Dark energy instabilities induced by gravitational waves

Vicharit Yingcharoenrat

The International School for Advanced Studies (SISSA)

P. Creminelli, G. Tambalo, F. Vernizzi and VY, JCAP 05 (2020) 002

August 20th, 2020

Introduction

• Universe undergoes accelerating expansion $\implies$ Many models of dark energy (DE) e.g. Quintessence, $\mathcal{P}(X)$, (beyond) Horndeski, etc.

• Gravitational wave (GW) observations $\implies$ New test of GR and modified gravity theories

• Use GW propagation (LIGO/Virgo) to constrain those DE models

Dark enery models: Scalar-tensor theories

- One extra scalar field:

$$\mathcal{L} = R - \frac{1}{2}X - V(\phi) \qquad \text{Quintessence}$$

$$\mathcal{L} = f(\phi)R - \frac{1}{2}X - V(\phi) \qquad \text{Brans-Dicke}$$

$$\mathcal{L} = R - \mathcal{P}(\phi, X) \qquad \text{k-essence}$$

$X = g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi$

Scalar fluctuation: $\phi = \phi_0(t) + \pi(t, \boldsymbol{x})$ leads to a sound speed c_s

$$X^2 \supset \phi_0^2(t)\dot{\pi}^2 \quad \Longrightarrow \quad \mathcal{L}_\pi \sim \dot{\pi}^2 - c_s^2(\partial_i\pi)^2$$

- Most general scalar-tensor theories with 2nd order EoM: (Beyond) Horndeski

$$
\begin{aligned}
\mathcal{L}_2 =& G_2(\phi, X) \\
\mathcal{L}_3 =& G_3(\phi, X)\Box\phi \\
\mathcal{L}_4 =& G_4(\phi, X)R - 2G_{4,X}(\phi, X)[(\Box\phi)^2 - \phi_{\mu\nu}\phi^{\mu\nu}] \\
&- F_4(X, \phi)\epsilon^{\mu\nu\rho}{}_{\sigma}\epsilon^{\mu'\nu'\rho'\sigma}\phi_\mu\phi_{\mu'}\phi_{\nu\nu'}\phi_{\rho\rho'} \\
\mathcal{L}_5 =& G_5(\phi, X)G_{\mu\nu}\phi^{\mu\nu} + \frac{1}{3}G_{5,X}(\phi, X)[(\Box\phi)^3 - 3(\Box\phi)\phi_{\mu\nu}\phi^{\mu\nu} + 2\phi_{\mu\nu}\phi^{\sigma\mu}\phi^{\nu}{}_{\sigma}] \\
&- F_5(\phi, X)\epsilon^{\mu\nu\rho\sigma}\epsilon^{\mu'\nu'\rho'\sigma'}\phi_\mu\phi_{\mu'}\phi_{\nu\nu'}\phi_{\rho\rho'}\phi_{\sigma\sigma'}
\end{aligned}
$$

$$\phi_\mu \equiv \nabla_\mu\phi$$

Horndeski 74, Deffayet et al. 11,
Zumalacárregui and García-Bellido 14, Gleyzes et al. 14

Dark enery models

• The cosmological background $\phi_0(t)$ spontaneously breaks Lorentz invariance

• Interesting phenomena for tensor perturbation γ_{ij} from second derivatives

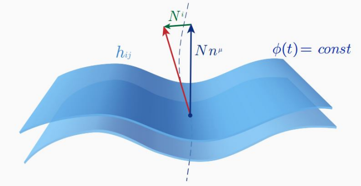
For example

$$(\nabla_\mu \nabla_\nu \phi)^2 \supset \dot{\phi}_0^2 \dot{\gamma}_{ij}^2$$

$$\mathcal{L}_\gamma \sim \dot{\gamma}_{ij}^2 - c_T^2 (\partial_l \gamma_{ij})^2$$

EFT of Dark Energy

- Efficient way to study a perturbation around fixed background
- Spontaneously break time diffeomorphism



$$ds^2 = -N^2 dt^2 + h_{ij}(N^i dt + dx^i)(N^j dt + dx^j)$$

$$S = \int d^4x \sqrt{-g}\; L[t; N, K^i_j, {}^{(3)}R, \ldots] \qquad g^{00} = -N^{-2}$$

- The action contains all possible invariances under 3d diffs

Cheung et al. 08,
Gubitosi et al. 12, and many others

EFT of Dark Energy

$$
\begin{aligned}
S^{\mathrm{EFT}} &= \int d^4x\sqrt{-g}\left[\frac{M_*^2}{2}f(t)^{(4)}R - \Lambda(t) - c(t)g^{00}\right. \\
&+ \frac{m_2^2(t)}{2}(\delta g^{00})^2 - \frac{m_3^3(t)}{2}\delta K\delta g^{00} - m_4^2(t)\delta\mathcal{K}_2 + \frac{\tilde{m}_4^2(t)}{2}\delta g^{00(3)}R \\
&- \left.\frac{m_5^2(t)}{2}\delta g^{00}\delta\mathcal{K}_2 - \frac{m_6(t)}{3}\delta\mathcal{K}_3 - \tilde{m}_6(t)\delta g^{00}\delta\mathcal{G}_2 - \frac{m_7(t)}{3}\delta g^{00}\delta\mathcal{K}_3\right]
\end{aligned}
$$

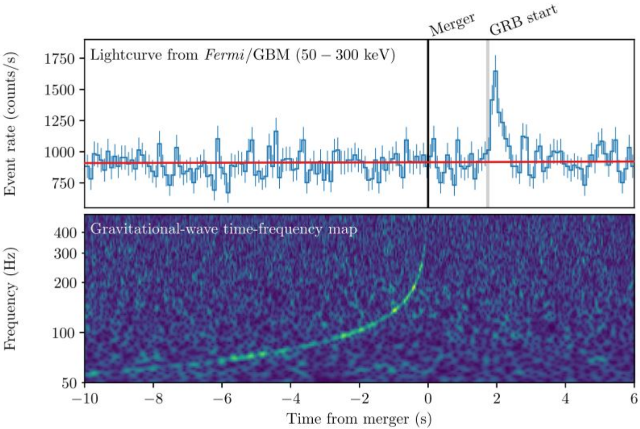
This term changes the speed of GWs

$$\delta\mathcal{K}_2 = \delta K^2 - \delta K^\mu_\nu\delta K^\nu_\mu \supset \dot{\gamma}_{ij}^2, \qquad \delta K^\mu_\nu = K^\mu_\nu - H\delta^\mu_\nu$$

$$\delta\mathcal{G}_2 = \delta K^{\mu(3)}_\nu R^\nu_\mu - \delta K^{(3)}R/2$$

$$\delta\mathcal{K}_3 = \delta K^3 - 3\delta K\delta K^\mu_\nu\delta K^\nu_\mu + 2\delta K^\nu_\mu\delta K^\mu_\rho\delta K^\rho_\nu$$

GW170817 = GRB170817A



$$|c_T^2 - 1| \lesssim 10^{-15}$$

LIGO/Virgo + Fermi/GBM + INTEGRAL 17

EFT of DE after GW170817 & GRB170817A

- The speed of GWs can be expressed as

$$c_T^2 = 1 - \frac{2m_4^2}{M_*^2 f + 2m_4^2}$$

- $c_T^2 = 1 \Rightarrow m_4^2 = 0$

EFT of DE after GW170817 & GRB170817A

- The speed of GWs can be expressed as

$$c_T^2 = 1 - \frac{2m_4^2}{M_*^2 f + 2m_4^2}$$

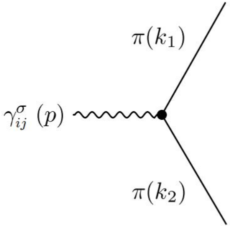
- $c_T^2 = 1 \Rightarrow m_4^2 = 0$
- The EFT action becomes

$$\mathcal{L}^{\rm EFT}_{c_T=1} = \frac{M_{\rm Pl}^2}{2} f(t)^{(4)}R - \Lambda(t) - c(t)g^{00} + \frac{m_2^2(t)}{2}(\delta g^{00})^2 - \frac{m_3^3(t)}{2}\delta K \delta g^{00} + \frac{\tilde{m}_4^2(t)}{2}\delta g^{00}\left({}^{(3)}R - \delta\mathcal{K}_2\right)$$

Creminelli and Vernizzi 17,
Ezquiaga and Zumalacárregui 17,
Baker et al. 17, Sakstein and Jain 17

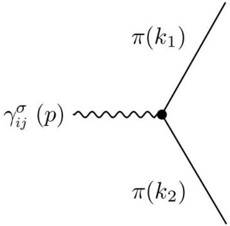
Pertubative decay of GWs due to $\tilde{m}_4^2$-term

- Spontaneous breaking of Lorentz allows the decay
- $\tilde{m}_4^2$: $\delta g^{00}({}^{(3)}R - \delta\mathcal{K}_2) \Leftrightarrow$ Beyond Horndeski ($F_4 \& F_5$)



Pertubative decay of GWs due to $\tilde{m}_4^2$-term

- Spontaneous breaking of Lorentz allows the decay
- $\tilde{m}_4^2$: $\delta g^{00}({}^{(3)}R - \delta\mathcal{K}_2) \Leftrightarrow$ Beyond Horndeski (F_4&F_5)



- The interaction term:

$$S_{\gamma\pi\pi} = \frac{1}{\Lambda_\star^3}\int d^4x\,\ddot{\gamma}_{ij}\partial_i\pi\partial_j\pi \quad , \quad \Lambda_\star^3 \simeq \sqrt{2}\frac{\alpha}{\alpha_{\rm H}}\Lambda_3^3$$

$\alpha_H = 2\tilde{m}_4^2/M_{\rm Pl}^2$, $\Lambda_3 = (M_{\rm Pl}H_0^2)^{1/3}$

- The perturbative decay rate: $\Gamma_{\gamma\to\pi\pi} \simeq \left(\frac{\alpha_{\rm H}}{\Lambda_3^3}\right)^2 \frac{\omega^7(1-c_s^2)^2}{480\pi c_s^7}$

Creminelli et al. 18

Constraint from no pert. decay

- At LIGO/Virgo, take $\omega \sim \Lambda_3,\ \Lambda_3 \sim 10^{-13}\ \mathrm{eV}$
- Compare the decay rate with the cosmological distances $\sim H_0^{-1}$

$$\frac{\Gamma_{\gamma\to\pi\pi}}{H_0} \sim 10^{20}\alpha_{\mathrm{H}}^2 \frac{(1-c_s^2)^2}{480\pi c_s^7} \lesssim 1$$

$H_0 \sim 10^{-33}$ eV

$\alpha_{\mathrm{H}} \lesssim 10^{-10} \implies$ beyond Horndeski is highly constrained

Constraint from no pert. decay

- At LIGO/Virgo, take $\omega \sim \Lambda_3,\ \Lambda_3 \sim 10^{-13}\ \mathrm{eV}$
- Compare the decay rate with the cosmological distances $\sim H_0^{-1}$

$$\frac{\Gamma_{\gamma\to\pi\pi}}{H_0} \sim 10^{20}\alpha_{\mathrm{H}}^2 \frac{(1-c_s^2)^2}{480\pi c_s^7} \lesssim 1$$

$H_0 \sim 10^{-33}$ eV

$\alpha_{\mathrm{H}} \lesssim 10^{-10} \implies$ beyond Horndeski is highly constrained

- Large occupation number of GWs $\implies$ non-perturbative effect, resonant π-production ?

EFT of DE with constraints

Recap most general EFT action with $c_T^2 = 1$: $S = S_0 + S_{m_3} + S_{\tilde{m}_4}$

$$S_0 = \int \mathrm{d}^4x\sqrt{-g}\left[\frac{M_{\rm Pl}^2}{2}\,{}^{(4)}R - \lambda(t) - c(t)g^{00} + \frac{m_2^4(t)}{2}(\delta g^{00})^2\right]$$

$$S_{m_3} = -\int \mathrm{d}^4x\sqrt{-g}\,\frac{m_3^3(t)}{2}\delta K \delta g^{00}$$ Cubic Horndeski

$$S_{\tilde{m}_4} = \int \mathrm{d}^4x\sqrt{-g}\,\frac{\tilde{m}_4^2(t)}{2}\delta g^{00}\left({}^{(3)}R + \delta K_\mu^\nu \delta K_\nu^\mu - \delta K^2\right)$$

Quartic beyond Horndeski

$\delta g^{00} = 1 + g^{00}, \delta K_\nu^\mu = K_\nu^\mu - H\delta_\nu^\mu$

Classification of instabilities

- $\mathcal{L}_0 + \mathcal{L}_{m_3} = \frac{1}{2}\dot{\gamma}_{ij}^2 - \frac{1}{2}(\partial_k \gamma_{ij})^2 + \frac{1}{2}\dot{\pi}^2 - \frac{c_s^2}{2}(\partial_i \pi)^2 + \frac{1}{\Lambda^2}\dot{\gamma}_{ij}\partial_i \pi \partial_j \pi$
- Treat GW as a classical background: $\gamma_{ij} = M_{\rm Pl} h_0^+ \sin(\omega(t - z))\epsilon_{ij}^+$

Classification of instabilities

- $\mathcal{L}_0 + \mathcal{L}_{m_3} = \frac{1}{2}\dot{\gamma}_{ij}^2 - \frac{1}{2}(\partial_k \gamma_{ij})^2 + \frac{1}{2}\dot{\pi}^2 - \frac{c_s^2}{2}(\partial_i \pi)^2 + \frac{1}{\Lambda^2}\dot{\gamma}_{ij}\partial_i \pi \partial_j \pi$
- Treat GW as a classical background: $\gamma_{ij} = M_{\rm Pl} h_0^+ \sin(\omega(t-z))\epsilon_{ij}^+$
- The Lagrangian of π reads

$$\mathcal{L}_\pi = \frac{1}{2}\dot{\pi}^2 - \frac{c_s^2}{2}(1-\beta)(\partial_i \pi)^2$$

where

$$\beta = \frac{2\omega M_{\rm Pl} h_0^+}{c_s^2 |\Lambda^2|}, \quad \Lambda^2 \simeq -\frac{\alpha \Lambda_2^2}{\sqrt{2}\alpha_{\rm B}}, \quad \alpha_{\rm B} \equiv -\frac{m_3^3}{2M_{\rm Pl}^2 H}$$

$\beta < 1$: Resonant instability $\Rightarrow$ Not applicable for m_3 and improve bound for $\alpha_{\rm H}$ (Creminelli, Tambalo, Vernizzi and VY 19)

$\beta > 1$: Gradient instability

Gradient/Ghost instabilities ($\beta > 1$)

- Let's consider

$$\mathcal{L}_\pi = \frac{1}{2}\left[\dot{\pi}^2 - c_s^2(\partial_i\pi)^2\right] + \frac{1}{\Lambda^2}\dot{\gamma}_{ij}\partial_i\pi\partial_j\pi + \dots$$
$$= \frac{1}{2}\dot{\pi}^2 - \frac{c_s^2}{2}(1-\beta)(\partial_i\pi)^2 + \text{NL self-couplings} + \text{Source terms}$$

Generally, this leads to the gradient instability of π.

Gradient/Ghost instabilities ($\beta > 1$)

- Let's consider

$$\mathcal{L}_\pi = \frac{1}{2}\left[\dot{\pi}^2 - c_s^2(\partial_i\pi)^2\right] + \frac{1}{\Lambda^2}\dot{\gamma}_{ij}\partial_i\pi\partial_j\pi + \dots$$
$$= \frac{1}{2}\dot{\pi}^2 - \frac{c_s^2}{2}(1-\beta)(\partial_i\pi)^2 + \text{NL self-couplings} + \text{Source terms}$$

Generally, this leads to the gradient instability of π.

- Can the non-linearity quench the instability ?

- Study the stability at NL level with the bg. of π induced by GWs

Classical stability conditions

- Consider a generic Lagrangian for π

$$\mathcal{L}_\pi \xrightarrow{\pi=\hat{\pi}+\delta\pi} \mathcal{L}_{\delta\pi} = Z^{\mu\nu}(x)\,\partial_\mu\delta\pi\partial_\nu\delta\pi$$

- Free of instability $\Rightarrow$ Conditions on $Z^{\mu\nu}$

Classical stability conditions

- Consider a generic Lagrangian for π

$$\mathcal{L}_\pi \xrightarrow{\pi=\hat{\pi}+\delta\pi} \mathcal{L}_{\delta\pi} = Z^{\mu\nu}(x)\,\partial_\mu\delta\pi\partial_\nu\delta\pi$$

- Free of instability $\Rightarrow$ Conditions on $Z^{\mu\nu}$
- Absence of ghost: $Z^{00} > 0$
- Absence of gradient: $Z^{0i}Z^{0j} - Z^{ij}Z^{00}$ positive definite

Classical stability conditions

- Consider a generic Lagrangian for π

$$\mathcal{L}_\pi \xrightarrow{\pi=\hat{\pi}+\delta\pi} \mathcal{L}_{\delta\pi} = Z^{\mu\nu}(x)\,\partial_\mu\delta\pi\partial_\nu\delta\pi$$

- Free of instability $\Rightarrow$ Conditions on $Z^{\mu\nu}$
- Absence of ghost: $Z^{00} > 0$
- Absence of gradient: $Z^{0i}Z^{0j} - Z^{ij}Z^{00}$ positive definite
- Cubic Galileon w/o GWs: no ghost/gradient inst. for non-relativistic source (Nicolis and Rattazzi 04)

Instability in the presence of GWs

- The Lagrangian for π now is

$$\mathcal{L} = -\frac{1}{2}\bar{\eta}^{\mu\nu}\partial_\mu\pi\partial_\nu\pi - \frac{1}{\Lambda_{\rm B}^3}\Box\pi(\partial\pi)^2 + \frac{1}{\Lambda^2}\dot{\gamma}_{ij}\partial_i\pi\partial_j\pi + \frac{\Lambda_{\rm B}^3}{2\Lambda^4}\pi\dot{\gamma}_{ij}^2$$

$\bar{\eta}_{\mu\nu} \equiv \mathrm{diag}(-1, c_s^2, c_s^2, c_s^2)$, The parameter $\beta \sim \dot{\gamma}_{ij}/\Lambda^2 > 1$

Instability in the presence of GWs

- The Lagrangian for π now is

$$\mathcal{L} = -\frac{1}{2}\bar{\eta}^{\mu\nu}\partial_\mu\pi\partial_\nu\pi - \frac{1}{\Lambda_{\rm B}^3}\Box\pi(\partial\pi)^2 + \frac{1}{\Lambda^2}\dot{\gamma}_{ij}\partial_i\pi\partial_j\pi + \frac{\Lambda_{\rm B}^3}{2\Lambda^4}\pi\dot{\gamma}_{ij}^2$$

$\bar{\eta}_{\mu\nu} \equiv \mathrm{diag}(-1, c_s^2, c_s^2, c_s^2)$, The parameter $\beta \sim \dot{\gamma}_{ij}/\Lambda^2 > 1$

- $\pi = \hat{\pi} + \delta\pi$. The kinetic matrix $Z^{\mu\nu}$ for $\delta\pi$ is

$$Z^{\mu\nu} \equiv -\frac{1}{2}\bar{\eta}^{\mu\nu} - 2\left(\mathcal{K}^{\mu\nu} - \eta^{\mu\nu}\mathcal{K}\right) + \frac{\dot{\gamma}_{\mu\nu}}{\Lambda^2}\,, \quad \mathcal{K}_{\mu\nu} = -\frac{1}{\Lambda_{\rm B}^3}\partial_\mu\partial_\nu\hat{\pi}$$

- The EoM for $\hat{\pi}$ is

$$\bar{\Box}\hat{\pi} - \frac{2}{\Lambda_{\rm B}^3}\left[(\partial_\mu\partial_\nu\hat{\pi})^2 - \Box\hat{\pi}^2\right] - \frac{2}{\Lambda^2}\dot{\gamma}_{\mu\nu}\partial^\mu\partial^\nu\hat{\pi} - \frac{\Lambda_{\rm B}^3}{2\Lambda^4}\dot{\gamma}_{\mu\nu}^2 = 0$$

Instability in the presence of GWs with $c_s < 1$

- Assume that $\gamma_{\mu\nu} = \gamma_{\mu\nu}(u)$
- One can solve the EoM for $\hat{\pi}$ analytically

$$\hat{\pi}''(u) = -\frac{\Lambda_{\mathrm{B}}^3 \dot{\gamma}_{\mu\nu}^2}{2(1-c_s^2)\Lambda^4}$$

Instability in the presence of GWs with $c_s < 1$

- Assume that $\gamma_{\mu\nu} = \gamma_{\mu\nu}(u)$
- One can solve the EoM for $\hat{\pi}$ analytically

$$\hat{\pi}''(u) = -\frac{\Lambda_{\rm B}^3 \dot{\gamma}_{\mu\nu}^2}{2(1-c_s^2)\Lambda^4}$$

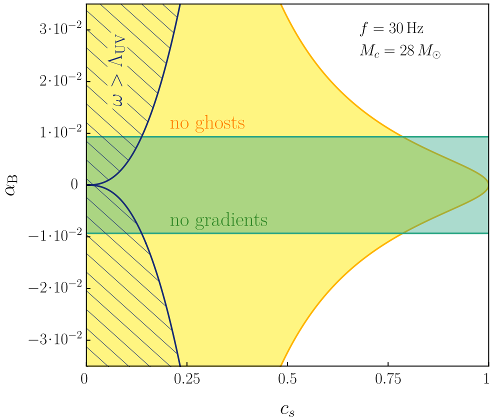
- The components of $Z^{\mu\nu}$ are

$$Z^{00} = \frac{1}{2} + 2\frac{\hat{\pi}''(u)}{\Lambda_{\rm B}^3}\,, \quad Z^{03} = Z^{30} = 2\frac{\hat{\pi}''(u)}{\Lambda_{\rm B}^3}\,, \quad Z^{33} = -\frac{1}{2}c_s^2 + 2\frac{\hat{\pi}''(u)}{\Lambda_{\rm B}^3}$$

$$Z^{11} = -\frac{1}{2}c_s^2 + \frac{\dot{\gamma}^{11}}{\Lambda^2}\,, \quad Z^{22} = -\frac{1}{2}c_s^2 + \frac{\dot{\gamma}^{22}}{\Lambda^2}$$

Phenomenological consequences

- Free of gradient int.: $Z^{11}, Z^{22} < 0 \Rightarrow \beta < 1$
- Free of ghost int.: $Z^{00} > 0 \Rightarrow \beta^2 < (1 - c_s^2)c_s^{-4}$



Free of instabilities: $|\alpha_\mathrm{B}| \lesssim 10^{-2}$

Fate of the instability

• The instabiliy occurs: the fluctuation grows at rate of the cutoff

• What happens next to the instability relies on the UV completion, so does the fate of $\gamma_{\mu\nu}$

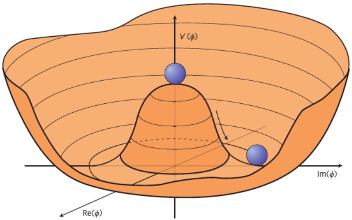
Fate of the instability

• The instabiliy occurs: the fluctuation grows at rate of the cutoff

• What happens next to the instability relies on the UV completion, so does the fate of $\gamma_{\mu\nu}$

• $\mathcal{L}_{\rm IR} = \mathcal{P}(X)$ with constant X background $\Rightarrow$ Ghost + gradient inst.

• $\mathcal{L}_{UV} = -|\partial\phi|^2 - \lambda(|\phi|^2 - v^2)^2$
$\phi = \phi_0 e^{i\pi}$, $\langle\phi_0\rangle = v^2 - \frac{X}{2\lambda}$,
$X = (\partial\pi)^2$



Ellis, et al. 15

All the modes are stable in the UV theory

Conclusion

- Perturbative/Resonant decays of GWs $\Rightarrow$ a strong bound on quartic beyond Horndeski (α_{H})

- Ghost/Gradient instabilities of π in GWs bg. $\Rightarrow$ a bound on Cubic Horndeski (α_{B})

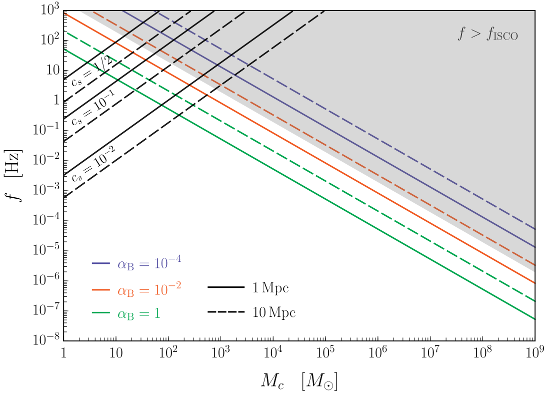
-The surviving scalar-tensor theory: $g_{\mu\nu} \to C(\phi, X) g_{\mu\nu}$

$$\mathcal{L} = G_2(\phi, X) + C(\phi, X) R + \frac{6 C_{,X}(\phi, X)^2}{C(\phi, X)} \phi^{;\mu} \phi_{;\mu\nu} \phi_{;\lambda} \phi^{;\nu\lambda}$$

- Fate of instability relies on the UV completion

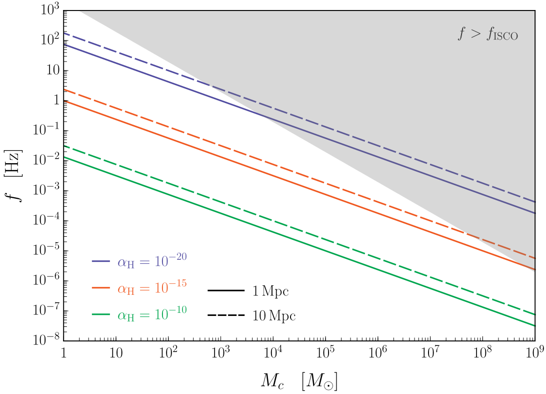
Backup

Gradient-instability lines α_B



- Gradient-instability lines, $\beta = 1$, for different value of α_B as a function of M_c of the binary system
- The black lines indicate frequencies $\omega > \Lambda_{\rm UV}$

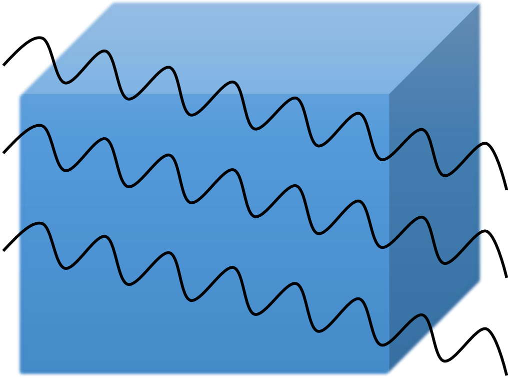
Gradient-instability lines α_{H}



- Gradient-instability lines, $\beta = 1$, for different value of α_{H} as a function of M_c of the binary system
- The black lines indicate frequencies $\omega > \Lambda_{\mathrm{UV}}$

Dark enery models

Extra scalar field: Lorentz violating medium $\Rightarrow c_T \neq 1$



Perturbative decay of GWs due to m_3^3-term

- m_3^3: $\delta K \delta g^{00} \Leftrightarrow$ Cubic Horndeski G_3
- The interaction term:

$$S_{\gamma\pi\pi} = \frac{1}{\Lambda^2}\int d^4x \dot{\gamma}_{ij}\partial_i\pi\partial_j\pi \quad , \quad \Lambda^2 = -\frac{\alpha}{\sqrt{2}\alpha_{\rm B}}\Lambda_2^2$$

$\alpha_{\rm B} \equiv -m_3^3/2M_{\rm Pl}^2 H$

- The perturbative decay rate

$$\Gamma_{\gamma\to\pi\pi} \simeq \left(\frac{\alpha_B}{\Lambda_2^2}\right)^2 \frac{\omega^5(1-c_s^2)^2}{480\pi c_s^7}$$

$\Gamma/H_0 \lesssim 1 \Rightarrow |\alpha_{\rm B}| \lesssim 10^{10}$, $\Lambda_2 \sim 10^{-3}$ eV

Resonant decay of GWs

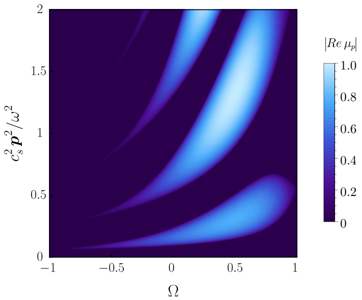
- In Fourier space f_p satisfies the Mathieu eq.

$$\frac{d^2 f}{d\tau^2} + [A - 2q\cos(2\tau)]f = 0$$

- $f_p \sim e^{\mu_k \tau}$

- the exponent $\mu \sim \beta$ for $\beta < 1$ (Narrow resonance)

- Need ~ 700 cycles to reach $\rho_\pi \sim \rho_\gamma$



$$\tau = \frac{\omega u}{2} \quad , \quad \Omega = p_z/|\boldsymbol{p}|$$

$$A = 4\frac{c_s^2\,\boldsymbol{p}^2}{\omega^2}\frac{(1-c_s\Omega)^2}{(1-c_s^2)^2}$$

$$q = 2\beta\frac{c_s^2\,\boldsymbol{p}^2}{\omega^2}\frac{(1-\Omega^2)\cos(2\phi)}{1-c_s^2}$$

Resonant decay of GWs

- EoM of π for $\tilde{m}_4^2$-operator

$$\ddot{\pi} - c_s^2 \partial^2 \pi - \beta \sin(\omega(t - z))(\partial_x^2 - \partial_y^2)\pi = 0$$

Resonant decay of GWs

- EoM of π for $\tilde{m}_4^2$-operator

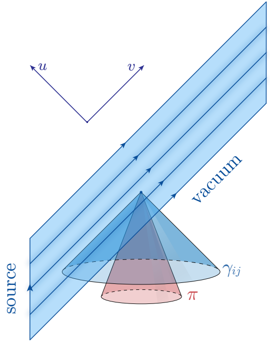
$$\ddot{\pi} - c_s^2 \partial^2 \pi - \beta \sin(\omega(t-z))(\partial_x^2 - \partial_y^2)\pi = 0$$

- Light-cone coord.

$$\frac{\mathrm{d}^2 f}{\mathrm{d}\tau^2} + [A - 2q\cos(2\tau)]f = 0$$

$$\pi(u, \tilde{\boldsymbol{x}}) \sim \int e^{i\tilde{\boldsymbol{p}}\cdot\tilde{\boldsymbol{x}}} f_{\tilde{p}}(u)\hat{a}_{\tilde{\boldsymbol{p}}} + \text{h.c.}$$

- $f_p \sim e^{\mu_k \tau}$, $\mu \sim \beta < 1$ (Narrow resonance)



Observational signature for $\tilde{m}_4$

- Modification of GWs signal: $\Delta\gamma_{ij} \sim -A \exp(\beta\omega u/4)\epsilon^{+}_{ij}$
- Effect of G_4 (Quartic Galileon), $\Lambda_c^6 \sim \Lambda_3^6/(\alpha_{\mathrm{H}} c_s^4)$ for $m_3^3 = 0$

$$G_4 = \frac{1}{\Lambda_c^6}(\partial\pi)^2[(\Box\pi)^2 - \pi_{\mu\nu}\pi^{\mu\nu}] \sim \frac{1}{\Lambda_\star^3}\ddot{\gamma}_{ij}\partial_i\pi\partial_j\pi$$

Observational signature for $\tilde{m}_4$

- Modification of GWs signal: $\Delta\gamma_{ij} \sim -A\exp(\beta\omega u/4)\epsilon^{+}_{ij}$

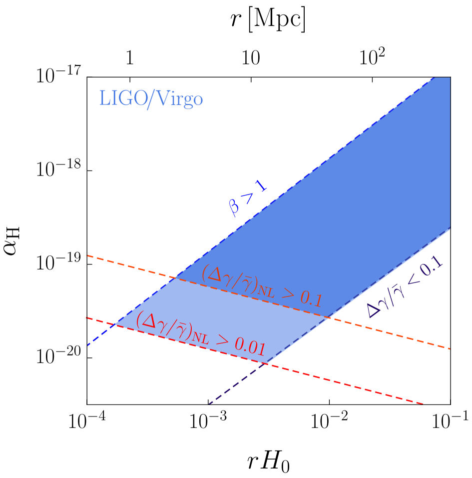
- Effect of G_4 (Quartic Galileon), $\Lambda_c^6 \sim \Lambda_3^6/(\alpha_{\mathrm{H}} c_s^4)$ for $m_3^3 = 0$

$$G_4 = \frac{1}{\Lambda_c^6}(\partial\pi)^2[(\Box\pi)^2 - \pi_{\mu\nu}\pi^{\mu\nu}] \sim \frac{1}{\Lambda_\star^3}\ddot{\gamma}_{ij}\partial_i\pi\partial_j\pi$$

We obtain

$$\frac{\Delta\gamma}{\bar{\gamma}} \lesssim (\beta N_{\mathrm{cyc}})^{3/2}(rH_0)^2 \equiv \left(\frac{\Delta\gamma}{\bar{\gamma}}\right)_{NL}$$

Observational signature for $\tilde{m}_4$



$f = 30$ Hz, $\mathrm{M_c} = 1.2\mathrm{M}_\odot$
GW170817,
40 Mpc ($\mathrm{rH}_0 \sim 5 \cdot 10^{-3}$)

perturbative bound: $\alpha_\mathrm{H} \lesssim 10^{-10}$

What about the resonant effect from m_3^3-term ?

- One can run the same procedure with $m_3^3 \delta K \delta g^{00}$

$$m_3^3 \delta K \delta g^{00} \supset \frac{1}{\Lambda^2} \dot{\gamma}_{ij} \partial_i \pi \partial_j \pi \ , \quad \Lambda^2 \simeq -\frac{\alpha \Lambda_2^2}{\sqrt{2} \alpha_{\rm B}}$$

$\alpha_{\rm B} \equiv -m_3^3/(2M_{\rm Pl}^2 H)$, $\beta = 2\omega M_{\rm Pl} h_0^+ / (c_s^2 |\Lambda^2|)$, $\Lambda_2 \sim 10^{-3}$ eV

What about the resonant effect from m_3^3-term ?

- One can run the same procedure with $m_3^3 \delta K \delta g^{00}$

$$m_3^3 \delta K \delta g^{00} \supset \frac{1}{\Lambda^2} \dot{\gamma}_{ij} \partial_i \pi \partial_j \pi \ , \quad \Lambda^2 \simeq -\frac{\alpha \Lambda_2^2}{\sqrt{2} \alpha_{\rm B}}$$

$\alpha_{\rm B} \equiv -m_3^3/(2M_{\rm Pl}^2 H)$, $\beta = 2\omega M_{\rm Pl} h_0^+/(c_s^2|\Lambda^2|)$, $\Lambda_2 \sim 10^{-3}$ eV

- Once the resonance happens ($\beta < 1$), the cubic self-interaction quickly becomes important

$$G_3 \sim \frac{1}{\Lambda_{\rm B}^3} \Box\pi \, (\partial_i \pi)^2 \ , \quad \Lambda_{\rm B}^3 \sim \alpha_{\rm B}^{-1} \Lambda_3^3$$

- No sizable effect on GWs signal: $\Delta\gamma/\bar{\gamma} \ll 1$

What about the resonant effect from m_3^3-term ?

- One can run the same procedure with $m_3^3 \delta K \delta g^{00}$

$$m_3^3 \delta K \delta g^{00} \supset \frac{1}{\Lambda^2} \dot{\gamma}_{ij} \partial_i \pi \partial_j \pi \ , \quad \Lambda^2 \simeq -\frac{\alpha \Lambda_2^2}{\sqrt{2}\alpha_{\rm B}}$$

$\alpha_{\rm B} \equiv -m_3^3/(2M_{\rm Pl}^2 H)$, $\beta = 2\omega M_{\rm Pl} h_0^+ / (c_s^2 |\Lambda^2|)$, $\Lambda_2 \sim 10^{-3}$ eV

- Once the resonance happens ($\beta < 1$), the cubic self-interaction quickly becomes important

$$G_3 \sim \frac{1}{\Lambda_{\rm B}^3} \Box \pi \left(\partial_i \pi\right)^2 \ , \quad \Lambda_{\rm B}^3 \sim \alpha_{\rm B}^{-1} \Lambda_3^3$$

- No sizable effect on GWs signal: $\Delta\gamma/\bar{\gamma} \ll 1 \Rightarrow$ Need to study $\beta > 1$

Vainshtein effect on the instability

• Suppose $\hat{\pi}$ is generated by a non-relativistic astrophysical object. The object possibly gives a large kinetic matrix Z for $\delta\pi$ and healthy (shown by Nicolis & Rattazzi) within r_V ($\sim$ kpc). One sees that within this region the coupling $\delta\pi T$ is suppressed and GR is recovered at small scales (non-linear).

• Can this happen to the instability induced by GWs ? Suppose again $\hat{\pi}$ is sourced by an astrophysical object. $\delta\pi$ seems to acquire a large Z. The parameter β of $\gamma\pi\pi$ seems to get suppressed due to a large Z and the instability might be stopped by this screening mechanism. But this is not the case for the GWs traveling over the cosmo. distances ($\gg$ the typical r_V) since at large distances one expects the linear perturbation theory is recovered, so that the Vainshtein mechanism is negligible. Hence, the argument of having large Z to suppress the instability is not applicable in the presence of GWs traveling over cosmo. distances and the instability still remains active.

Fate of instability

• Once the instability happens, a huge amount is damped into π's until their backreaction stops the instability. Now it's quite hard to imagine the new π state will resemble to the original one and make the same predictions. We expect the unstable π's at some point make the thoery healthy again but this would affect the other predictions of the theory.

• It might be that EFT breaks down at instability and one cannot make prediction unless UV is known. But the frequency of π's can be as low as 10^{10} km that means the small scale experiment cannot be explained by this EFT.

Instability of plane wave $\hat{\pi}$

- Let $\hat{\pi} = Af(u)$ (consider u-direction)
- $\Box\hat{\pi}(u) = -4\partial_u\partial_v\hat{\pi}(u) = 0$
- $\partial_t^2\hat{\pi}(u) = Af''(u)$, $\partial_z^2\hat{\pi}(u) = Af''(u)$, $\partial_t\partial_z\hat{\pi}(u) = -Af''(u)$
- Without GWs: $Z^{\mu\nu} \equiv -\frac{1}{2}\eta^{\mu\nu} - 2\left(\mathcal{K}^{\mu\nu} - \eta^{\mu\nu}\mathcal{K}\right)$, we have

$$Z^{00} = \frac{1}{2} + 2\frac{Af''(u)}{\Lambda_{\mathrm{B}}^3}\,, \quad Z^{33} = -\frac{1}{2} + 2\frac{Af''(u)}{\Lambda_{\mathrm{B}}^3}\,, \quad Z^{03} = 2\frac{Af''(u)}{\Lambda_{\mathrm{B}}^3}$$

$Z^{11} = Z^{22} = -1/2$

- Ghost: $Z^{00} < 0 \Rightarrow Af'' < -\Lambda_{\mathrm{B}}^3/4$.
- No gradient: $Z^{11}, Z^{22} < 0$ and $(Z^{03})^2 - Z^{33}Z^{00} = 1/4 > 0$
- Non-diagonalizable $Z^{\mu\nu}$: $2|Z^{03}| = |Z^{00} + Z^{33}|$ (avoid the theorem)

Observational signature for $\tilde{m}_4$

- For a binary system (M_c, f, r): $h_0^+ \sim 10^{-3}/(fN_{\rm cyc}r)$
- Sizeable effect in GW waveform requires $\exp(\beta\omega u/4) \sim \mathcal{O}(10^2)$

$$\frac{\Delta\gamma}{\bar{\gamma}} > 0.1 \quad \Rightarrow \quad \alpha_{\rm H} \gtrsim 10^{-17} \cdot rH_0 \cdot \frac{\Lambda_3}{2\pi f}\alpha c_s^2$$

- Our calculation is valid when $\beta < 1$

$$\alpha_{\rm H} \lesssim \frac{H_0}{f} \cdot N_{\rm cyc} \cdot rH_0 \quad , \quad N_{\rm cyc} \sim (GM_c f)^{-5/3}$$

- To neglect effect of NL, demands $(\Delta\gamma/\bar{\gamma})_{NL} > 0.1$

$$\alpha_{\rm H} \gtrsim \frac{H_0}{f}(rH_0)^{1/3}$$

$c_T^2 = 1$ from GW170817 & GRB170817A

$$\mathcal{L}_{c_T=1}^{\rm EFT} = \frac{M_{\rm Pl}^2}{2} f(t)^{(4)}R - \Lambda(t) - c(t)g^{00} + \frac{m_2^2(t)}{2}(\delta g^{00})^2 - \frac{m_3^3(t)}{2}\delta K \delta g^{00} + \frac{\tilde{m}_4^2(t)}{2}\delta g^{00}\left({}^{(3)}R - \delta\mathcal{K}_2\right)$$

In the covariant theory

$$\mathcal{L}_{c_T=1} = G_2(\phi, X) + G_3(\phi, X)\Box\phi + B_4(\phi, X)^{(4)}R - \frac{4}{X}B_{4,X}(\phi, X)(\phi^\mu\phi^\nu\phi_{\mu\nu}\Box\phi - \phi^\mu\phi_{\mu\nu}\phi_\lambda\phi^{\lambda\nu})$$

$B_4 \equiv G_4 + XG_{5,\phi}/2$

Summary of the results

EFT of DE operator	$\frac{1}{2}\tilde{m}_4^2\,\delta g^{00}\left({}^{(3)}R+\delta K^\nu_\mu\delta K^\mu_\nu-\delta K^2\right)$	$m_3^3\,\delta g^{00}\delta K$
GLPV theories with $c_T=1$ $\mathcal{L}=G_2+G_3\Box\phi+B_4R-\frac{4B_{4,X}}{X}(\phi^{;\mu}\phi^{;\nu}\phi_{;\mu\nu}\Box\phi-\phi^{;\mu}\phi_{;\mu\nu}\phi_{;\lambda}\phi^{;\lambda\nu})$	$-\frac{2XB_{4,X}}{B_4}$	$\frac{2XB_{4,X}}{B_4}+\frac{\dot{\phi}XG_{3,X}}{2HB_4}$
Dimensionless function α_i	$\alpha_{\rm H}$	$\alpha_{\rm B}$
Perturbative decay ($\Gamma_{\gamma\to\pi\pi}/H_0>1$)	$\lvert\alpha_{\rm H}\rvert\gtrsim 10^{-10}$	Irrelevant ($\lvert\alpha_{\rm B}\rvert\gtrsim 10^{10}$)
Narrow resonance ($\beta<1,\ \beta\omega u>1$)	$3\times10^{-20}\lesssim\lvert\alpha_{\rm H}\rvert\lesssim 10^{-17}$ with LIGO-Virgo $10^{-16}\lesssim\lvert\alpha_{\rm H}\rvert\lesssim 10^{-10}$ with LISA	Not applicable (large non-linearities)
Instability ($\beta>1,\ \beta\omega>1$)	$\lvert\alpha_{\rm H}\rvert\gtrsim 10^{-20}$	$\lvert\alpha_{\rm B}\rvert\gtrsim 10^{-2}$

The surviving scalar-tensor theory: $g_{\mu\nu}\to C(\phi,X)g_{\mu\nu}$

$$\mathcal{L}=G_2(\phi,X)+C(\phi,X)R+\frac{6C_{,X}(\phi,X)^2}{C(\phi,X)}\phi^{;\mu}\phi_{;\mu\nu}\phi_{;\lambda}\phi^{;\nu\lambda}$$

Stability in the absence of GWs

Here the Lagrangian is

$$\mathcal{L} = -\frac{1}{2}\eta^{\mu\nu}\partial_\mu\pi\partial_\nu\pi - \frac{1}{\Lambda_{\rm B}^3}\Box\pi(\partial\pi)^2 + \frac{\pi T}{2M_{\rm Pl}}$$

- $\pi = \hat{\pi} + \delta\pi$, the EoM for $\hat{\pi}$ is

$$\mathcal{K} + 2\left(\mathcal{K}_{\mu\nu}\mathcal{K}^{\mu\nu} - \mathcal{K}^2\right) = \frac{T}{2M_{\rm Pl}\Lambda_{\rm B}^3}\,, \quad \mathcal{K}_{\mu\nu} = -\frac{1}{\Lambda_{\rm B}^3}\partial_\mu\partial_\nu\hat{\pi}$$

- The kinetic matrix reads: $\mathcal{L}_{\delta\pi} = Z^{\mu\nu}\partial_\mu\delta\pi\partial_\nu\delta\pi$

$$Z^{\mu\nu} \equiv -\frac{1}{2}\eta^{\mu\nu} - 2\left(\mathcal{K}^{\mu\nu} - \eta^{\mu\nu}\mathcal{K}\right)$$

Nicolis and Rattazzi 04

Stability in the absence of GWs

• In terms of $Z_{\mu\nu}$ the EoM of $\hat{\pi}$ becomes

$$\frac{1}{3}Z^2 - (Z_{\mu\nu})^2 = \frac{1}{3} - \frac{T}{M_{\rm Pl}\Lambda_{\rm B}^3}$$

• For the non-relativistic source $v \ll 1$, the matrix $Z^{\mu\nu}$ is diagonalizable with a Lorentz boost, so that $Z^{\mu}_{\nu} = \mathrm{diag}(z_0, z_1, z_2, z_3)$ and $T \simeq -\rho \leq 0$

• Consider the plane $z_0 = 0$ in z_i-space

$$-\frac{1}{3}\left[(z_1 - z_2)^2 + (z_1 - z_3)^2 + (z_2 - z_3)^2\right] = \frac{1}{3} + \frac{\rho}{M_{\rm Pl}\Lambda_{\rm B}^3}$$

⇒ A solution crossing the plane doesn't exist

⇒ The initial stable solution ($Z^{\mu\nu} = -\eta^{\mu\nu}/2$) remains stable everywhere

DGP - Self-accelerating Universe

- Cubic Gal. $(\partial\pi)^2\Box\pi$ is inspired by the DGP model
- $5d$ gravity theory (∞-extra dim) - Bulk is Mink_5
- Two branches on the brane: self-accelerating and FRW (normal)

$$4M_4^2(R_{\mu\nu}-\frac{1}{2}g_{\mu\nu}R)-4M_5^3(K_{\mu\nu}-g_{\mu\nu}K)=T_{\mu\nu}$$

$\Rightarrow$ Acceleration happens w/o cosmological constant

- The brane bending mode π becomes ghost in self-accelerating branch

Dvali, Gabadadze and Porrati 2000,
Luty et al. 2003 and many others

DGP - Instability of π

- Study an instability of π in the presence of GWs

- Consider a curved $5d$ spacetime G_{MN} and $G_{MN} = \bar{G}_{MN} + H_{MN}$

- On the boundary, $S_{\rm bdy} = S_{\rm 4dEH} + S_{\rm In5dEH} + S_{\rm In5dGF}$ and $H_{\mu\nu}| = h_{\mu\nu}$, $H_{\mu y}| = \partial_\mu \pi$, $H_{yy}| = -2\bar{\triangle}\pi$, $\bar{\triangle} = \sqrt{-\bar{D}^2}$

$$S^{(2)}_{\rm bdy} \supset M_5^3 \pi (\bar{K}^{\mu\nu}\bar{D}_\mu \bar{D}_\nu - \bar{K}\bar{D}^2)\pi$$

neglecting $\bar{K}h^2$, $\bar{K}h\bar{\triangle}\pi$ at high energy limit

$\Rightarrow$ In the presence of GWs bg. $\bar{G}_{\mu\nu}$, the instability of π is solely determined by $\bar{K}_{\mu\nu} \sim \bar{\triangle}\bar{G}_{\mu\nu} + \nabla_\mu N_\nu$

How large is this new piece ?

in progress

DGP - Modification of Green function

- To tackle this problem, let's consider a propagator of a massless scalar field on the brane

$$G_E(p) \sim \frac{1}{p^2 + 2\kappa p}\,, \quad p = \sqrt{p_4^2 + \boldsymbol{p}^2}$$

The linear in p is due to the induced action on the brane.

- The y-derivative of Lorentzian Green function in terms of (ω, r) is

$$\partial_y G(\omega, r) \sim \kappa G(\omega, r) + \frac{\omega^2}{(\omega r)^{3/2}} e^{-i\omega r}$$

$\Rightarrow$ Both terms are suppressed in the limit $\omega r \gg 1$

Any other example where
the modification of propagator is sizeable ?

in progress

Some formulas

Canonical normalizations

$$\pi_c \equiv \sqrt{\alpha} M_{\rm Pl} H \pi \ , \qquad \gamma_{ij}^c \equiv \frac{M_{\rm Pl}}{\sqrt{2}} \gamma_{ij} \ . \tag{1}$$

$$\alpha \equiv \frac{4 M_{\rm Pl}^2 (c + 2 m_2^4) + 3 m_3^6}{2 M_{\rm Pl}^4 H^2} \tag{2}$$

$$c_s^2 = \frac{2}{\alpha} \left[(1+\alpha_{\rm H})^2 \frac{c}{M_{\rm Pl}^2 H^2} + \alpha_{\rm H} + \alpha_{\rm H}(1+\alpha_{\rm H}) \frac{\dot{H}}{H^2} \right] \tag{3}$$

α_B: scalar-gravity mixing, α_H: scalar-matter mixing.

(beyond) (beyond) Horndeski

$$g_{\mu\nu} \to C(\phi, X) g_{\mu\nu} + D(\phi, X) \partial_\mu \phi \partial_\mu \phi \quad (4)$$

- Horn is preserved under this transf. with $C(\phi)$ and $D(\phi)$
- If $D = D(\phi, X) \implies$ Beyond Horn. which is preserved under this with $C(\phi)$ and $D(\phi, X)$
- If $C = C(\phi, X) \implies$ DHOST

Beyond Hondeski structure is preseved under disformal tranf. $g_{\mu\nu} \to C(\phi) g_{\mu\nu} + D(\phi, X) \partial_\mu \phi \partial_\mu \phi$. Once we perform this field redefinition (the disformal coefficient D depends on derivative of ϕ) we obtain a kinetic mixing between scalar and matter: α_{H}

Some formulas

$$
\begin{aligned}
M^2 &\equiv M_*^2 f + 2m_4^2 = 2G_4 - 4XG_{4,X} - X\left(G_{5,\phi} + 2H\dot{\phi}G_{5,X}\right) + 2X^2F_4 - 6H\dot{\phi}X^2F_5 \ , \\
m_4^2 &= \tilde{m}_4^2 + X^2F_4 - 3H\dot{\phi}X^2F_5 \ , \\
\tilde{m}_4^2 &= -\left[2XG_{4,X} + XG_{5,\phi} + \left(H\dot{\phi} - \ddot{\phi}\right)XG_{5,X}\right] \ , \\
m_5^2 &= X\left[2G_{4,X} + 4XG_{4,XX} + H\dot{\phi}(3G_{5,X} + 2XG_{5,XX}) + G_{5,\phi}\right. \\
&\quad \left. + XG_{5,X\phi} - 4XF_4 - 2X^2F_{4,X} + H\dot{\phi}X\left(15F_5 + 6XF_{5,X}\right)\right] \ , \\
m_6 &= \tilde{m}_6 - 3\dot{\phi}X^2F_5 \ , \\
\tilde{m}_6 &= -\dot{\phi}XG_{5,X} \ , \\
m_7 &= \frac{1}{2}\dot{\phi}X\left(3G_{5,X} + 2XG_{5,XX} + 15XF_5 + 6X^2F_{5,X}\right) \ .
\end{aligned}
$$

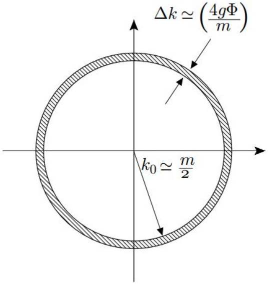
- Boltzmann equation: $\phi \to \chi\chi$

$$\frac{1}{a^3}\frac{d(a^3 n_\chi)}{dt} = \frac{2}{V}\Gamma_{\phi\to\chi\chi}[(n_k^\chi + 1)(n_{-k}^\chi + 1)n_{\mathbf{0}}^\phi - n_k^\chi n_{-k}^\chi (1 + n_{\mathbf{0}}^\phi)]$$
$$\simeq 2\Gamma_{\phi\to\chi\chi} n_\phi (1 + 2n_k^\chi)$$

$$n_{k=m/2}^\chi \simeq \frac{n_\chi}{(4\pi k_0^2 \Delta k)/(2\pi)^3} \simeq \frac{\pi^2\Phi}{g}\frac{n_\chi}{n_\phi}$$

$$n_\chi \propto \exp(\frac{\pi^2 g\Phi}{m^2}N) \propto \exp(2\pi\mu_B N)$$

$$N = mt/2\pi \;,\; \mu_B \equiv \frac{\pi g\Phi}{2m^2}$$



Mukhanov 05