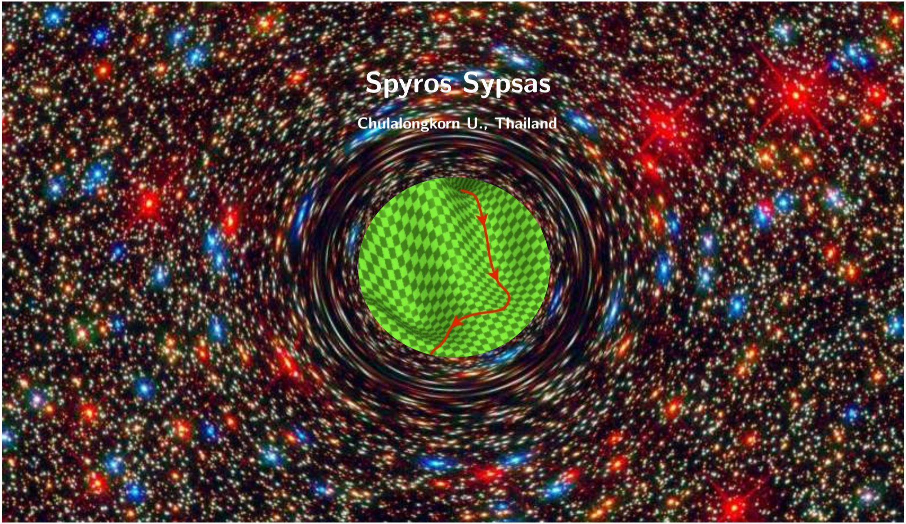
# Seeding Primordial Black Holes in Multifield Inflation



**Spyros Sypsas**

Chulalongkorn U., Thailand

Based on

2004.06106 (to appear in PRL)

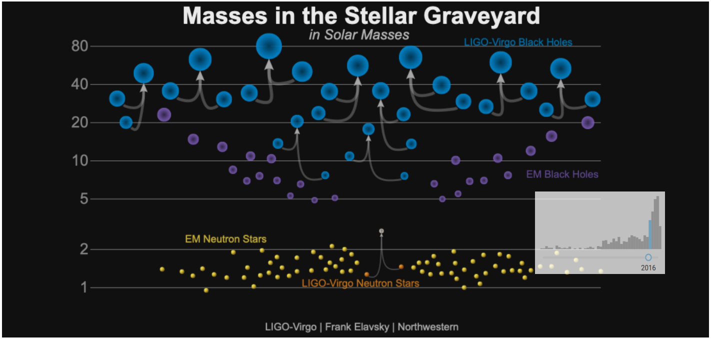
in collaboration with:

Gonzalo Palma, Cristobal Zenteno

Closely related: 2004.08369 by Fumagalli et al.

# Outline

1. Intro
   - PBHs: Why & How
   - Perturbations in 2-field Inflation
2. Black Hole Seeds
3. Concluding remarks

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**Did LIGO Detect Dark Matter?**

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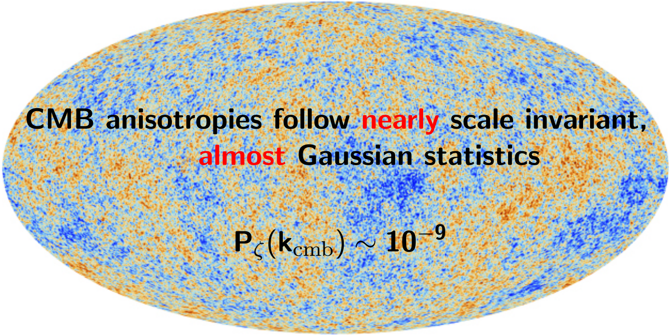
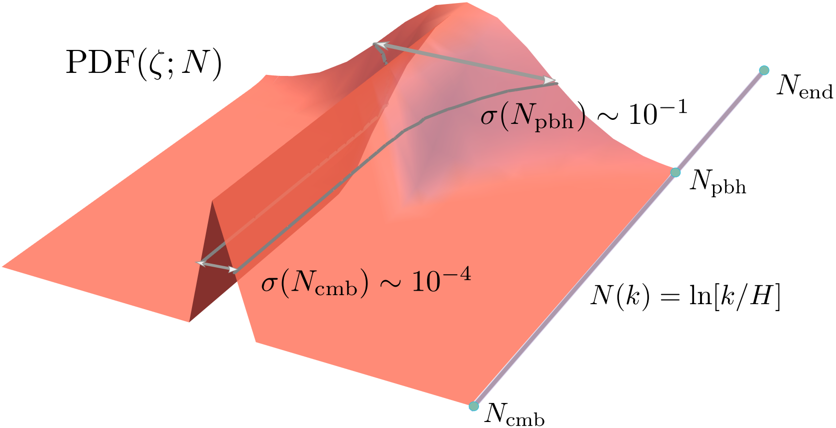
We consider the possibility that the black-hole (BH) binary detected by LIGO may be a signature of dark matter. Interestingly enough, there remains a window for masses $20M_\odot \lesssim M_{\rm bh} \lesssim 100M_\odot$ where primordial black holes (PBHs) may constitute the dark matter. If two BHs in a galactic halo pass sufficiently close,




For collapse into BHs we need

$$\zeta_{\mathrm{cr}}(k_{\mathrm{pbh}}) \simeq 10^{-1}$$

If we keep the scale invariance, this is an outlier $10^3\sigma$ away...


PDF(ζ; N)
σ(N_pbh) ∼ 10^−1
σ(N_cmb) ∼ 10^−4
N_end
N_pbh
N_cmb
N(k) = ln[k/H]

Numerous ways to achieve this either with one or many fields:

$$P_\zeta \propto \frac{H^2}{\epsilon c_s}$$

inflection points (regions) in the potential (USR, running mass), oscillatory (resonant) features in the sound speed (potential), reheating, gauge fields...
In addition PBH can stem from bubble/soliton collisions...

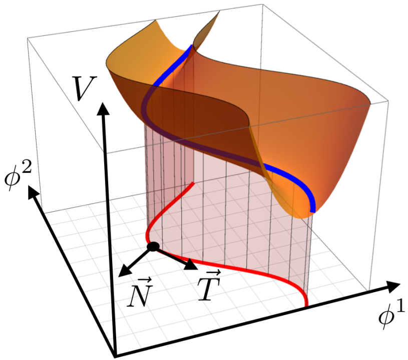
Is there any mechanism that operates only in multifield inflation?

$$P_\zeta \propto \frac{H^2}{\epsilon} f(\lambda)$$

$$S = S_{\mathrm{EH}} - \int \mathrm{d}^4x\sqrt{-g}\left[\frac{1}{2}\gamma_{ab}(\phi)\,\partial_\mu\phi^a\partial^\mu\phi^b + V(\phi)\right]$$

$$\phi^a(x,t) = \phi^a_0(t) + T^a(t)\varphi(x,t) + N^a(t)\psi(x,t)$$

$$\mathrm{d}s^2 = -N^2\mathrm{d}t^2 + a^2e^{2\zeta}(\mathrm{d}\vec{x} + \vec{N}\mathrm{d}t)^2 \quad (\varphi = 0)$$



$$\mathcal{L}^{(2)} \propto (D_t\zeta)^2 - \frac{1}{a^2}(\nabla\zeta)^2 + \dot{\psi}^2 - \frac{1}{a^2}(\nabla\psi)^2 + \cancelto{0}{U(\psi)}$$

with

$$D_t\zeta \equiv \dot{\zeta} - \lambda H\psi, \qquad \lambda \equiv \frac{2\Omega}{H} = \frac{2}{H}\frac{\delta\theta}{\delta t}$$

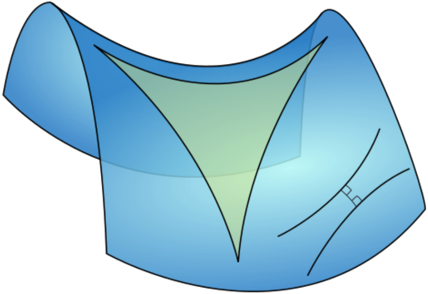
Result:

$$\boxed{P_{\zeta,\mathrm{pbh}} \sim \frac{e^{2\delta\theta}}{4(1+4\delta\theta^2)} \times P_{\zeta,\mathrm{cmb}}}$$

This implies $\delta\theta = \lambda\delta N/2 \sim 4\pi$ for $10^7$ enhancement.

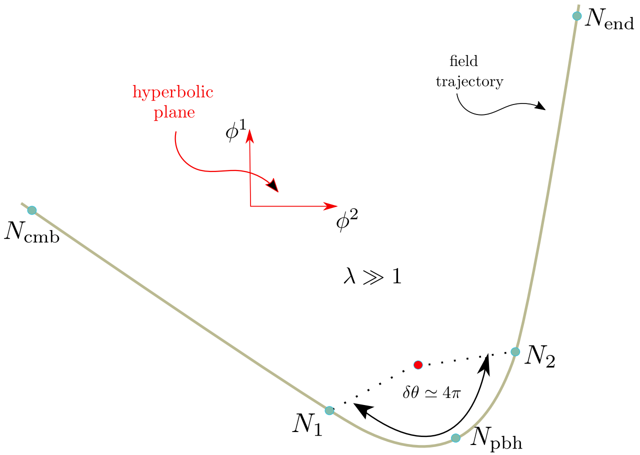
No canonical model can do that!

On a hyperbolic surface, parallel lines deviate, so the trajectory can turn for $\Delta\theta > \pi$ without self crossing!



$$L_{\rm circ} = 2\pi \frac{\sinh[K]}{K} \gg 2\pi$$

(curvature $K$ controled by the coupling $\lambda$)


$N_{\rm end}$
field
trajectory
hyperbolic
plane
$\phi^1$
$\phi^2$
$N_{\rm cmb}$
$\lambda \gg 1$
$N_2$
$\delta\theta \simeq 4\pi$
$N_1$
$N_{\rm pbh}$

# Analytical solution

$$\begin{aligned} \frac{\mathrm{d}}{\mathrm{d}t} D_t\zeta + 3HD_t\zeta + \frac{k^2}{a^2}\zeta &= 0, \\ \ddot{\psi} + 3H\dot{\psi} + \frac{k^2}{a^2}\psi + \lambda H D_t\zeta &= 0. \end{aligned}$$

Choise:

$$\lambda(t) = \lambda_0 \left[\theta(t - t_1) - \theta(t - t_2)\right]$$

Piecewise solution:

$t < t_1$: dS mode functions with BD ICs

$t_1 < t < t_2$: assume $\delta t \ll H^{-1}$ plane waves with ICs set by region I

$t_2 < t$: dS mode functions but with ICs set by region II

# Analytical solution

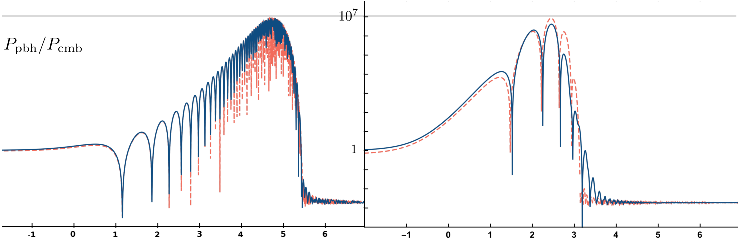
$$\zeta(k) = \frac{iH}{\sqrt{2k^3}} \sum_{\pm} \Bigg\{ \left[ A_{\pm}[k,\omega_{\pm},\delta N] \cos\left(\frac{\omega_{\pm}\delta N}{k_0}\right) + B_{\pm}[k,\omega_{\pm},\delta N] \sin\left(\frac{\omega_{\pm}\delta N}{k_0}\right) \right] a_{\zeta}(k) + \left[ C_{\pm}[k,\omega_{\pm},\delta N] \cos\left(\frac{\omega_{\pm}\delta N}{k_0}\right) + D_{\pm}[k,\omega_{\pm},\delta N] \sin\left(\frac{\omega_{\pm}\delta N}{k_0}\right) \right] a_{\psi}(k) \Bigg\} + \mathrm{h.c.}(-k) \,,$$

(A,B,C,D such that $[\zeta, P_{\zeta}] = i\delta$ etc)

$\omega_{\pm} = \sqrt{k^2 \pm k k_0 \lambda_0}$ : $\omega_{-}$ has an unstable region

# Power spectrum

$$\frac{P_{\rm pbh}}{H^2/\epsilon} \sim \begin{cases} 1 + 4\delta\theta^2 & \text{if } \; k \ll k_0\lambda/2 \quad \text{CMB} \\ \frac{1}{4} e^{2\delta\theta} & \text{if } \; k \sim k_0\lambda/2 \quad \text{PBH} \\ 1 & \text{if } \; k \gg k_0\lambda/2 \end{cases}$$




- ⋆ Evades theorems [Byrnes et al. '18, Carrilho et al. '19, Palma, SS, Patil '20]: $P_{\rm pbh} \propto k^5$ vs $k^4$
- ⋆ Does not break slow-roll conditions [Hertzberg, Yamada '17]

- ⋆ Purely multifield mechanism for PBH formation
  - ⋆ Need for nontrivial field geometry
- ⋆ Completely decoupled from slow-roll dynamics
  - ⋆ Evades known theorems of steepest growth
- ⋆ First analytic solution in the rapid turn regime $\lambda(t) \gg 1$

[Cremonini et al. '10, Achúcarro et al. '10, '12, '19, Assassi et al. '14, Iyer et al. '17, An et al. '17, Fumagalli et al. '19, Bjorkmo '19, Christodoulidis et al. '19]

To do:

- ⋆ Compute the NG PBH mass function
- ⋆ GW spectrum

# Thanks!