha_0 &= \alpha_0)
\end{aligned}$$

- Redefine to use **spin-connection** rather than **torsion** as field variables, and **conformal time**:

$$U = 3(X + \partial_t R)/R, \quad Q = Y/R, \quad \rho_{\mathrm{r}} = \varrho_{\mathrm{r}}/R^4, \quad \rho_{\mathrm{m}} = \varrho_{\mathrm{m}}/\kappa^{1/2}R^3, \quad \mathrm{d}\tau = \mathrm{d}t/R$$

- Can obtain (relatively compact) statement of cosmological equations:

$$\begin{aligned}
0 &= (\upsilon_2 + \alpha_0)R(RX + \partial_\tau R) - 8\kappa\sigma_3\partial_\tau^2 X - 4\kappa\sigma_1 Y\partial_\tau Y - 4\kappa X(\sigma_2 Y^2 - 4\sigma_3(X^2 + k)), \\
0 &= (4\upsilon_1 - \alpha_0)R^2 Y - 4\kappa(\sigma_3 - \sigma_2)\partial_\tau^2 Y + 16\kappa\sigma_1 Y\partial_\tau X + 4\kappa Y(\sigma_3 Y^2 - 4\kappa(\sigma_2 X^2 + \sigma_3 k)), \\
0 &= 12\upsilon_2\partial_\tau^2 R + 12(\upsilon_2 + \alpha_0)R(\partial_\tau X - X^2) - 3(4\upsilon_1 - \alpha_0)RY^2 - 12\alpha_0 kR + 2\kappa^{\frac{1}{2}}\varrho_{\mathrm{m}} + 8\Lambda R^3, \\
0 &= 12\upsilon_2(2R\partial_\tau^2 R - (\partial_\tau R)^2) + 12(\upsilon_2 + \alpha_0)R^2(2\partial_\tau X - X^2) - 3(4\upsilon_1 - \alpha_0)R^2 Y^2 - 12\alpha_0 kR^2 \\
&\quad + 6\kappa\sigma_3(16X^2(X^2 + 2k) + Y^2(Y^2 - 8k) + 16k^2 - 2(\partial_\tau Y)^2 - 16(\partial_\tau X)^2) \\
&\quad + 12\kappa\sigma_2((\partial_\tau Y)^2 - 2X^2Y^2) - 4\kappa\varrho_{\mathrm{r}} + 12\Lambda R^4
\end{aligned}$$

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k-SCREENING

- First pair of equations are **torsion** field equations, second pair are **generalised Friedmann** equations:

$$0 = (\upsilon_2 + \alpha_0)R(RX + \partial_\tau R) - 8\kappa\sigma_3\partial_\tau^2 X - 4\kappa\sigma_1 Y\partial_\tau Y - 4\kappa X(\sigma_2 Y^2 - 4\sigma_3(X^2 + k)),$$

$$0 = (4\upsilon_1 - \alpha_0)R^2 Y - 4\kappa(\sigma_3 - \sigma_2)\partial_\tau^2 Y + 16\kappa\sigma_1 Y\partial_\tau X + 4\kappa Y(\sigma_3 Y^2 - 4\kappa(\sigma_2 X^2 + \sigma_3 k)),$$

$$0 = 12\upsilon_2\partial_\tau^2 R + 12(\upsilon_2 + \alpha_0)R(\partial_\tau X - X^2) - 3(4\upsilon_1 - \alpha_0)RY^2 - 12\alpha_0 kR + 2\kappa^{\frac{1}{2}}\varrho_{\mathrm{m}} + 8\Lambda R^3,$$

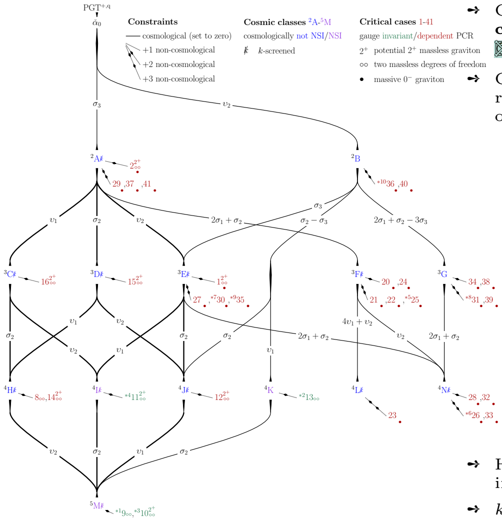
$$0 = 12\upsilon_2(2R\partial_\tau^2 R - (\partial_\tau R)^2) + 12(\upsilon_2 + \alpha_0)R^2(2\partial_\tau X - X^2) - 3(4\upsilon_1 - \alpha_0)R^2Y^2 - 12\alpha_0 kR^2$$
$$+ 6\kappa\sigma_3(16X^2(X^2 + 2k) + Y^2(Y^2 - 8k) + 16k^2 - 2(\partial_\tau Y)^2 - 16(\partial_\tau X)^2)$$
$$+ 12\kappa\sigma_2((\partial_\tau Y)^2 - 2X^2Y^2) - 4\kappa\varrho_{\mathrm{r}} + 12\Lambda R^4$$

- Consider switching off **Einstein-Hilbert** and **quadratic** $\mathcal{R}^a{}_{bcd}$ combination:

$$\alpha_0 = \sigma_3 = 0$$

- **Spatial curvature** $k \in \{\pm 1, 0\}$ is **eliminated** from field equations.
  - Can have **arbitrary** open/flat/closed universes.
  - But that geometry **won't** affect **expansion** H or **torsion** U, Q.
  - Call this 'k**-screening**'.

33 CRITICAL CASES SPAN 14 COSMIC CLASSES



- Group **critical cases** by $\{\sigma_i, \upsilon_i\}$ into **cosmic classes** (partial results H. Goenner et al. (1984)).
- Can recover many 0^- massive literature results on RHS of diagram, though these often include **Einstein-Hilbert** term:
  - Minkevich: A. V. Minkevich (1980), A. V. Minkevich et al. (2000), 0310060 [gr-qc], 0512123 [gr-qc], 0512130 [gr-qc], 0902.2860 [gr-qc], 1107.1566 [gr-qc], 1302.2578 [gr-qc].
  - Nester *et al* 0805.3834 [gr-qc], 0908.3323 [gr-qc], 1009.5112 [gr-qc], 1105.5001 [gr-qc], Fei-Hung Ho et al. (2011), 1512.01202 [gr-qc].
  - Zhang: 1904.03545 [gr-qc], 1906.04340 [gr-qc].
  - Lasenby: 0509014 [gr-qc].
- Hilighted 'cube' on LHS far more interesting. . .
- *k*-**screened**.
- All possible 2^+ gravitons.

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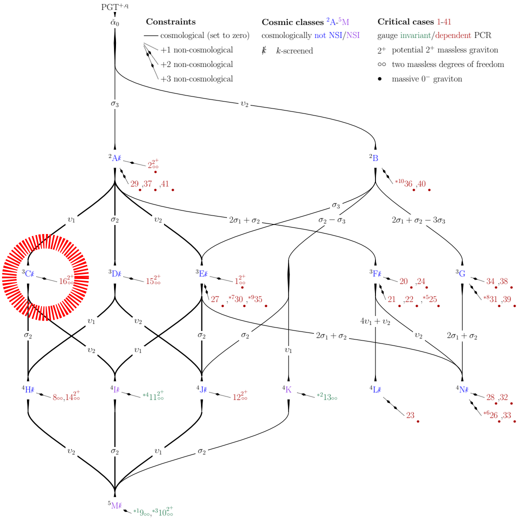
CLASS ^{3}C: MOTIVATION



- Pick **tractable** but **promising** cosmology like **Class ^{3}C**:

$$\alpha_0 = \sigma_3 = \upsilon_1 = 0$$

- Contains **Case 16**:
  - 2 **massless** gravitons. . .
  - . . . **potentially** 2^+
  - No **massive** gravitons.

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CLASS ^{3}C: EINSTEIN-FREEZING

- Useful consequence of $\upsilon_1 = 0$ is elimination of **scalar** torsion:

$$U = \frac{12\kappa Q\left(\left(\sigma_2 - \sigma_1\right) QH - \sigma_1 \partial_t Q\right)}{4\kappa\sigma_2 Q^2 - \upsilon_2}$$

- Only **two** remaining field equations of interest are now **density** Friedman equation and **pseudoscalar** torsion equation:

$$\Omega_{\mathrm{r}} + \Omega_{\mathrm{m}} + \Omega_{\Lambda} + \Omega_{\Psi} + \Omega_{\Phi} = 0, \quad f_1 \frac{\partial_t^2 Q}{Q} + f_2 \frac{(\partial_t Q)^2}{Q^2} + f_3 \frac{\partial_t Q}{Q} H + f_4 \partial_t H + f_5 H^2 = 0$$

- The effects of modified gravity are encoded in rational (but cumbersome) functions of the form:

$$f_i = f_i\left(\kappa^{\frac{1}{2}} Q \middle| \sigma_1, \sigma_2, \upsilon_2\right), \quad \Omega_{\Phi} = \Omega_{\Phi}\left(\kappa^{\frac{1}{2}} Q \middle| \sigma_1, \sigma_2, \upsilon_2\right), \quad \Omega_{\Psi} = \Omega_{\Psi}\left(\kappa^{\frac{1}{2}} \partial_t Q H^{-1}, \kappa^{\frac{1}{2}} Q \middle| \sigma_1, \sigma_2, \upsilon_2\right)$$

- Since we have k-**screening**, we might think to try **flat-GR** solutions – these are consistent when $\partial_t^2 Q = \partial_t Q = 0$ and $H \propto t^{-1}$.
- In this case it turns out **Class ^{3}C** is actually described by $\{\varsigma, \upsilon_1\}$ where $\varsigma = \sigma_1/\sigma_2$.
- If a **cosmological fluid** with **equation of state parameter** w_i is **dominant**, the **flat-GR** solution looks like:

$$-\Omega_{\Psi} - \Omega_{\Phi} = g_i(\varsigma, \upsilon_1, w_i) = \Omega_i, \quad \Omega_i = \frac{\kappa\rho_i}{3H^2}$$

- The **constant** g_i modifies the Einstein constant to $\breve{\kappa} = \kappa/g_i$, the **pseudoscalar torsion** freezes out at **constant** $Q = Q_i(\varsigma, \upsilon_1, w_i)$.

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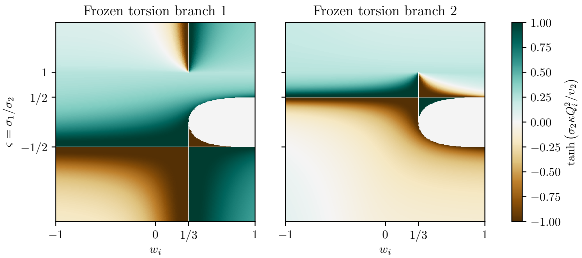
Class ^{3}C: Einstein-freezing

- **Class ^{3}C** depends on $\{\varsigma, \upsilon_1\}$ where $\varsigma = \sigma_1/\sigma_2$
- If a **cosmological fluid** with **equation of state parameter** w_i is **dominant**, the modified cosmological equations share solutions with **flat-GR**:

$$g_i(\varsigma, \upsilon_1, w_i) = \Omega_i, \quad Q = Q_i(\varsigma, \upsilon_1, w_i)$$

- The **constant** g_i modifies the Einstein constant to $\breve{\kappa} = \kappa/g_i$, while Q_i is given by:

$$\begin{aligned}(4\sigma_2/\upsilon_2)(12\varsigma^2 w_i - 4\varsigma^2 - 3w_i + 1)\kappa Q_i^2 = \\ 6w_i\varsigma^2 + 2\varsigma^2 + 6w_i\varsigma - 6\varsigma - 3w_i + 1 \\ \pm 2\big[9\varsigma^4 w_i^2 + 6\varsigma^4 w_i - 18\varsigma^3 w_i^2 + \varsigma^4 \\ - 12\varsigma^3 w_i + 9\varsigma^2 w_i^2 - 2\varsigma^3 + 3\varsigma^2 \\ + 12\varsigma w_i - 4\varsigma - 6w_i + 2\big]^{1/2}\end{aligned}$$



- Might be impossible under certain $\{\varsigma, \upsilon_1\}$ for **stiff matter** $w_{\mathrm{s}} = 1$.
- **Radiation** with $w_{\mathrm{r}} = 1/3$ is **special**.
- Note **scalar** torsion $U \propto H$, not constant.

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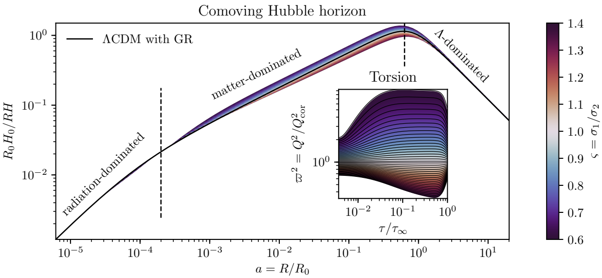
Class ^{3}C → Class ^{3}C*: motivation



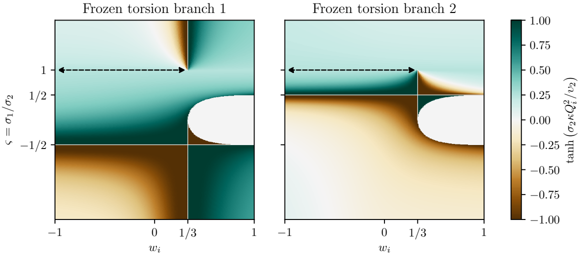
- **Flat-GR**-like expansion easy to obtain numerically.
- **Exact flat-GR** solution emerges at $\varsigma = 1$, call this **Class ^{3}C***.

- Can see that $\varsigma = 1$ is a **contour** of frozen Q_i:

$$\kappa Q_i^2 \equiv \kappa Q^2_{\rm cor} = \upsilon_2/4\sigma_1$$

- Also a **contour** of modified Einstein constant κ/g_i:

$$g_i \equiv g_{\rm cor} = -4/3\upsilon_2$$



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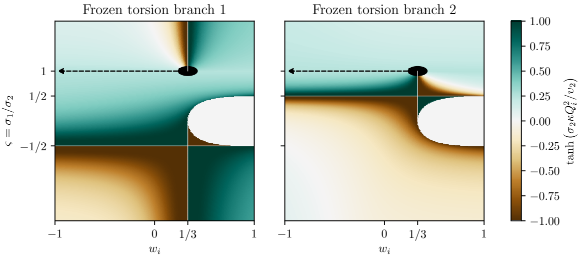
CLASS ^{3}C*: CORRESPONDANCE SOLUTION

- Call this the **correspondence solution** to **Class ^{3}C***:

$$\sigma_3 = \upsilon_1 = \alpha_0 = 0, \quad \sigma_2 = \sigma_1, \quad \kappa Q_i^2 \equiv \kappa Q_{\text{cor}}^2 = \upsilon_2/4\sigma_1, \quad g_i \equiv g_{\text{cor}} = -4/3\upsilon_2$$

  - Set $\upsilon_2 = -4/3$ to recover expected modified Einstein constant $\breve{\kappa}_i \equiv \kappa$
  - Without a measurement of Q we can't constrain $\sigma_2 = \sigma_1$
  - Nice surprise (given **k-screening** and $\alpha_0 = 0$)...
  - ...but expansion history contains **no new physics** beyond ΛCDM

- Can relax the **correspondence solution** by noticing that Q_{r} is **arbitrary** in **Class ^{3}C***
- This affects the expansion by tweaking g_{r} and $\breve{\kappa}_{\text{r}}$ at the Big Bang



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Class ${}^3\mathrm{C}^*$: arbitrary-ϖ_{R} solution

- The **flat-GR** series expansion for $a = R/R_0$ in **conformal time** $\mathrm{d}\tilde{\tau} = H_0 \mathrm{d}t/a$:

$$a = \sqrt{\Omega_{\mathrm{r},0}}\tilde{\tau} + \frac{\Omega_{\mathrm{m},0}}{4}\tilde{\tau}^2 + \frac{\Omega_{\Lambda,0}}{10}{\Omega_{\mathrm{r},0}}^{\frac{3}{2}}\tilde{\tau}^5 + \mathcal{O}(\tilde{\tau}^6).$$

- Define deviation from **correspondence torsion** $\varpi = Q/Q_{\mathrm{cor}}$ to give analogous series expansion for **arbitrary-ϖ_{r} solution**:

$$\begin{aligned} a =& \frac{g_{\mathrm{cor}}}{\varpi_{\mathrm{r}}}\sqrt{\Omega_{\mathrm{r},0}}\tilde{\tau} + \frac{\Omega_{\mathrm{m},0}\left(3\,{\varpi_{\mathrm{r}}}^2+1\right){g_{\mathrm{cor}}}^2}{16\,{\varpi_{\mathrm{r}}}^2}\tilde{\tau}^2 + \frac{5\,{\Omega_{\mathrm{m},0}}^2{g_{\mathrm{cor}}}^3\left({\varpi_{\mathrm{r}}}^2-1\right)}{512\,{\varpi_{\mathrm{r}}}^3}\frac{1}{\sqrt{\Omega_{\mathrm{r},0}}}\tilde{\tau}^3 \\ &+ \frac{{\Omega_{\mathrm{m},0}}^3\left(27\,{\varpi_{\mathrm{r}}}^2-121\right){g_{\mathrm{cor}}}^4\left({\varpi_{\mathrm{r}}}^2-1\right)}{49152\,{\varpi_{\mathrm{r}}}^4\Omega_{\mathrm{r},0}}\tilde{\tau}^4 \\ &+ \frac{\left(-441\,{\varpi_{\mathrm{r}}}^4{\Omega_{\mathrm{m},0}}^4 + 98304\,{\varpi_{\mathrm{r}}}^2\Omega_{\Lambda,0}\,{\Omega_{\mathrm{r},0}}^3 + 1421\,{\varpi_{\mathrm{r}}}^2{\Omega_{\mathrm{m},0}}^4 + 32768\,\Omega_{\Lambda,0}\,{\Omega_{\mathrm{r},0}}^3 - 980\,{\Omega_{\mathrm{m},0}}^4\right){g_{\mathrm{cor}}}^5}{1310720\,{\varpi_{\mathrm{r}}}^5}{\Omega_{\mathrm{r},0}}^{-\frac{3}{2}}\tilde{\tau}^5 \\ &+ \mathcal{O}(\tilde{\tau}^6), \end{aligned}$$

- Torsion no longer **constant** if $\varpi_{\mathrm{r}} \neq 1$ so has its own series:

$$\begin{aligned} \varpi =& \varpi_{\mathrm{r}} + \frac{3\,\Omega_{\mathrm{m},0}\,g_{\mathrm{cor}}\left({\varpi_{\mathrm{r}}}^2-1\right)}{16}\frac{1}{\sqrt{\Omega_{\mathrm{r},0}}}\tilde{\tau} + \frac{{\Omega_{\mathrm{m},0}}^2{g_{\mathrm{cor}}}^2\left(18\,{\varpi_{\mathrm{r}}}^2+13\right)\left({\varpi_{\mathrm{r}}}^2-1\right)}{512\,\Omega_{\mathrm{r},0}\,\varpi_{\mathrm{r}}}\tilde{\tau}^2 \\ &+ \frac{{\Omega_{\mathrm{m},0}}^3{g_{\mathrm{cor}}}^3\left(324\,{\varpi_{\mathrm{r}}}^4+279\,{\varpi_{\mathrm{r}}}^2+299\right)\left({\varpi_{\mathrm{r}}}^2-1\right)}{49152\,{\varpi_{\mathrm{r}}}^2}{\Omega_{\mathrm{r},0}}^{-\frac{3}{2}}\tilde{\tau}^3 \\ &- \frac{{g_{\mathrm{cor}}}^4\left(-1620\,{\Omega_{\mathrm{m},0}}^4{\varpi_{\mathrm{r}}}^6 - 1620\,{\varpi_{\mathrm{r}}}^4{\Omega_{\mathrm{m},0}}^4 - 1462\,{\varpi_{\mathrm{r}}}^2{\Omega_{\mathrm{m},0}}^4 + 98304\,\Omega_{\Lambda,0}\,{\Omega_{\mathrm{r},0}}^3 - 2327\,{\Omega_{\mathrm{m},0}}^4\right)\left({\varpi_{\mathrm{r}}}^2-1\right)}{1310720\,{\Omega_{\mathrm{r},0}}^2{\varpi_{\mathrm{r}}}^3}\tilde{\tau}^4 \\ &+ \mathcal{O}(\tilde{\tau}^5). \end{aligned}$$

Class ^{3}C*: Dark radiation

- We now want to package this in a form cosmologists can efficiently use, such as an **extra component model** for small ε in each dominant cosmic fluid:

$$\varpi = 1 + \varepsilon\delta\varpi + \mathcal{O}(\varepsilon^2), \quad a = \left(\frac{3w_i+1}{2}\tilde{\tau}\right)^{\frac{2}{3w_i+1}} + \varepsilon\delta a + \mathcal{O}(\varepsilon^2)$$

  - Deviation from correspondence torsion generally has **two decaying modes**:

$$\delta\varpi = \begin{cases} \left(c_1\tilde{\tau}^{-1} + c_2\right)^2 & w_i = 1/3 \\ \left(c_1\tilde{\tau}^{-\frac{3+\sqrt{3}}{2}} + c_2\tilde{\tau}^{-\frac{3-\sqrt{3}}{2}}\right)^2 & w_i = 0 \\ \left(c_1\tilde{\tau}^{\frac{3+\sqrt{3}}{2}} + c_2\tilde{\tau}^{\frac{3-\sqrt{3}}{2}}\right)^2 & w_i = -1 \end{cases}$$

  - Implying density of **extra component** in terms of scale factor a:

$$a^4\delta\rho = \begin{cases} c_3 + c_4 a^{-2} & w_i = 1/3 \\ c_3 a^{-\frac{1+\sqrt{3}}{2}} + c_4 a^{-\frac{1-\sqrt{3}}{2}} & w_i = 0 \\ c_3 a^{1+\sqrt{3}} + c_4 a^{1-\sqrt{3}} & w_i = -1 \end{cases}$$

- Approximating extra component by **slowest-decaying** mode, we can find its effective equation-of-state parameter under each dominant cosmic fluid:

$$w_{\mathrm{r,eff}} = 1/3, \quad w_{\mathrm{m,eff}} = (1 - 1/\sqrt{3})/2 \approx 0.211, \quad w_{\Lambda,\mathrm{eff}} = -1/\sqrt{3} \approx -0.577$$

- Since $w_{\mathrm{m,eff}} > w_{\mathrm{m}}$ and $w_{\Lambda,\mathrm{eff}} > w_{\Lambda}$ extra component **redshifts away** at **late times**...
- ...but since $w_{\mathrm{r,eff}} = w_{\mathrm{r}}$ extra component **co-dominant** with radiation at early times: call this **dark radiation**
- Crudest way to import effect to ΛCDM is to ignore late times:

$$\Delta N_{\mathrm{dr,eff}} = \left(\varpi_{\mathrm{r}}^{-2} - 1\right)\left(\frac{8}{7}\left(\frac{11}{4}\right)^{4/3} + N_{\nu,\mathrm{eff}}\right)$$

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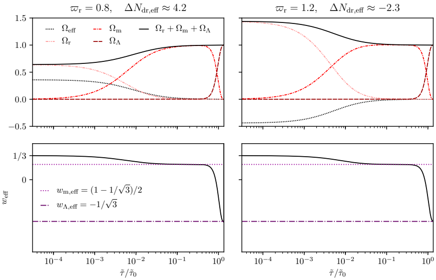
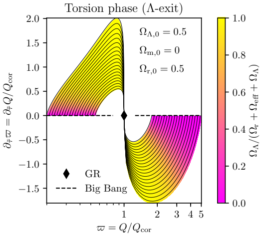
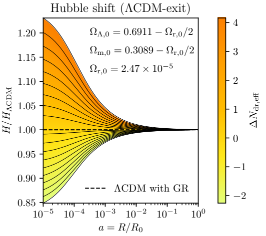
Class ^{3}C*: Dark radiation



- Physical $\Sigma\Omega_i \neq 1$ during radiation-dominated epoch
- Numerically, effective equation-of-state parameters behave as expected
- Since extra component is effective, it can have **positive** or **negative** densities...
- ...leads to **advanced** or **retarded** epoch of equality...
- ...and **enhanced** or **suppressed** early H
- Elimination of **dark radiation** at late times is reliable over a wide range of $\Delta N_{dr,eff}$
- Especially striking in pure $\Omega_r + \Omega_\Lambda$ universes: **correspondence solution** resembles an **attractor** state in ϖ phase space





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H_0 TENSION AND DARK RADIATION

General Idea

- **H_0 tension** (e.g. as much as 4.4σ 1903.07603 [astro-ph.CO]): **Low** CMB-inferred value 0310723 [astro-ph], 1807.06209 [astro-ph.CO] vs **High** locally-observed value 1908.00993 [astro-ph.GA], 1907.05922 [astro-ph.CO], 1907.04869 [astro-ph.CO].
- Aim to revise CMB-inferred H_0 upward without changing CMB characteristics, e.g. **multipole position** l_a of first peak:
$$l_\mathrm{a} = \pi D_\mathrm{A}(z_\mathrm{rec})/r_\mathrm{s}$$
- Depends on **angular diameter distance** to z_rec, and **sound horizon**:
$$D_\mathrm{A}(z_\mathrm{rec}) = (1+z_\mathrm{rec})d_\mathrm{A}(z_\mathrm{rec}) = \frac{\sin\left(\sqrt{-\Omega_{k,0}}\int_0^{z_\mathrm{rec}}\frac{H_0\mathrm{d}z}{H}\right)}{H_0\sqrt{-\Omega_{k,0}}},$$
$$r_\mathrm{s} = \int_0^{t_\mathrm{rec}}\frac{c_\mathrm{s}\mathrm{d}t}{a}$$
- **Constant** z_rec, **increase** H for $z < z_\mathrm{rec}$, **increase** H for $z_\mathrm{rec} < z$

Methods

- Many ways to increase early H, e.g. review 1801.07260 [astro-ph.CO]
- **dark radiation** is an augmentation of radiation density:
$$\Sigma_i\Omega_i = 1 \implies \Omega_\mathrm{r} = 1, \quad \Omega_i = \frac{\kappa\rho_i}{3H^2}$$
- This can be done by introducing extra **relativistic degrees of freedom** e.g. **sterile neutrinos**:
$$\Omega_{\mathrm{r},0} = \left(1+\frac{7}{8}\left(\frac{4}{11}\right)^{4/3}N_\mathrm{eff}\right)\Omega_{\gamma,0},$$
$$N_\mathrm{eff} = N_{\nu,\mathrm{eff}} + \Delta N_{\mathrm{dr,eff}}$$
- **early dark energy/varying-Λ** models 0702343 [astro-ph], 1608.01309 [astro-ph.CO]
- **decaying dark matter** 1902.10636 [astro-ph.CO],

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Part 2: Mapping to a non-canonial bi-scalar-tensor theory

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Metrical Analogues

- Of course, **metrical** theories are understood better than **gauge** theories!
- Is there some way to cast $\text{PGT}^{\text{q},+}$ as a **metrical** gravity?
- The gauge structure of PGT manifestly guarantees **second order equations** if we use the field strengths as building blocks:

$$\mathcal{R}^{ab}{}_{cd} \equiv 2h_c{}^{\mu}h_d{}^{\nu}\left(\partial_{[\mu}A^{ab}{}_{\nu]}+A^{a}{}_{e[\mu}A^{eb}{}_{\nu]}\right), \quad \mathcal{T}^{a}{}_{bc} \equiv 2h_b{}^{\mu}h_c{}^{\nu}\left(\partial_{[\mu}b^{a}{}_{\nu]}+A^{a}{}_{d[\mu}b^{d}{}_{\nu]}\right)$$

- In PGT cosmology, only the **two** 0^+ and 0^- torsional modes are dynamical:

$$\mathcal{T}^{a}{}_{bc} = (\hat{\boldsymbol{e}}_t)^d\left(\tfrac{2}{3}U\delta^a_{[c}\eta_{db]} - Q\epsilon^{a}{}_{dbc}\right)$$

The bi-galileon

- Many simple modifications of GR also guarantee **second order equations** due to the odd structure of the metrical Ricci: the $g^{-2}\partial^2 g$ terms happen not to cause a problem, though removing them through surface transformations can break covariance 1811.09844 [gr-qc].
- We need **two** torsional freedoms, so we will try a simple **bi-Galileon** Gregory Walter Horndeski (1974) (note that the multi-Galileon is **not** the most general scalar-tensor action without ghostly characteristics, but a sufficiently general start for our purposes 1901.07183 [gr-qc]):

$$U = 3(\tfrac{1}{2}\varphi + H), \quad Q = \psi,$$
$$X_{\varphi\varphi} = \tfrac{1}{2}g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi, \quad X_{\varphi\psi} = \tfrac{1}{2}g^{\mu\nu}\partial_\mu\varphi\partial_\nu\psi, \quad X_{\psi\psi} = \tfrac{1}{2}g^{\mu\nu}\partial_\mu\psi\partial_\nu\psi$$

- The **multi-Galileon action** depends on **arbitrary** G-functions of φ, ψ, $X_{\varphi\varphi}$, $X_{\varphi\psi}$ and $X_{\psi\psi}$:

$$\begin{aligned} L_{\mathrm{T}} =& G_2 + G_{3,\varphi}\Box\varphi + G_{3,\psi}\Box\psi + G_4 R + G_{5,\varphi}G^{\mu\nu}\nabla_\mu\nabla_\nu\varphi + G_{5,\psi}G^{\mu\nu}\nabla_\mu\nabla_\nu\psi \\ &+\partial_{X_{\varphi\varphi}}G_4(\ldots) + \partial_{X_{\varphi\psi}}G_4(\ldots) + \partial_{X_{\psi\psi}}G_4(\ldots) \\ &+\partial_{X_{\varphi\varphi}}G_{5,\varphi}(\ldots) + \partial_{X_{\varphi\psi}}G_{5,\varphi}(\ldots) + \partial_{X_{\psi\psi}}G_{5,\varphi}(\ldots) \\ &+\partial_{X_{\psi\psi}}G_{5,\psi}(\ldots) + \partial_{X_{\psi\psi}}G_{5,\psi}(\ldots) + \partial_{X_{\psi\psi}}G_{5,\psi}(\ldots) + L_{\mathrm{m}} \end{aligned}$$

- The **counterterms** can get a bit involved, and we need some symmetries to ward off **ghosts**:

$$\partial_{X_{\varphi\psi}}G_{3,\varphi} = 2\partial_{X_{\varphi\varphi}}G_{3,\psi}, \quad \partial_{X_{\varphi\psi}}G_{3,\psi} = 2\partial_{X_{\psi\psi}}G_{3,\varphi}, \quad \partial^2_{X_{\varphi\psi}}G_4 = 4\partial_{X_{\varphi\varphi}}\partial_{X_{\psi\psi}}G_4$$

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The general metrical analogue

- Fortunately, **no counterterms** are needed to replicate $\mathrm{PGT}^{\mathrm{q},+}$.
- Unfortunately, we do need to add a **third scalar** χ and a **neutral vector** B^{μ}:

$$L_{\mathrm{T}} = G_2 + G_4 R + [G_6^{\phi}\partial_{\mu}\phi + G_6^{\psi}\partial_{\mu}\psi]B^{\mu} + m_{\mathrm{p}}({m_{\mathrm{p}}}^2 - B_{\mu}B^{\mu})\chi + L_{\mathrm{m}}$$

- We notice that χ is a multiplier which turns B^{μ} into a **Lorentz-violating vector field** `0407149 [hep-th]`, quite a surprising thing to find buried in $\mathrm{PGT}^{\mathrm{q},+}$, and a well-known source of interesting cosmology.
- An alternative picture is obtained by integrating out both χ and B^{μ} (which are non-dynamical), to give a **non-canonical** bi-scalar-tensor theory:

$$\begin{aligned} L_{\mathrm{T}} = {} & [\tfrac{1}{2}{m_{\mathrm{p}}}^2\upsilon_2 + \sigma_3\phi^2 + \tfrac{1}{2}(\sigma_3 - \sigma_2)\psi^2]R + 12\sigma_3 X^{\phi\phi} + 6(\sigma_3 - \sigma_2)X^{\psi\psi} + \sqrt{|J_{\mu}J^{\mu}|} \\ & + \tfrac{3}{4}{m_{\mathrm{p}}}^2(\alpha_0 + \upsilon_2)\phi^2 - \tfrac{3}{4}{m_{\mathrm{p}}}^2(\alpha_0 - 4\upsilon_1)\psi^2 + \tfrac{3}{2}\sigma_3\phi^4 - 3\sigma_2\phi^2\psi^2 + \tfrac{3}{2}\sigma_3\psi^4 + L_{\mathrm{m}} \end{aligned}$$

- Here, $J_{\mu} \equiv 4\sigma_1\psi^3\partial_{\mu}(\phi/\psi) - {m_{\mathrm{p}}}^2(\alpha_0 + \upsilon_2)\partial_{\mu}\phi$, so the **non-canonical/Lorentz-violating** part vanishes if $\alpha_0 + \upsilon_2 = \sigma_1 = 0$.
- There are also **canonical kinetic terms**, **mass terms** and **quartic potentials**: this clearly accounts for **hybrid inflation** found by other authors `1904.03545 [gr-qc]`, `1906.04340 [gr-qc]`.
- While $\mathrm{PGT}^{\mathrm{q},+}$ is **quadratic**, the **metrical analogue** contains only the **Einstein–Hilbert** term.
- Note the **metrical analogue** is naturally in the **Jordan frame**.

Metrical analogues of well-known theories

- What about well-known examples of $\text{PGT}^{\text{q},+}$?
- The **teleparallel** theory $\mathbb{T} \equiv \frac{1}{4}\mathcal{T}_{abc}\mathcal{T}^{abc} + \frac{1}{2}\mathcal{T}_{abc}\mathcal{T}^{bac} - \mathcal{T}_a\mathcal{T}^a$ is dynamically equivalent to **GR**:

$$\frac{1}{2}{m_{\text{p}}}^2\beta\mathbb{T} \mapsto -\frac{1}{2}{m_{\text{p}}}^2\beta R + {m_{\text{p}}}^2\beta\left[\sqrt{2|X^{\phi\phi}|} - \frac{3}{4}\phi^2 + \frac{3}{4}\psi^2\right]$$
$$\approx -\frac{1}{2}{m_{\text{p}}}^2\beta R$$

- Note ψ and ϕ vanish on-shell because Weitzenböck spacetime is curvature-free.
- Thus, the **metrical analogue** of **teleparallel** theory is the **Einstein–Hilbert** action.
- The **ECKS** theory is also dynamically equivalent to **GR**:

$$-\frac{1}{2}{m_{\text{p}}}^2\alpha_0\mathcal{R} \mapsto -{m_{\text{p}}}^2\alpha_0\left[\sqrt{2|X^{\phi\phi}|} - \frac{3}{4}\phi^2 + \frac{3}{4}\psi^2\right]$$
$$\approx -{m_{\text{p}}}^2\alpha_0\left[\sqrt{2|X^{\phi\phi}|} - \frac{3}{4}\phi^2\right]$$

- The **Einstein-Hilbert** action is eliminated from the **metrical analogue** completely!
- This is not such a disaster: the remaining **non-canonical** term is a **Cuscuton** field 0702002 [astro-ph]. With a quadratic potential, the **Cuscuton** reproduces the Friedmann equations all by itself.

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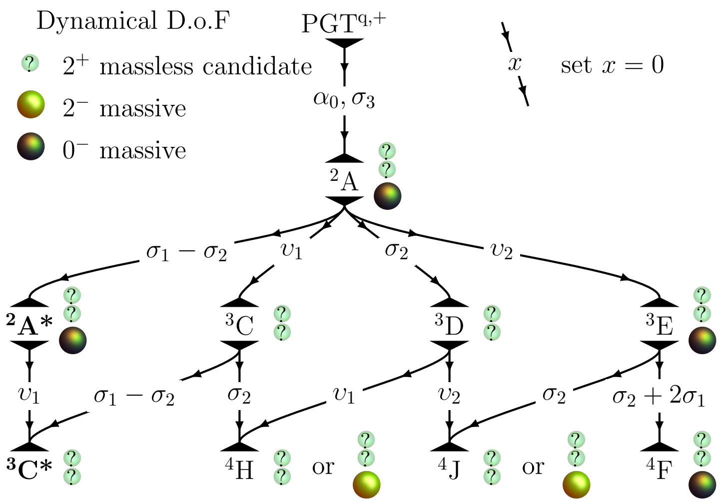
Metrical analogues of novel theories



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Moving into the Einstein frame

- Impose constraints for Class ^{2}A*: $\sigma_3 = \alpha_0 = 0$ (from 1910.14197 [gr-qc]) and $\sigma_2 = \sigma_1$ and $\upsilon_2 = -4/3$ (from 2003.02690 [gr-qc]).
- Transition from **physical Jordan frame** to **Einstein frame** and reparameterise with new fields $\zeta(\phi,\psi)$ and $\xi(\psi)$:

$$L_{\mathrm{T}} = -\tfrac{1}{2}{m_{\mathrm{p}}}^2 R + X^{\xi\xi} - V(\xi) + {m_{\mathrm{p}}}^2\omega(\xi)^3\sqrt{|X^{\zeta\zeta}|} + \tfrac{3}{4}{m_{\mathrm{p}}}^2\omega(\xi)^4\zeta^2 + L_{\mathrm{m}}$$

- This is **GR** augmented with a **canonical scalar** ξ, coupled to a **quadratic Cuscuton** ζ.
- The **Cuscuton** coupling $\omega(\xi) \equiv \sqrt{\left|3\cosh\left(\sqrt{2/3}\,\xi/m_{\mathrm{p}}\right) - 5\right|}$ conveniently measures the degree to which the **physical Jordan frame** has drifted away from the **Einstein frame**:

$$g_{\mu\nu} \mapsto \left(1 + \tfrac{1}{8}\omega^2\right) g_{\mu\nu}$$

- The **canonical scalar** carries a potential:

$$V(\xi) \equiv \tfrac{\upsilon_1}{\sigma_1}{m_{\mathrm{p}}}^4\left(1 + \tfrac{1}{8}\omega^2\right)\left(1 + \tfrac{1}{2}\omega^2\right)$$

- But the **unitarity** conditions demand $\sigma_1 < 0$ (no **ghost**) and $\upsilon_1 < 0$ (no **tachyon**), so $V(\xi) > 0$ will look like **dark energy**!

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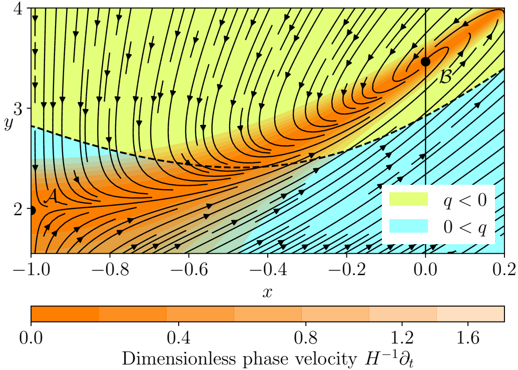
Inflation from negative vacuum energy

- In the **physical Jordan frame** pick a **negative bare** cosmological constant $\Lambda_\mathrm{b} < 0$, so $L_\mathrm{m} = -{m_\mathrm{p}}^2 \Lambda_\mathrm{b}$.
- In the **Einstein frame** we have $L_\mathrm{m} = -{m_\mathrm{p}}^2 \Lambda_\mathrm{b}\left(1 + \frac{1}{8}\omega^2\right)^2$ and soltion $\xi, \zeta, H \to$ const.
- In the **physical Jordan frame** this is a **de Sitter expansion** $H^2 = \Lambda/3$, where **effective**:

$$\Lambda = \frac{\upsilon_1}{2\sigma_1} {m_\mathrm{p}}^2$$

- Recalling **unitarity** $\sigma_1, \upsilon_1 < 0$: the **positive effective** $\Lambda > 0$ is **screened** from $\Lambda_\mathrm{b} < 0$.



- In the **Einstein frame** treat ξ as **canonical inflaton** with Hamiltonian coordinates:

$$x^2 \equiv \frac{{m_\mathrm{p}}^2 (\partial_t \xi)^2}{6H^2}, \quad y^2 \equiv \frac{V_\mathrm{T}}{3{m_\mathrm{p}}^2 H^2}$$

- Can apply usual **dynamical systems analysis** on 2D phase space.
- The **de Sitter expansion** is then an **attractor** state.

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Emergent dark energy

- Recall **Cuscuton** coupling $\omega(\xi)$ measures difference between **physical Jordan** and **Einstein frames** $g_{\mu\nu} \mapsto \left(1+\frac{1}{8}\omega^2\right)g_{\mu\nu}$.
- When $\omega \to 0$ these frames **coincide** and we again have $\xi, \zeta \to$ const:

$$L_{\mathrm{T}} = -\tfrac{1}{2}{m_{\mathrm{p}}}^2 R + X^{\xi\xi} - V(\xi) + {m_{\mathrm{p}}}^2\omega(\xi)^3\sqrt{|X^{\zeta\zeta}|} + \tfrac{3}{4}{m_{\mathrm{p}}}^2\omega(\xi)^4\zeta^2 + L_{\mathrm{m}}$$

- This is **GR** augmented with a **positive frozen potential**, which just adds to the **bare** cosmological constant. The **effective** cosmological constant is now:

$$\Lambda = \Lambda_{\mathrm{b}} + \frac{\upsilon_1}{\sigma_1}{m_{\mathrm{p}}}^2$$

- This is also an **attractor** solution. For matter in L_{m} with E.o.S parameter w, deviation from this solution looks like **GR** plus **extra component** ρ_{eff} with E.o.S parameter:

$$w_{\mathrm{eff}}(w) \equiv \tfrac{1}{2}(w+1) - \tfrac{1}{6}\sqrt{9w^2+3}, \quad -1 \le w \le \tfrac{1}{3}$$

- This is the **dark radiation** from earlier, since $w_{\mathrm{eff}}(1/3) = 1/3$.

Part 3: Nonlinear Hamiltonian and beyond

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Moving parts

- **Unitarity** was only confirmed at **linear** level. We can check nonlinear theory by analysing the constraint chain of the $\mathrm{PGT}^{q,+}$ Hamiltonian M. Blagojević et al. (1983), Hsin Chen et al. (1998), 9902032 [gr-qc], 0112030 [gr-qc], 1804.05556 [gr-qc].
- $\mathrm{PGT}^{q,+}$ contains 40 field D.o.F
$$16[h_a^{\ \mu}] + 24[A^{ab}_{\ \ \mu}] = 40$$
- Gauge fixing the Poincaré symmetry leaves 20 potentially propagating D.o.F, these are the 2^+ **massless graviton** and $0^\pm$, $1^\pm$, $2^\pm$ **massive rotons**:
$$40 - 2[\text{gauge}] \times 10[\mathbb{R}^{1,3} \rtimes \mathrm{SO}^+(1,3)] = 20$$
- At the level of the fields however, there are $40 - 10[\mathbb{R}^{1,3} \rtimes \mathrm{SO}^+(1,3)] = 30$ D.o.F which are accounted for by O(3) irreps under the $3+1$ decomposition:
$$30 = 1[0^+] + 3[1^+] + 3[1^-] + 5[2^+] + 1[0^+] + 1[0^-] + 3[1^+] + 3[1^-] + 5[2^+] + 5[2^-]$$
- We label these basic moving parts φ, $\hat{\varphi}_{\underline{kl}}$, $\varphi_{\perp\underline{k}}$, $\tilde{\varphi}_{\underline{kl}}$, $\varphi_{\perp}$, ${}^{\mathrm{P}}\varphi$, $\hat{\varphi}_{\perp\underline{kl}}$, $\vec{\varphi}_{\underline{k}}$, $\tilde{\varphi}_{\perp\underline{kl}}$, ${}^{\mathrm{T}}\varphi_{\underline{klo}}$, and note that they can be expressed as functions of **fields**, **field momenta** and **spatial field gradients** (i.e. **canonical variables**), e.g.:
$$\pi_i^{\ \mu} \equiv \frac{\partial bL}{\partial \partial_0 b^i_{\ \mu}}, \quad \pi_{ij}^{\ \ \mu} \equiv \frac{\partial bL}{\partial \partial_0 A^{ij}_{\ \ \mu}}, \quad {}^{\mathrm{P}}\varphi \equiv J^{-1}\,{}^{\mathrm{P}}\hat{\pi} + m_{\mathrm{p}}^{\ 2}\left(\hat{\alpha}_2 - \hat{\alpha}_3\right){}^{\mathrm{P}}\mathcal{R}_{\circ\perp}$$

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Dirac–Bergmann algorithm

- Some of the φ, $\hat{\varphi}_{\overline{kl}}$, $\varphi_{\perp\overline{k}}$, $\tilde{\varphi}_{\overline{kl}}$, $\varphi_{\perp}$, ${}^{\mathrm{P}}\varphi$, $\hat{\varphi}_{\perp\overline{kl}}$, $\vec{\varphi}_{\overline{k}}$, $\tilde{\varphi}_{\perp\overline{kl}}$, ${}^{\mathrm{T}}\varphi_{\overline{klo}}$ may become **primary constraints** when we impose relations among the $\{\alpha_i, \beta_i\}$.

$$L_{\mathrm{T}} = \sum_{I=1}^{3} m_{\mathrm{p}}{}^2 \hat{\beta}_I \mathcal{T}^i{}_{jk}\, {}^I\mathcal{P}_i{}^{jk}{}_l{}^{nm} \mathcal{T}^l{}_{nm} + \sum_{I=1}^{6} \hat{\alpha}_I \mathcal{R}^{ij}{}_{kl}\, {}^I\mathcal{P}_{ij}{}^{kl}{}_{nm}{}^{pq} \mathcal{R}^{nm}{}_{pq} + L_{\mathrm{m}}$$

- Analysis of the **Poisson brackets** between **primary constraints** leads to identification of **multipliers** and possible **secondary constraints** etc.
- Repeating the process, all constraints are identified. The **particle spectrum** reflects how the constraints encroach on the 20 propagating D.o.F.
- The constraint **classes** provide information about **emergent gauge symmetries**.
- By inspecting the **canonical Hamiltonian**, we can idenify any unconstrained **ghostly** or **tachyonic** D.o.F:

$$\begin{aligned}
\mathcal{H}_{\perp} \equiv{}& \hat{\pi}_i{}^{\overline{k}} \mathcal{T}^i{}_{\perp\overline{k}} + \frac{1}{2}\hat{\pi}_{ij}{}^{\overline{k}} \mathcal{R}^{ij}{}_{\perp\overline{k}} - JL - n^k D_\alpha \pi_k{}^\alpha \\
={}& \sum_{I=1}^{3} m_{\mathrm{p}}{}^2 J \hat{\beta}_I \left[4\mathcal{T}^i{}_{\perp\overline{k}}\, {}^I\mathcal{P}_i{}^{\perp\overline{k}}{}_j{}^{\perp\overline{l}} \mathcal{T}^j{}_{\perp\overline{l}} - \mathcal{T}^i{}_{\overline{mk}}\, {}^I\mathcal{P}_i{}^{\overline{mk}}{}_j{}^{\overline{nl}} \mathcal{T}^j{}_{\overline{nl}} \right] \\
&+ \sum_{I=1}^{6} J \hat{\alpha}_I \left[4\mathcal{R}^{ip}{}_{\perp\overline{k}}\, {}^I\mathcal{P}_{ip}{}^{\perp\overline{k}}{}_{jq}{}^{\perp\overline{l}} \mathcal{R}^{jq}{}_{\perp\overline{l}} - \mathcal{R}^{ip}{}_{\overline{mk}}\, {}^I\mathcal{P}_{ip}{}^{\overline{mk}}{}_{jq}{}^{\overline{nl}} \mathcal{R}^{jq}{}_{\overline{nl}} \right] - n^k D_\alpha \pi_k{}^\alpha
\end{aligned}$$

Primary Poisson matrices

- The **primary Poisson matrix** of brackets between all **primary constraints** provides a good (but non conclusive) indicator of the health of the theory. It is even suggested Hsin Chen et al. (1998), 9902032 [gr-qc], 0112030 [gr-qc] that the pseudodeterminant of this matrix has something to say about **tachyonic** content.
- Only a handful of the novel theories in 1812.02675 [gr-qc], 1910.14197 [gr-qc] have been considered so far, **none** of them are Class ^{3}C or Class ^{2}A: these must be tested!

- Propagates a 0^- **massive roton**

	$\widetilde{\varphi}_{\overline{kl}}$	$\varphi_{\perp}$	$\widetilde{\varphi}_{\perp\overline{kl}}$	${}^{\mathrm{T}}\varphi_{\overline{klo}}$	
$\widetilde{\varphi}_{\overline{kl}}$	$\hat{\pi}$	$\cdot$	$\hat{\pi}$	$\hat{\pi}$	5
$\varphi_{\perp}$	$\cdot$	$\cdot$	$\cdot$	$\cdot$	1
$\widetilde{\varphi}_{\perp\overline{kl}}$	$\hat{\pi}$	$\cdot$	$\cdot$	$\cdot$	5
${}^{\mathrm{T}}\varphi_{\overline{klo}}$	$\hat{\pi}$	$\cdot$	$\cdot$	$\cdot$	5
	5	1	5	5	

Case 25

- Propagates a 0^- **massive roton**

	$\widetilde{\varphi}_{\overline{kl}}$	$\varphi_{\perp}$	$\widehat{\varphi}_{\perp\overline{kl}}$	$\overrightarrow{\varphi}_{\overline{k}}$	$\widetilde{\varphi}_{\perp\overline{kl}}$	${}^{\mathrm{T}}\varphi_{\overline{klo}}$	
$\widetilde{\varphi}_{\overline{kl}}$	$\hat{\pi}$	·	·	·	·	$\hat{\pi}$	5
$\varphi_{\perp}$	·	·	·	·	·	·	1
$\widehat{\varphi}_{\perp\overline{kl}}$	·	·	·	·	·	·	3
$\overrightarrow{\varphi}_{\overline{k}}$	·	·	·	·	·	·	3
$\widetilde{\varphi}_{\perp\overline{kl}}$	·	·	·	·	·	·	5
${}^{\mathrm{T}}\varphi_{\overline{klo}}$	$\hat{\pi}$	·	·	·	·	·	5
	5	1	3	3	5	5	

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Case 28

- Propagates a 0^- **massive roton**

	φ	$\varphi_{\perp\overline{k}}$	$\widetilde{\varphi}_{\overline{kl}}$	$\varphi_{\perp}$	$\widetilde{\varphi}_{\perp\overline{kl}}$	${}^{\mathrm{T}}\varphi_{\overline{klo}}$	
φ	·	·	·	·	·	·	1
$\varphi_{\perp\overline{k}}$	·	$\hat{\pi}$	·	$\hat{\pi}$	$\hat{\pi}$	$\hat{\pi}$	3
$\widetilde{\varphi}_{\overline{kl}}$	·	·	$\hat{\pi}$	·	$\hat{\pi}$	$\hat{\pi}$	5
$\varphi_{\perp}$	·	$\hat{\pi}$	·	·	·	·	1
$\widetilde{\varphi}_{\perp\overline{kl}}$	·	$\hat{\pi}$	$\hat{\pi}$	·	·	·	5
${}^{\mathrm{T}}\varphi_{\overline{klo}}$	·	$\hat{\pi}$	$\hat{\pi}$	·	·	·	5
	1	3	5	1	5	5	

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Case 26

- Propagates a 0^- **massive roton**

	φ	$\varphi_{\perp\overline{k}}$	$\widetilde{\varphi}_{\overline{kl}}$	$\varphi_{\perp}$	$\widehat{\varphi}_{\perp\overline{kl}}$	$\overrightarrow{\varphi}_{\overline{k}}$	$\widetilde{\varphi}_{\perp\overline{kl}}$	${}^{\mathrm{T}}\varphi_{\overline{klo}}$	
φ	·	·	·	·	·	·	·	·	1
$\varphi_{\perp\overline{k}}$	·	$\hat{\pi}$	·	·	$\hat{\pi}$	·	·	·	3
$\widetilde{\varphi}_{\overline{kl}}$	·	·	$\hat{\pi}$	·	·	·	·	$\hat{\pi}$	5
$\varphi_{\perp}$	·	·	·	·	·	·	·	·	1
$\widehat{\varphi}_{\perp\overline{kl}}$	·	$\hat{\pi}$	·	·	·	·	·	·	3
$\overrightarrow{\varphi}_{\overline{k}}$	·	·	·	·	·	·	·	·	3
$\widetilde{\varphi}_{\perp\overline{kl}}$	·	·	·	·	·	·	·	·	5
${}^{\mathrm{T}}\varphi_{\overline{klo}}$	·	·	$\hat{\pi}$	·	·	·	·	·	5
	1	3	5	1	3	3	5	5	

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Case 20

- Propagates a 0^{-} **massive roton**

	$\varphi_{\perp}$	$\hat{\varphi}_{\perp\overline{kl}}$	$\vec{\varphi}_{\overline{k}}$	$\tilde{\varphi}_{\perp\overline{kl}}$	${}^{\mathrm{T}}\varphi_{\overline{klo}}$	
$\varphi_{\perp}$	·	·	·	·	·	1
$\hat{\varphi}_{\perp\overline{kl}}$	·	·	·	·	·	3
$\vec{\varphi}_{\overline{k}}$	·	·	·	·	·	3
$\tilde{\varphi}_{\perp\overline{kl}}$	·	·	·	·	·	5
${}^{\mathrm{T}}\varphi_{\overline{klo}}$	·	·	·	·	·	5
	1	3	3	5	5	

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Case 32

- Propagates a 0^- **massive roton**

	φ	$\varphi_{\perp}$	$\hat{\varphi}_{\perp\overline{kl}}$	$\vec{\varphi}_{\overline{k}}$	$\tilde{\varphi}_{\perp\overline{kl}}$	${}^{\mathrm{T}}\varphi_{\overline{klo}}$	
φ	·	·	·	·	·	·	1
$\varphi_{\perp}$	·	·	·	·	·	·	1
$\hat{\varphi}_{\perp\overline{kl}}$	·	·	·	·	·	·	3
$\vec{\varphi}_{\overline{k}}$	·	·	·	·	·	·	3
$\tilde{\varphi}_{\perp\overline{kl}}$	·	·	·	·	·	·	5
${}^{\mathrm{T}}\varphi_{\overline{klo}}$	·	·	·	·	·	·	5
	1	1	3	3	5	5	

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Case 3

- Propagates a 2^+ **massless roton** and 0^- **massive roton**

	φ	$\hat{\varphi}_{\overline{kl}}$	$\varphi_{\perp}$	$\tilde{\varphi}_{\perp\overline{kl}}$	${}^{\mathrm{T}}\varphi_{\overline{klo}}$	
φ	$\cdot$	$\hat{\pi}$	$\cdot$	$\cdot$	$\cdot$	1
$\hat{\varphi}_{\overline{kl}}$	$\hat{\pi}$	$\cdot$	$\hat{\pi}$	$\hat{\pi}$	$\hat{\pi}$	3
$\varphi_{\perp}$	$\cdot$	$\hat{\pi}$	$\cdot$	$\cdot$	$\cdot$	1
$\tilde{\varphi}_{\perp\overline{kl}}$	$\cdot$	$\hat{\pi}$	$\cdot$	$\cdot$	$\cdot$	5
${}^{\mathrm{T}}\varphi_{\overline{klo}}$	$\cdot$	$\hat{\pi}$	$\cdot$	$\cdot$	$\cdot$	5
	1	3	1	5	5	

Case 17

- Propagates a 2^+ **massless roton** and 0^- **massive roton**

	φ	$\hat{\varphi}_{\overline{kl}}$	$\varphi_\perp$	${}^{\mathrm{P}}\varphi$	$\tilde{\varphi}_{\perp\overline{kl}}$	${}^{\mathrm{T}}\varphi_{\overline{klo}}$	
φ	$\cdot$	$\hat{\pi}$	$\cdot$	$\cdot$	$\cdot$	$\cdot$	1
$\hat{\varphi}_{\overline{kl}}$	$\hat{\pi}$	$\cdot$	$\hat{\pi}$	$\hat{\pi}$	$\hat{\pi}$	$\hat{\pi}$	3
$\varphi_\perp$	$\cdot$	$\hat{\pi}$	$\cdot$	$\cdot$	$\cdot$	$\cdot$	1
${}^{\mathrm{P}}\varphi$	$\cdot$	$\hat{\pi}$	$\cdot$	$\cdot$	$\cdot$	$\cdot$	1
$\tilde{\varphi}_{\perp\overline{kl}}$	$\cdot$	$\hat{\pi}$	$\cdot$	$\cdot$	$\cdot$	$\cdot$	5
${}^{\mathrm{T}}\varphi_{\overline{klo}}$	$\cdot$	$\hat{\pi}$	$\cdot$	$\cdot$	$\cdot$	$\cdot$	5
	1	3	1	1	5	5	

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Summary

- Many **novel, purely quadratic torsion theories**, which appear **unitary** and **power-counting renormalisable**, were discovered 1812.02675 [gr-qc], 1910.14197 [gr-qc].
- Their **IR cosmology** was systematically surveyed. The 'promising' theories are k**-screened**, but **replicate GR** up to optional **dark radiation** component, with possible application to H_0 **tension** 2003.02690 [gr-qc].
- General **quadratic torsion theory** was mapped to a **non-canonical bi-scalar-tensor**, the IR phenomenology is thus accessible. Novel motivation for the **Cuscuton** field $\sqrt{|X^{\phi\phi}|}$ was found. The 'promising' theories account for **dark energy** even with external $\Lambda_{\mathrm{b}} \leq 0$ 2006.03581 [gr-qc].
- **Nonlinear Hamiltonian analysis** is ongoing.
- Future **UV** work may add rigour to **renormalisibility**. Plenty still to do in **IR**, such as **inflation**, and **perturbations**.

Thank you for listening, questions welcome!

(P.S. Currently seeking postdoc!)

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