A new approach to observational cosmology using the scattering transform

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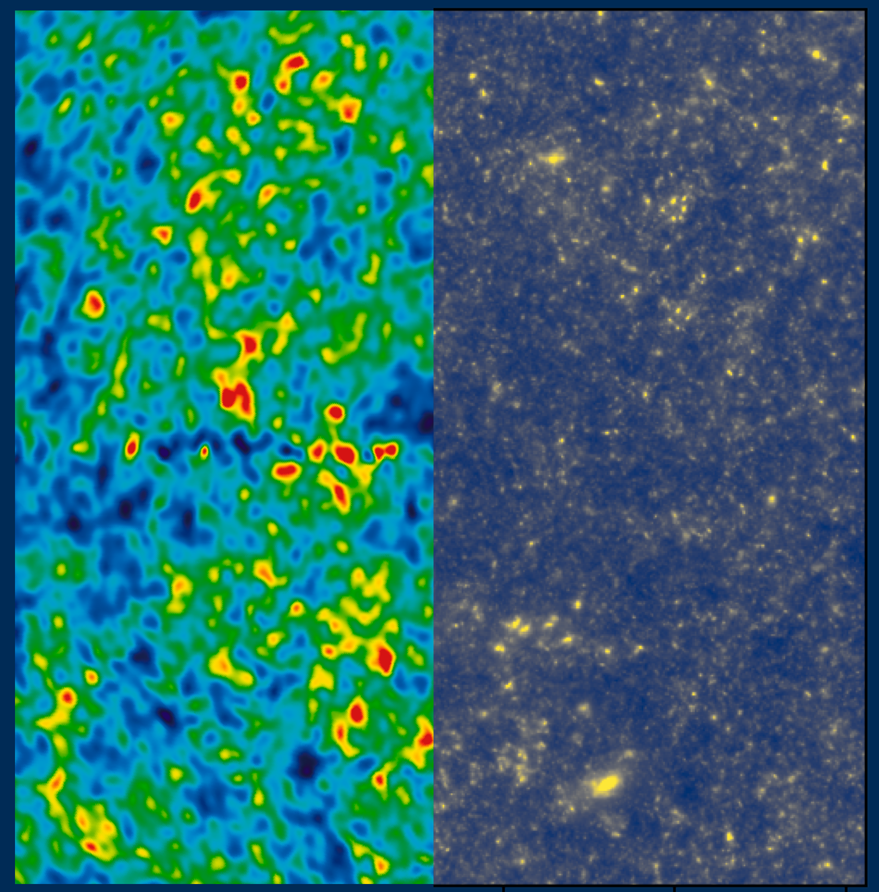
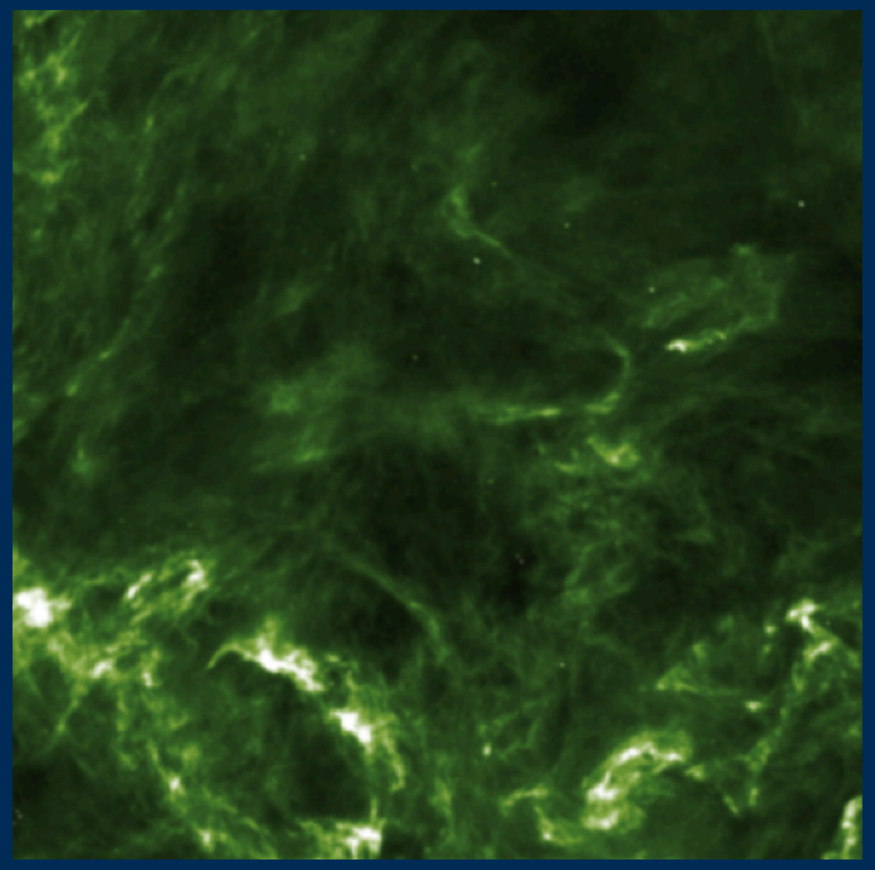
Brice Ménard (JHU), Joan Bruna (NYU)

How do we characterize a physical field?

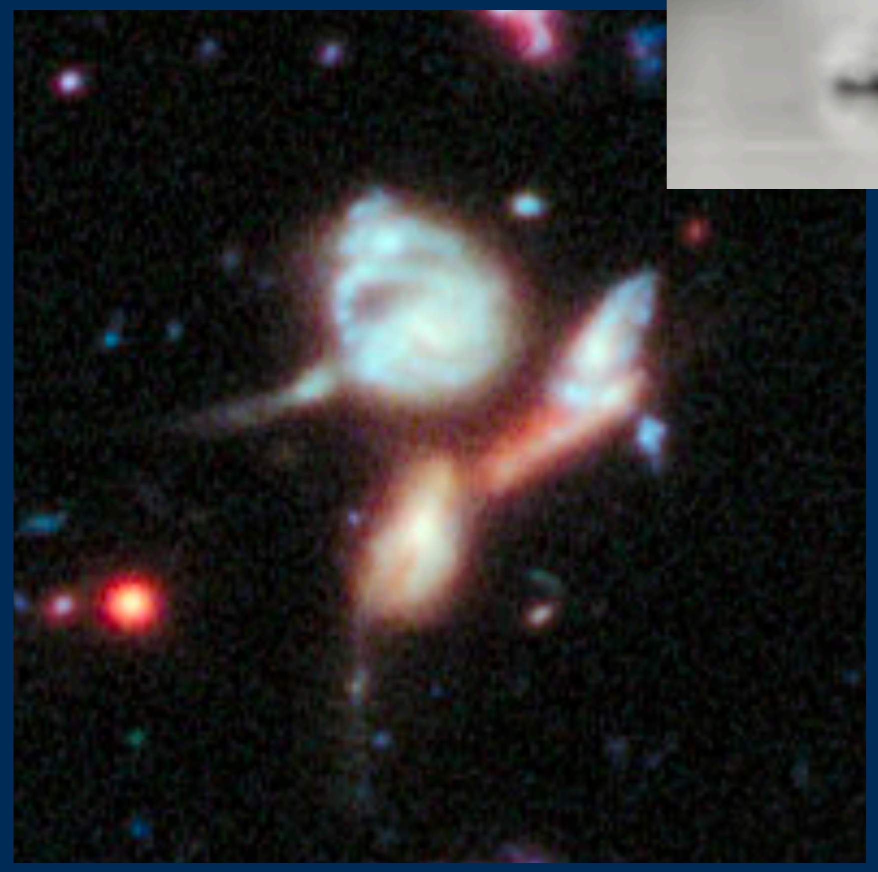
huge generic richness



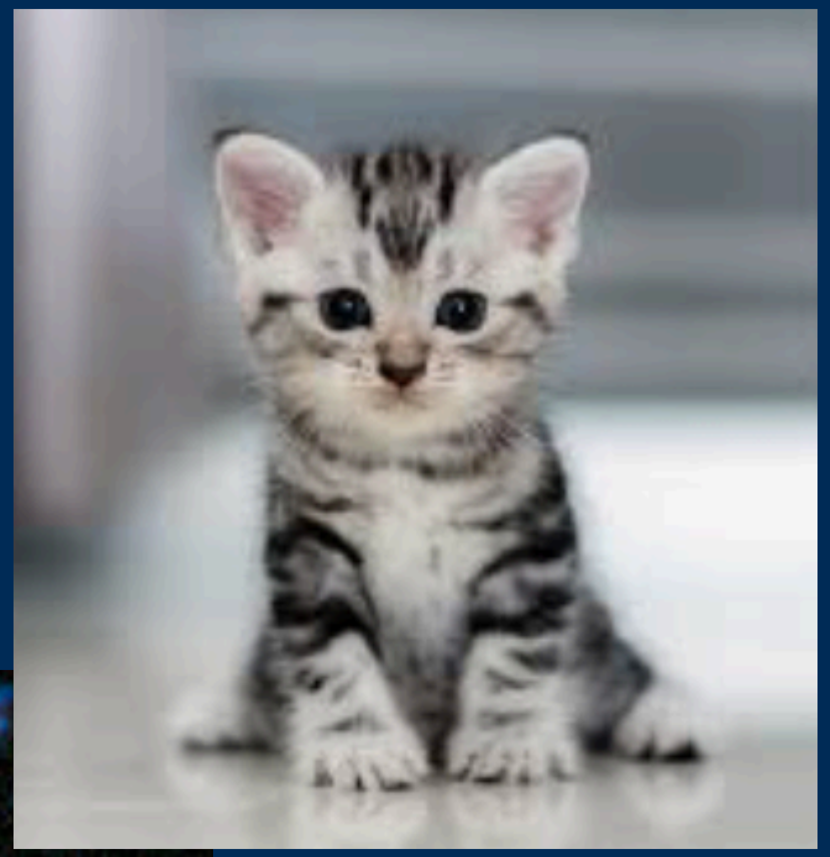
density fluctuation



ISM emission



blended galaxies

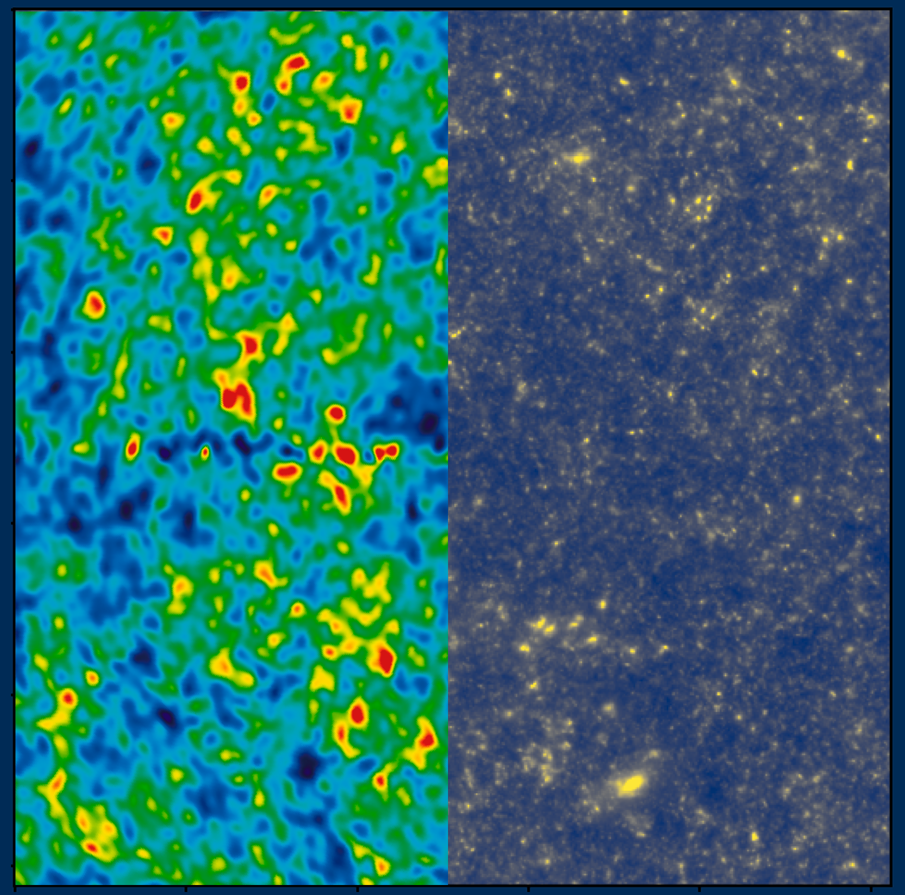


→ daily-life parameters

→ physical parameters

How do we characterize a physical field?

huge generic richness



data

→

extract features

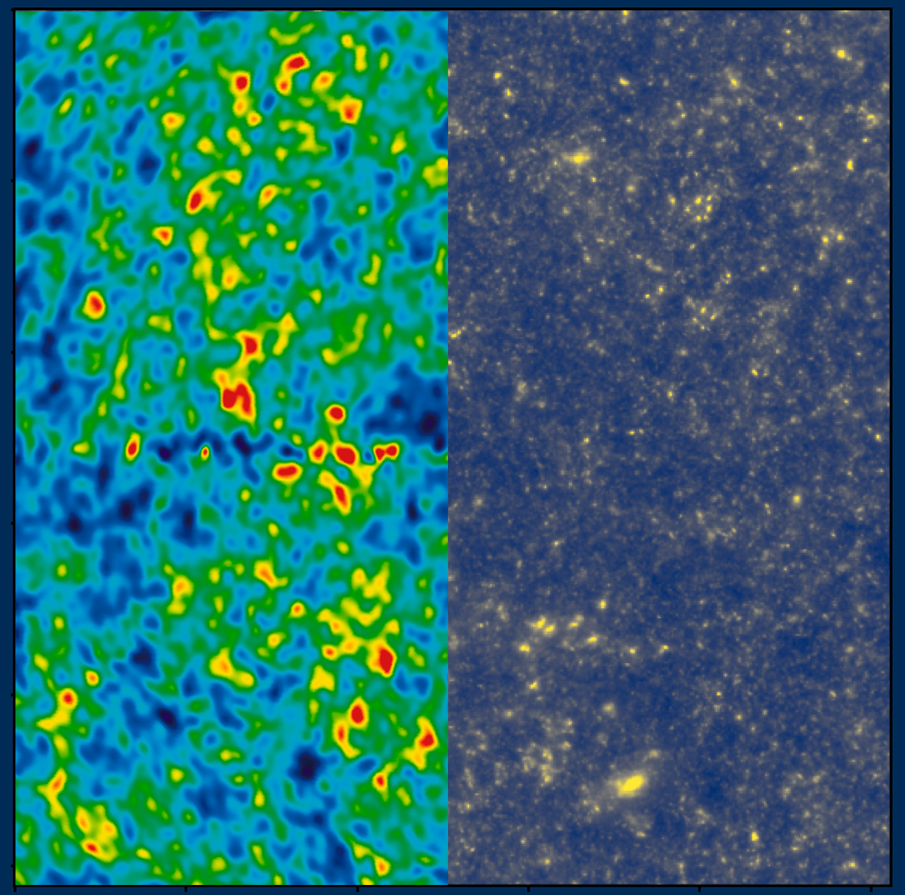
→ $\Omega_m, \sigma_8, w, M_\nu,$ etc.

physical parameters

How do we characterize a physical field?

huge generic richness

domain knowledge



data

→

power spectrum
n-point correlation expansion
cluster counting
CNN: learning features

extract features

→

$\Omega_m, \sigma_8, w, M_\nu,$ etc.

physical parameters

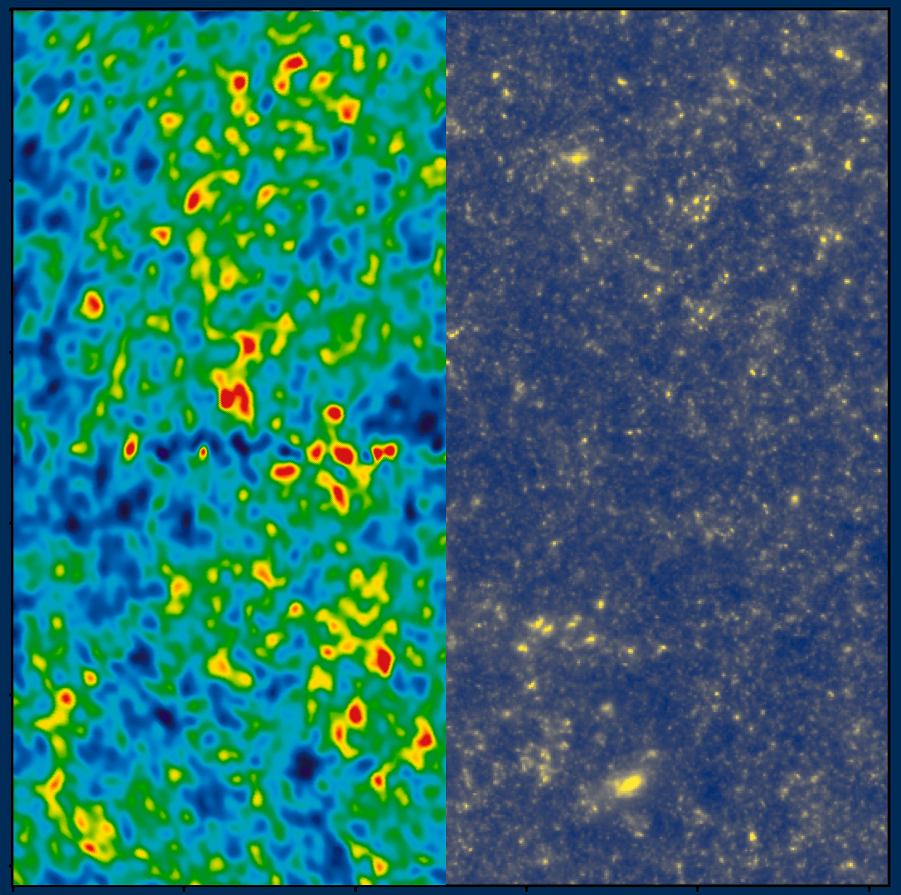
(representation, characterization, statistics, …)

How do we characterize a physical field?

huge generic richness

domain knowledge

localized features
multi-scale
long range interaction
hierarchical structure



→ CNN: learning features
Can we build a library of physical features?

data

extract features

(representation, characterization, statistics, …)

scattering transform

One operator:

$$UI = |I \star \psi^{j,l}|$$

Scattering transform =
wavelet convolution
+modulus
+mean

Input field I_0

Coefficients: $S_0 \equiv \langle I_0 \rangle$

Fields $I_1 \equiv \left| I_0 \star \psi_1 \right|$

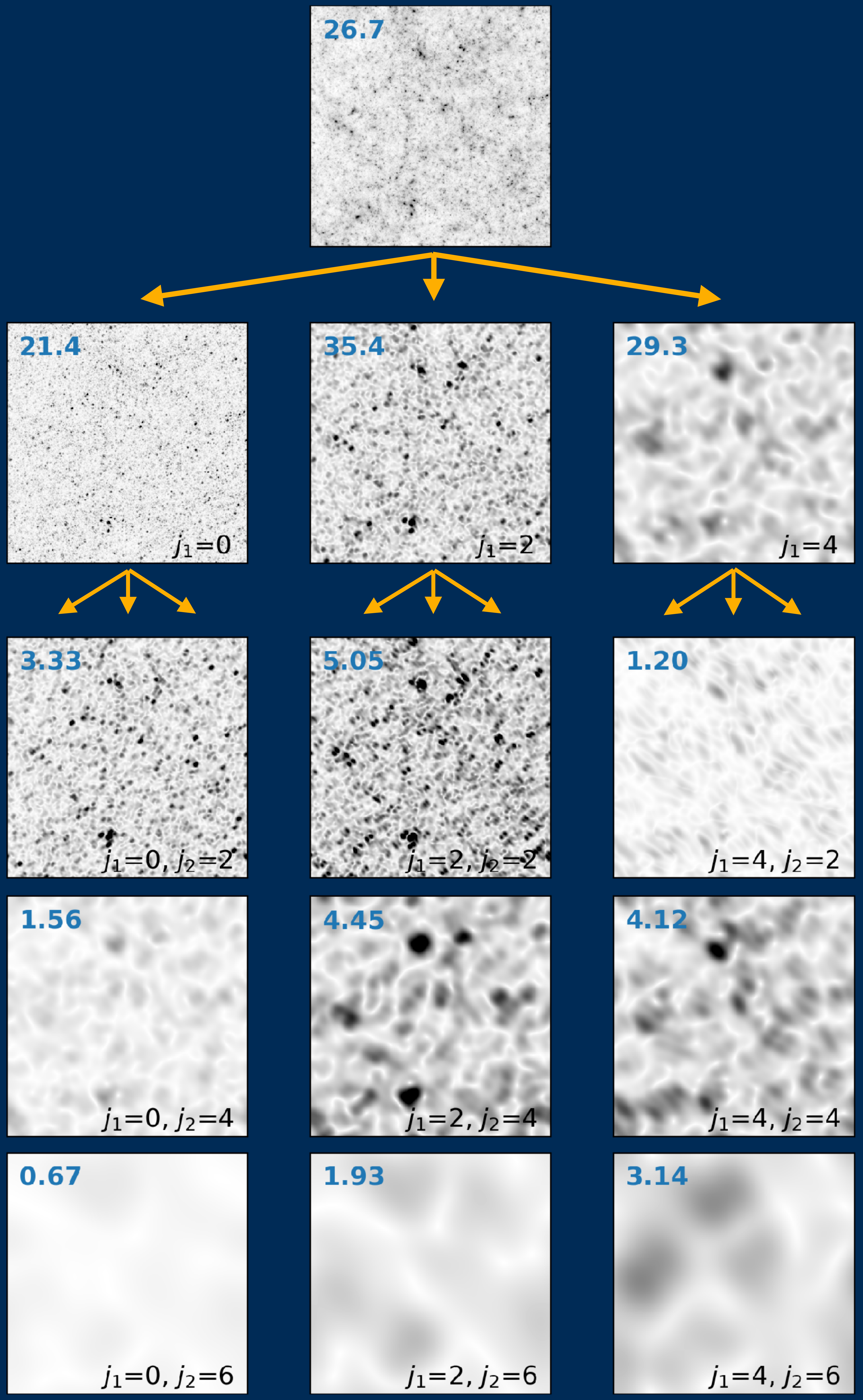
Coefficients: $S_1 \equiv \langle I_1 \rangle$

~power spectrum of I_0

Fields $I_2 \equiv \left| \, | I_0 \star \psi_1 | \star \psi_2 \right|$

Coefficients: $S_2 \equiv \langle I_2 \rangle$

~power spectrum of I_1



scattering transform

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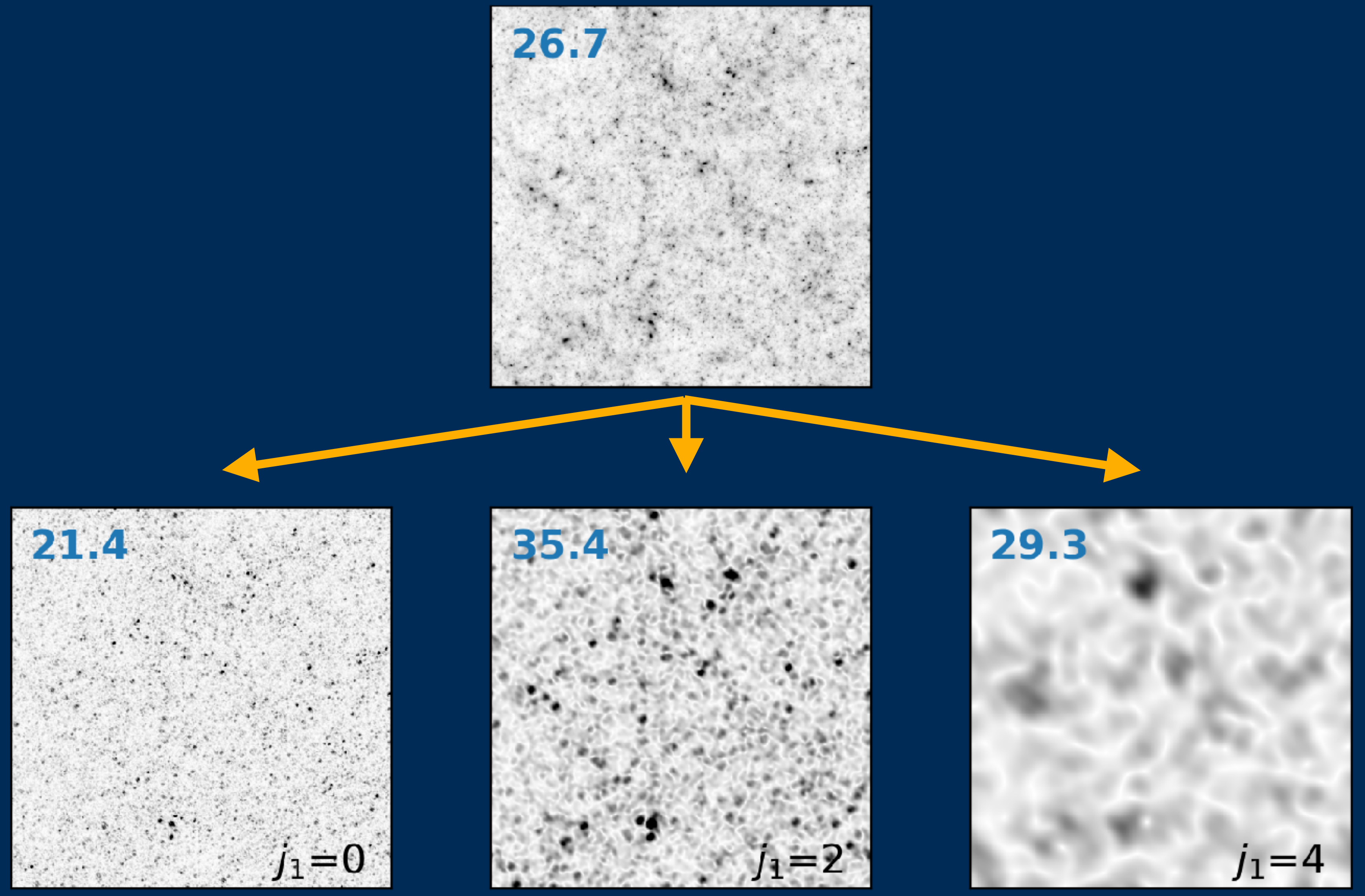
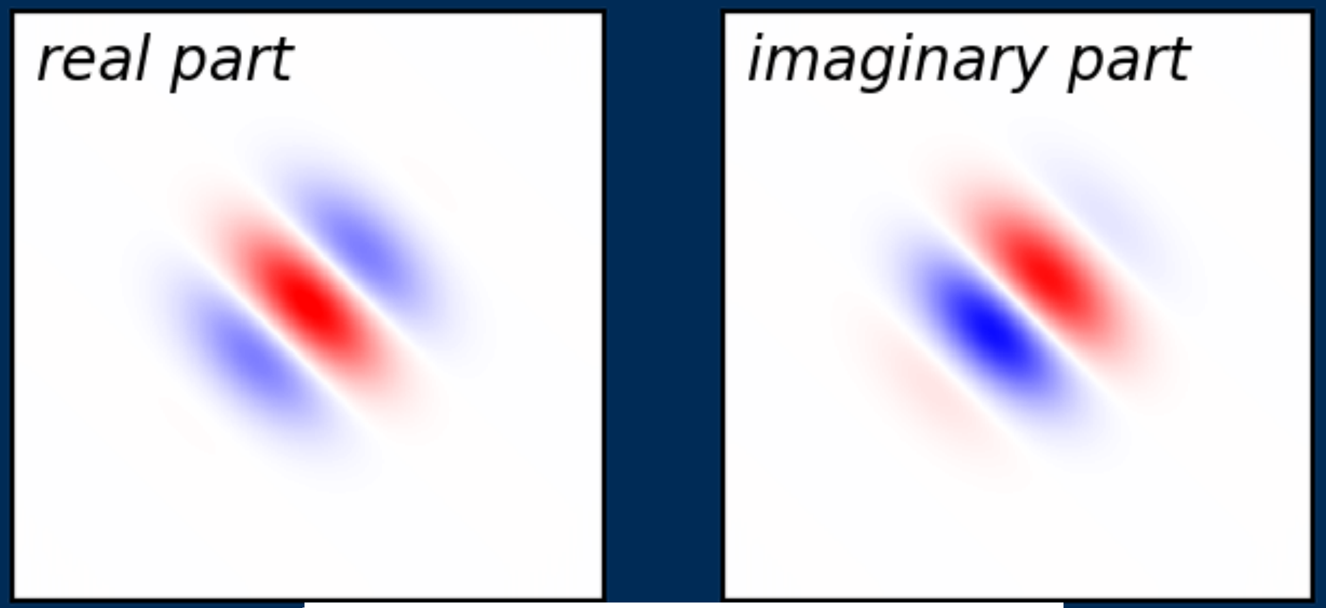
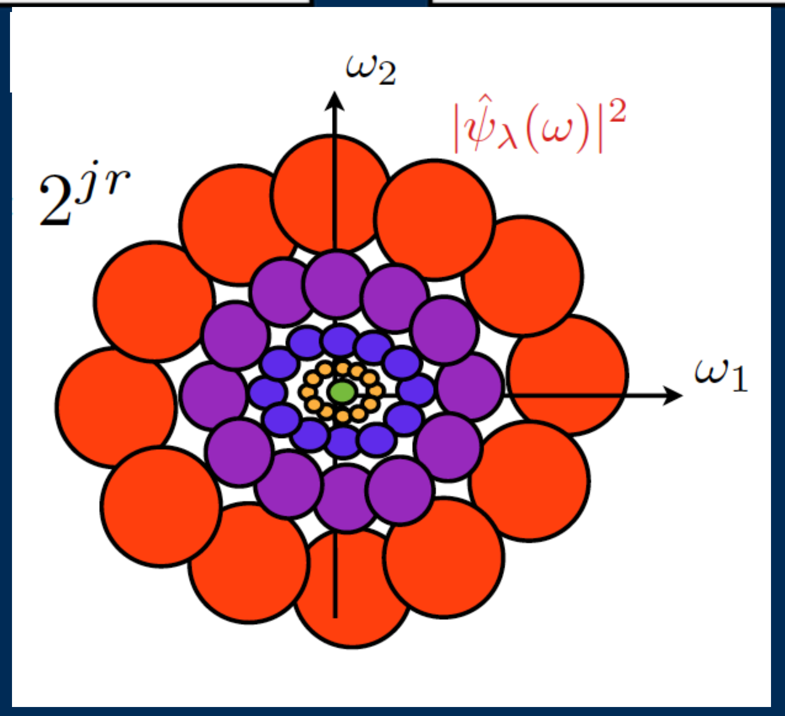


(Morlet) wavelets:

- j: scales
  (logarithmic spacing)
- l: directions

real space





Fourier space

scattering transform

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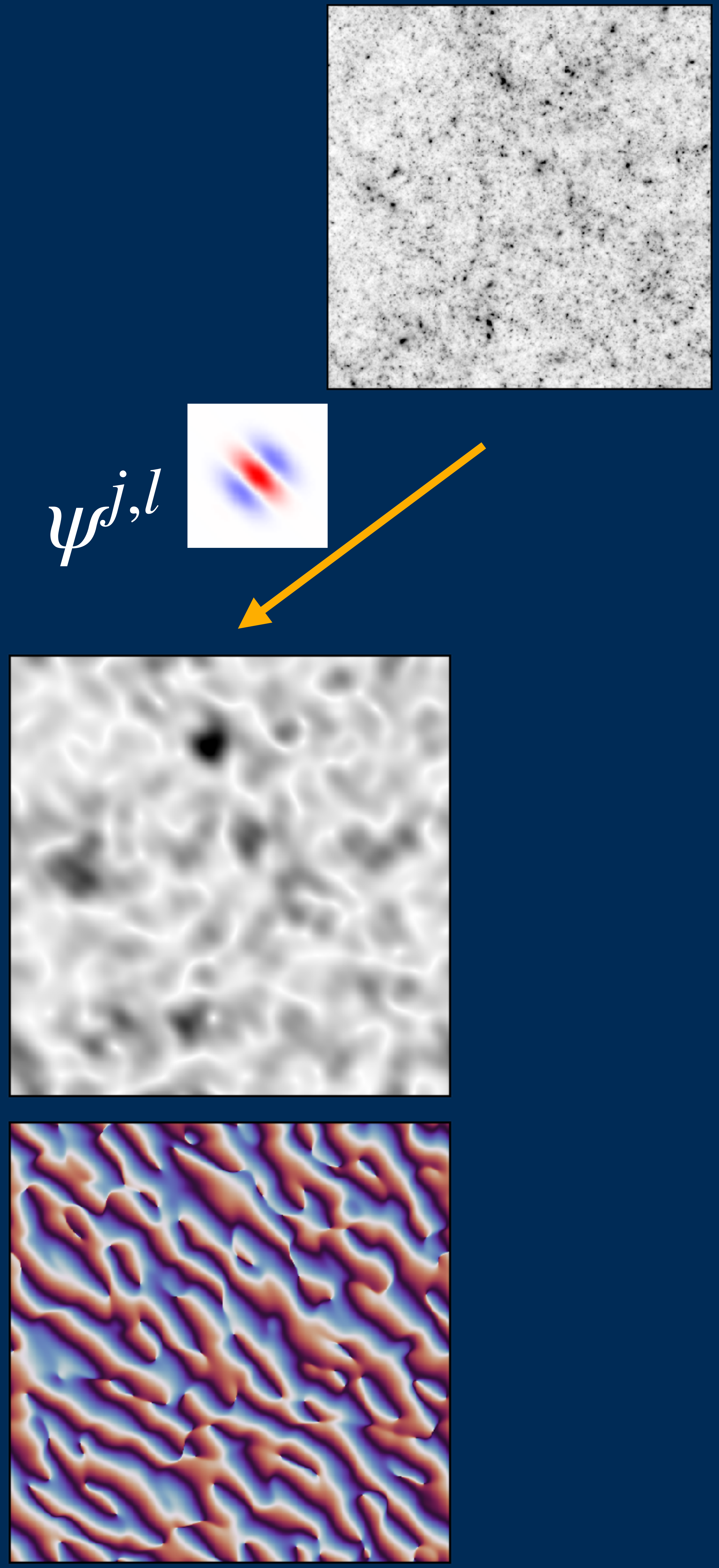
+mean

1. convolution

select a scale

$\psi^{j,l}$



modulus

phase

scattering transform

One operator:

$$UI = |I \star \psi^{j,l}|$$

Scattering transform =

wavelet convolution

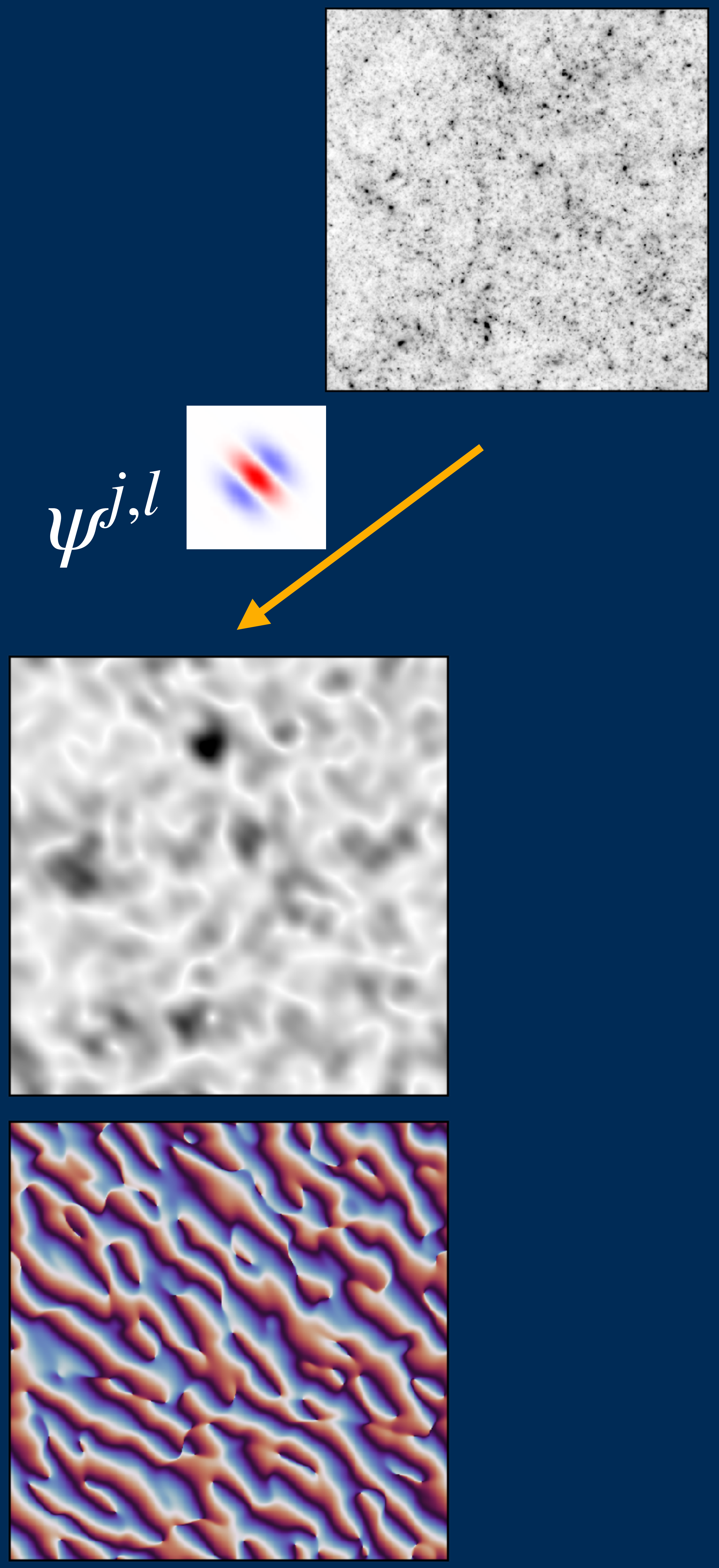
+modulus

+mean

1. convolution
select a scale

2. modulus
local strength

phase



scattering transform

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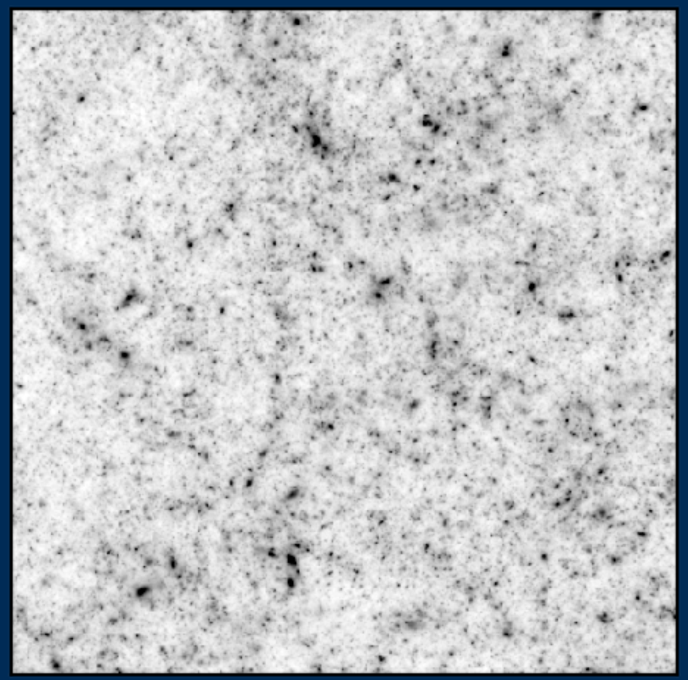
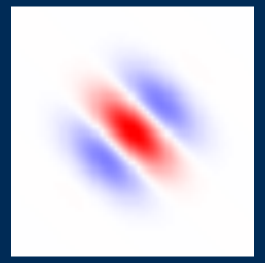


1. convolution

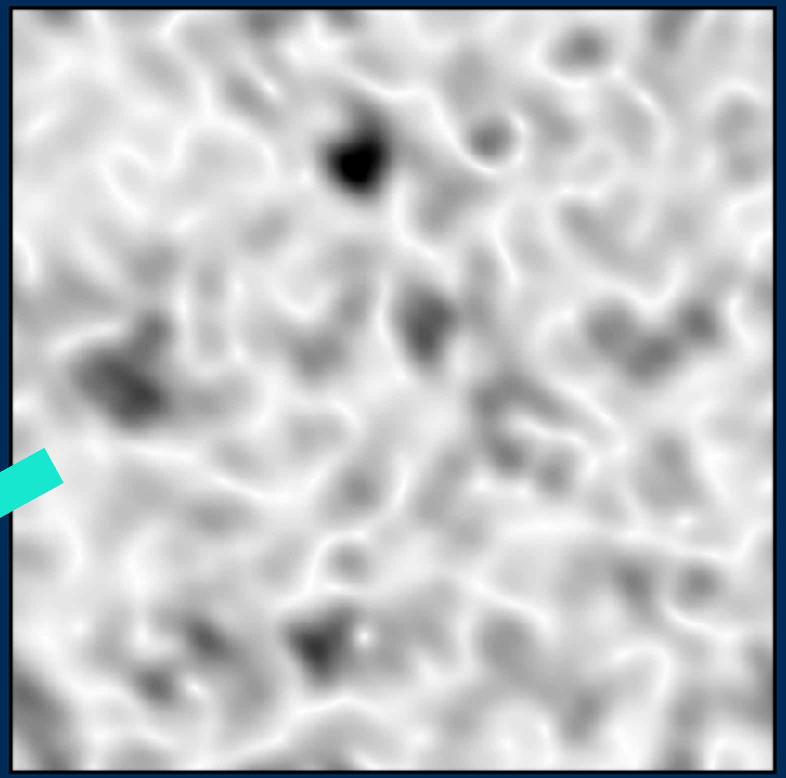
select a scale

$\psi^{j,l}$



2. modulus

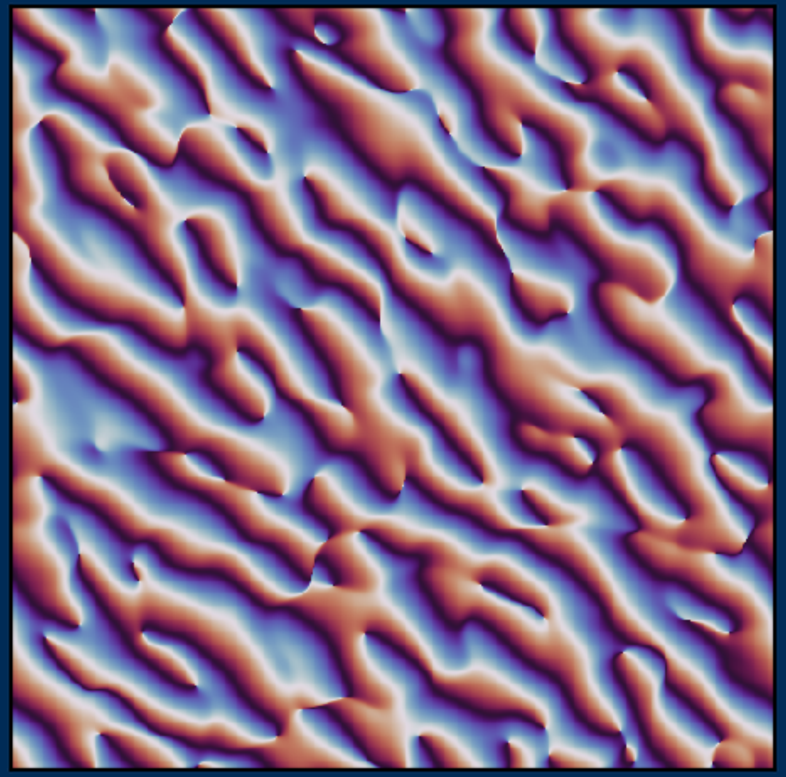
local strength



3. mean

scattering coefficients

phase



scattering vs. power spectrum

One operator:

$$UI = | I \star \psi^{j,l} |$$

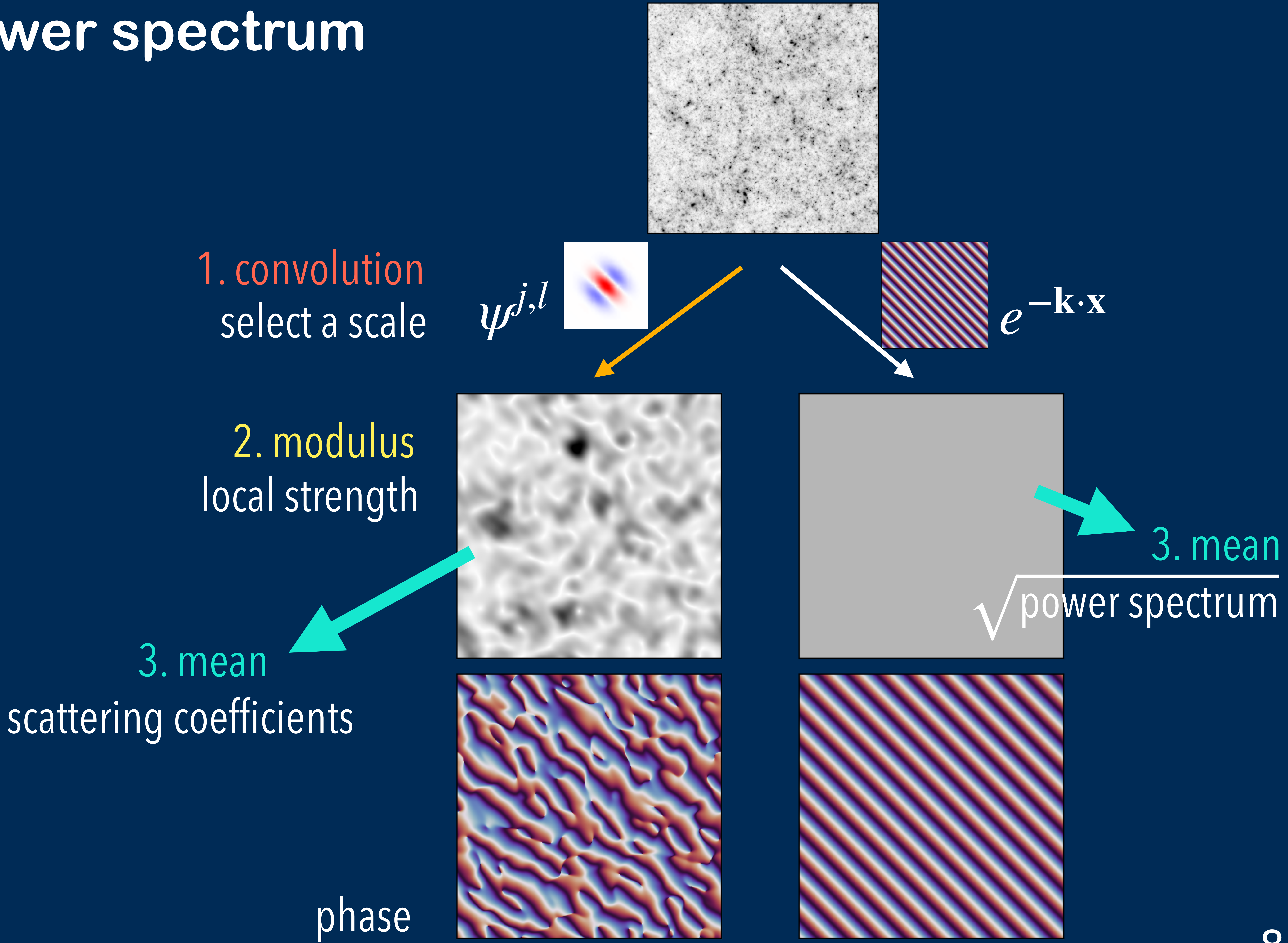
- similarity:

strength of fluctuations

- difference:

1st-order estimator

spatial information



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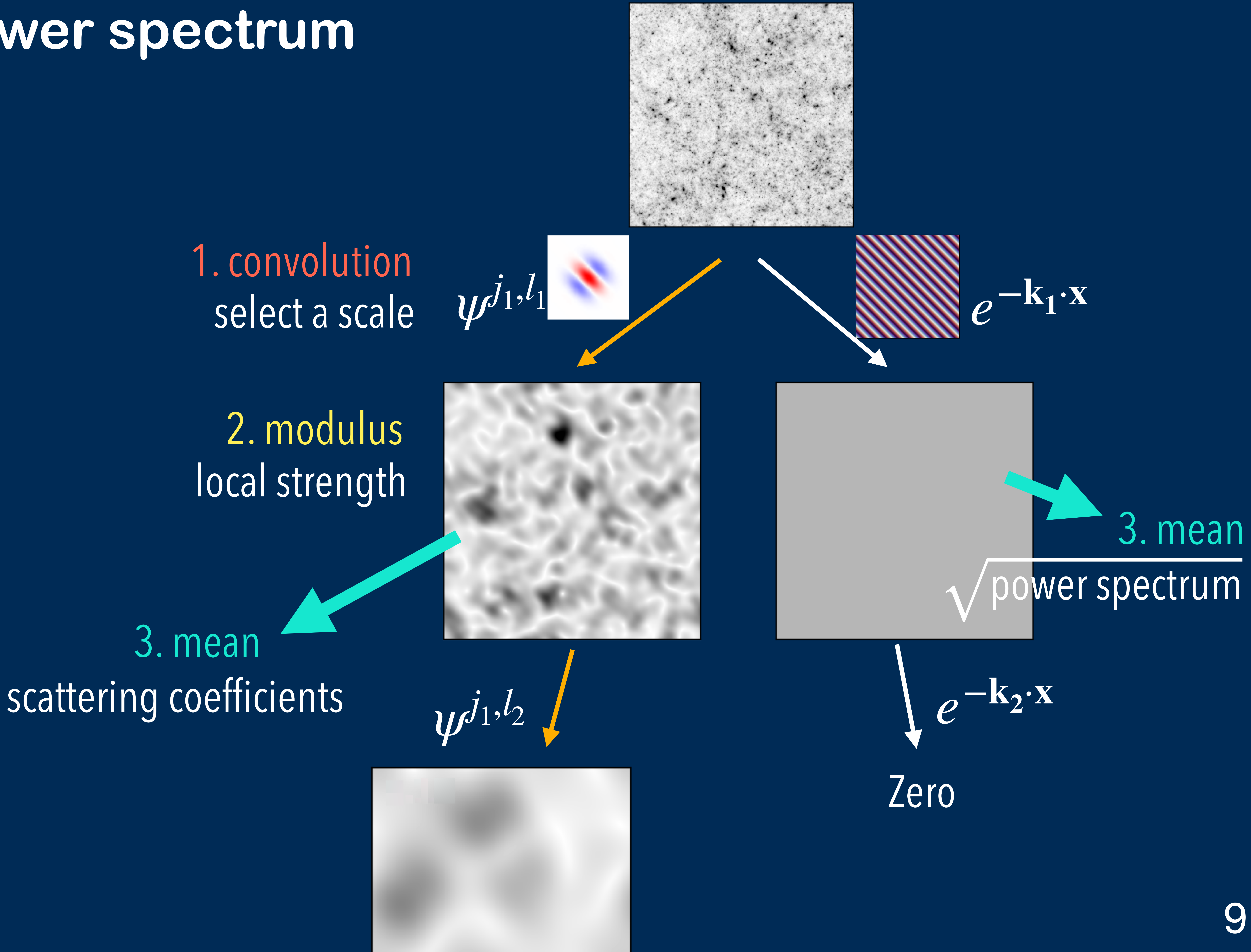


scattering coefficients

- 1st : clustering of particles

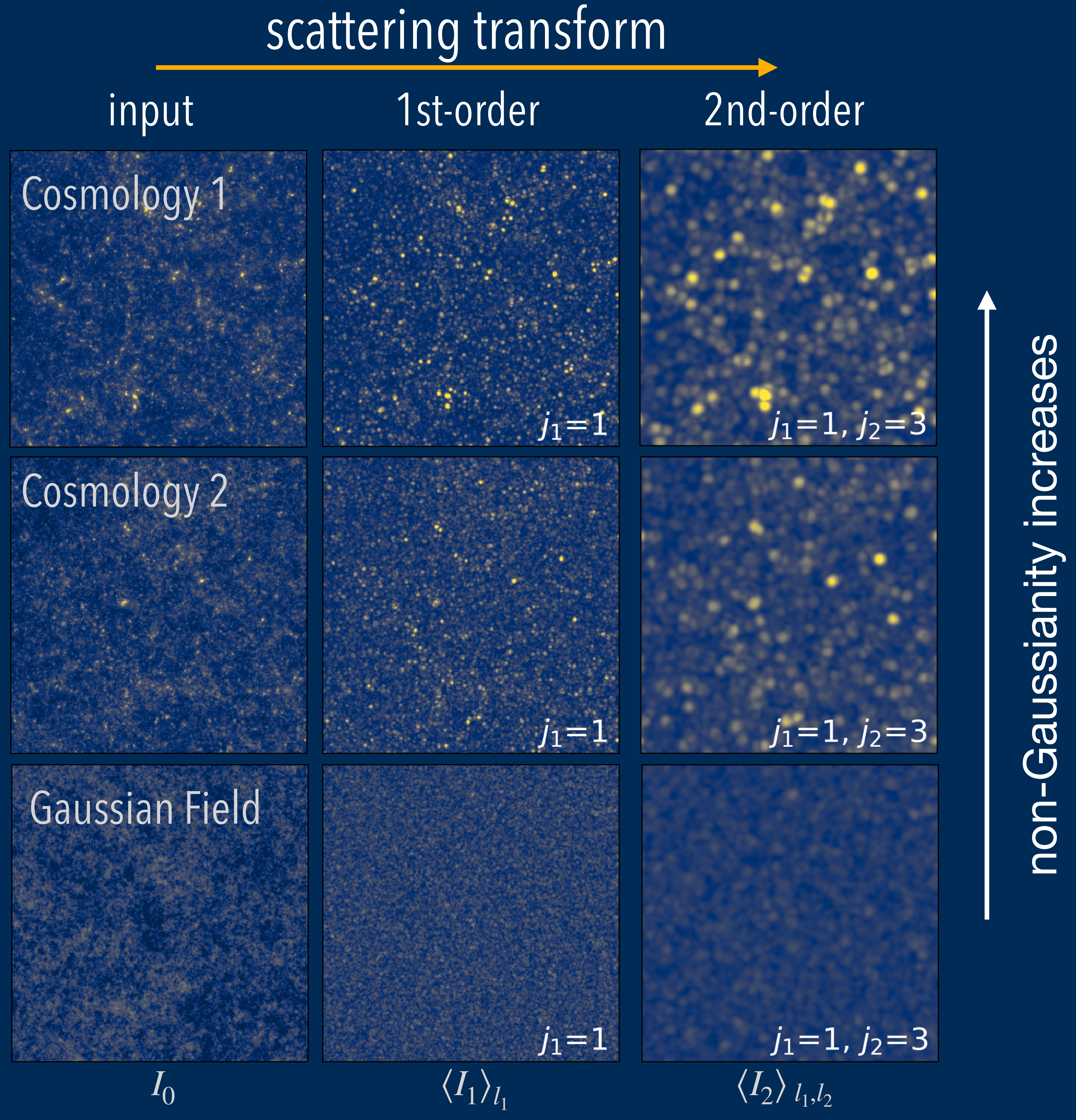
  ~power spectrum

- 2nd: clustering of structures

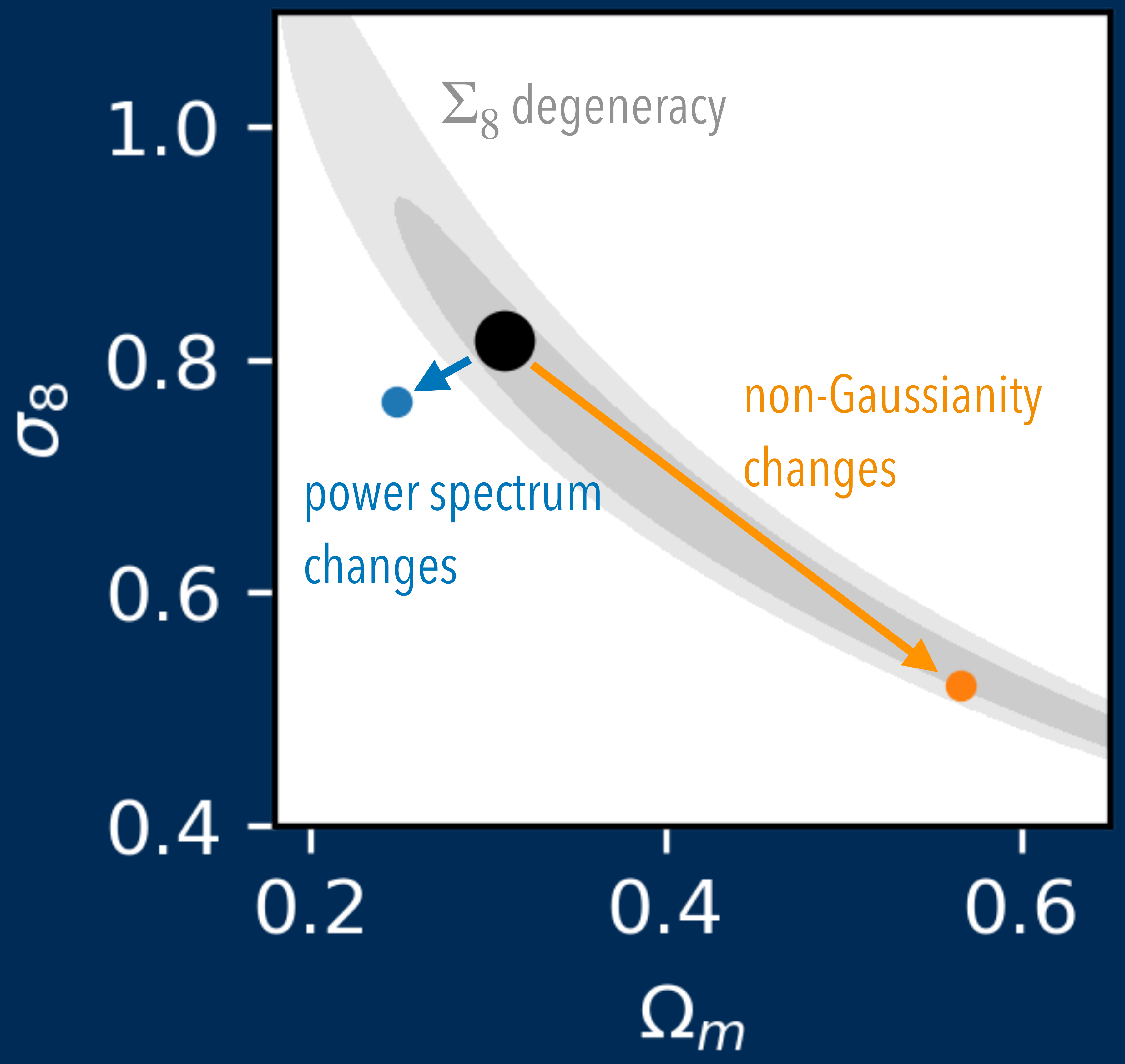
  ~3+4 point functions

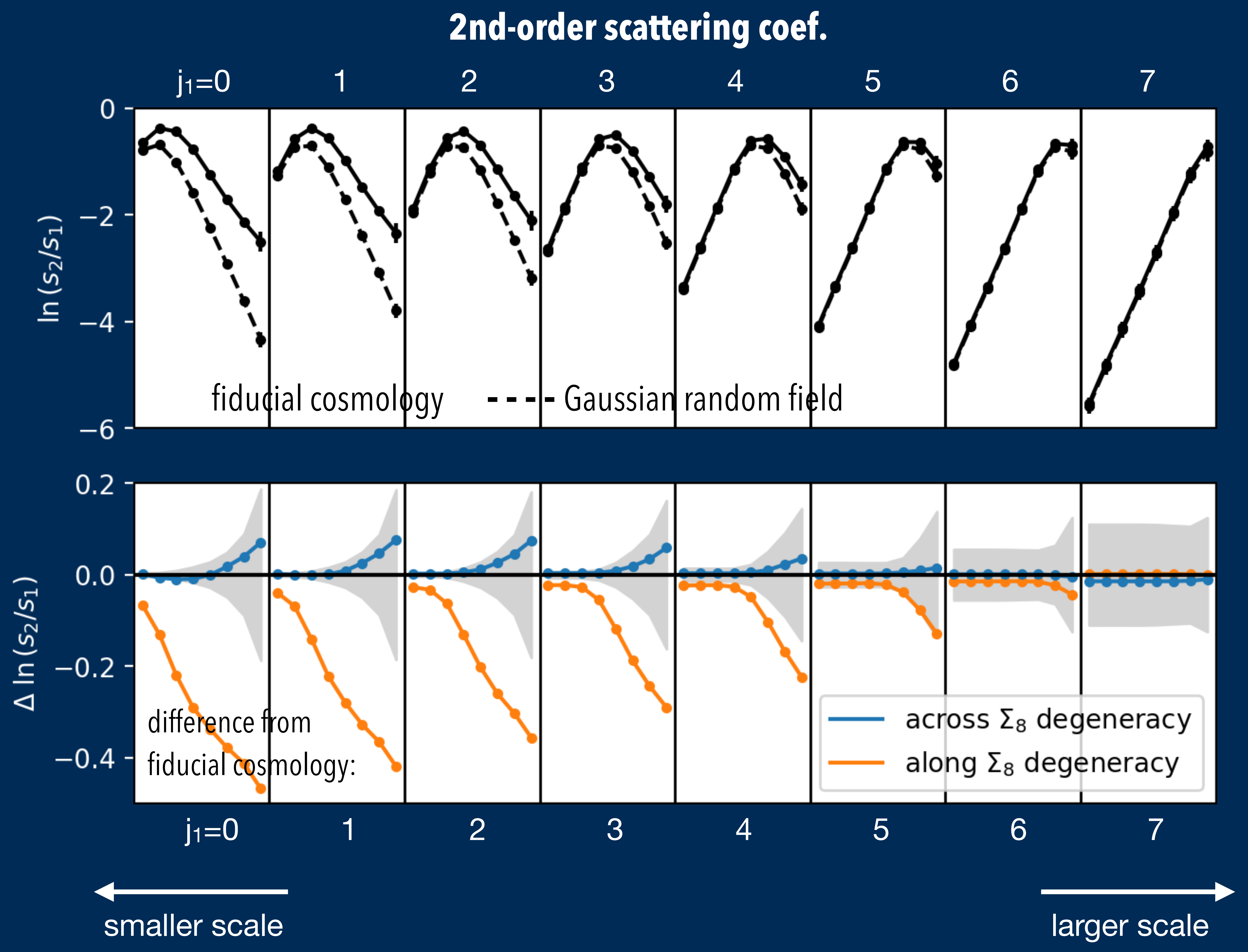
- nth: (clustering)^n

  ~up to 2^n point functions



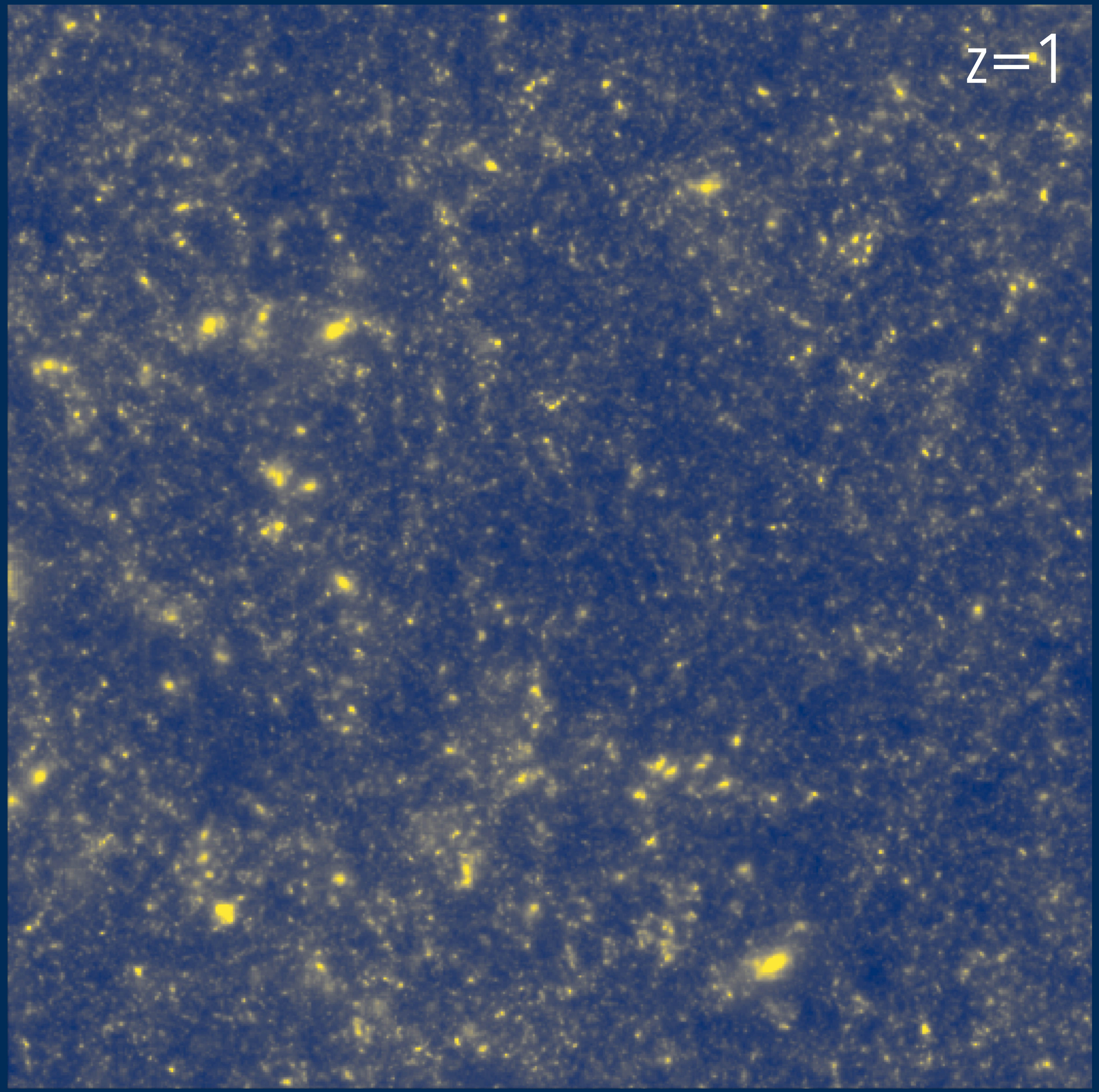
scattering coefficients





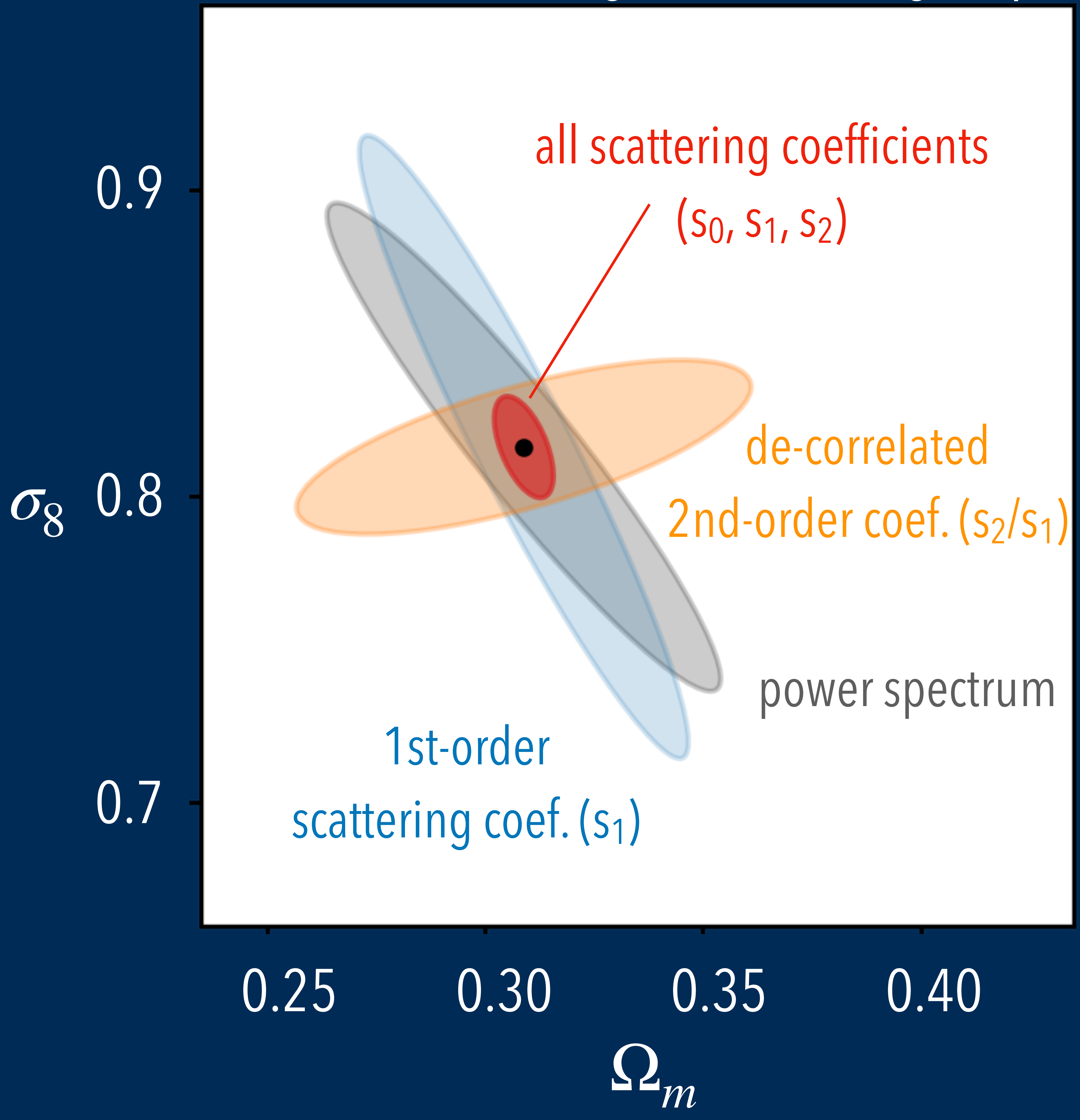
weak lensing application

simulated lensing κ map, ideal



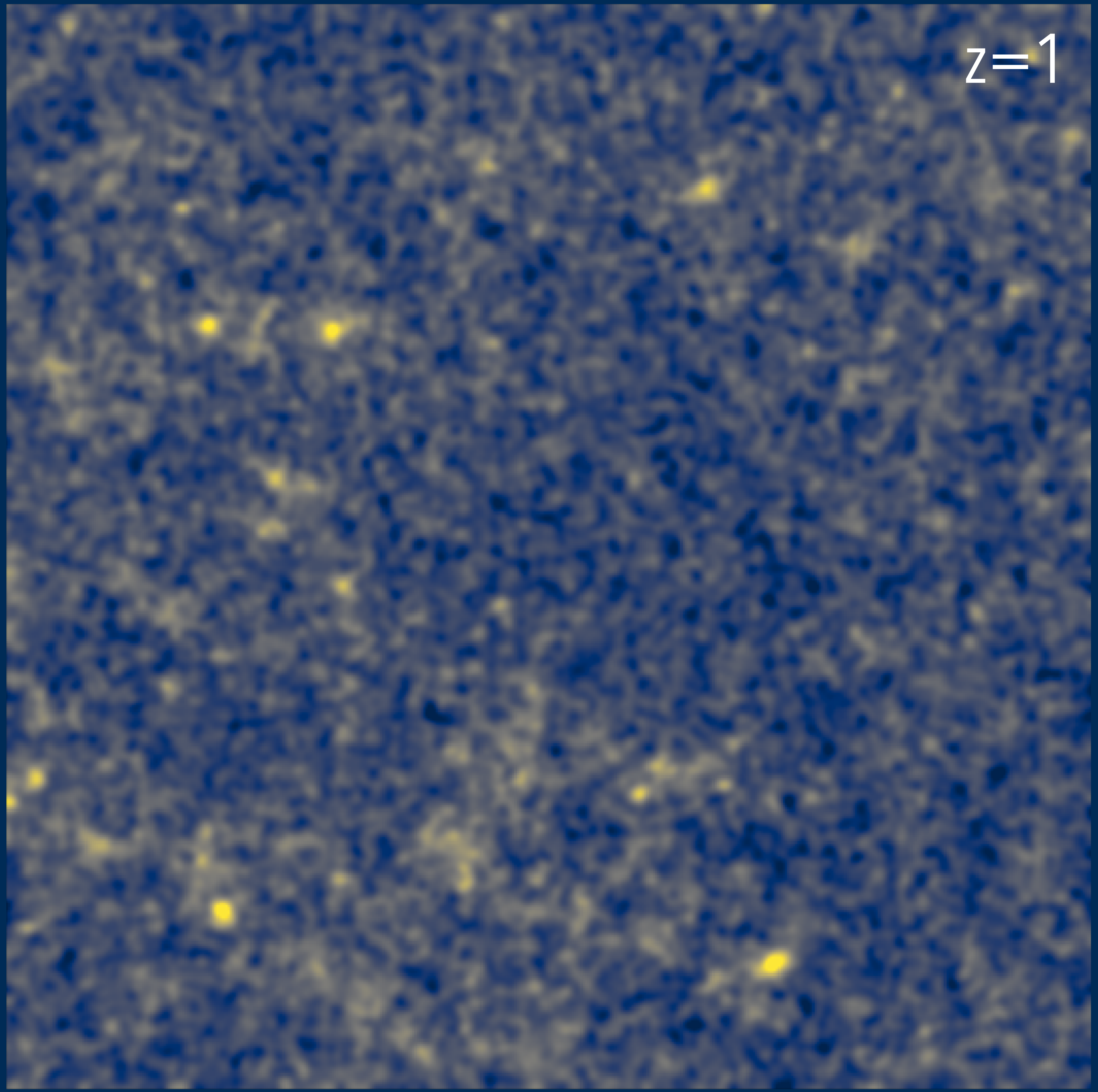
from Matilla et al. 2016 & Gupta et al. 2018

with a 3.5x3.5 deg^2 weak lensing map



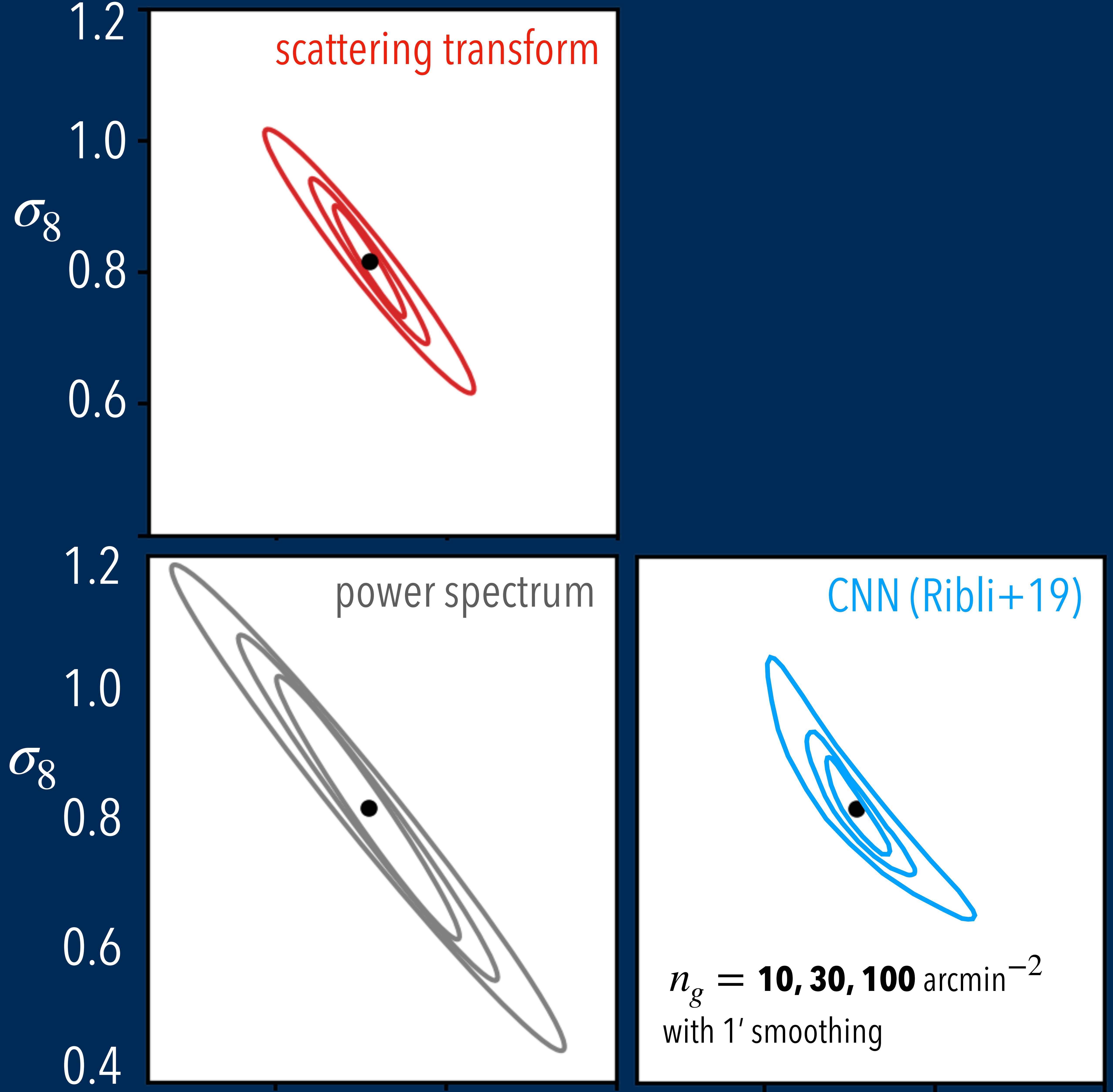
weak lensing application (z=1)

simulated lensing κ map, noisy and smoothed



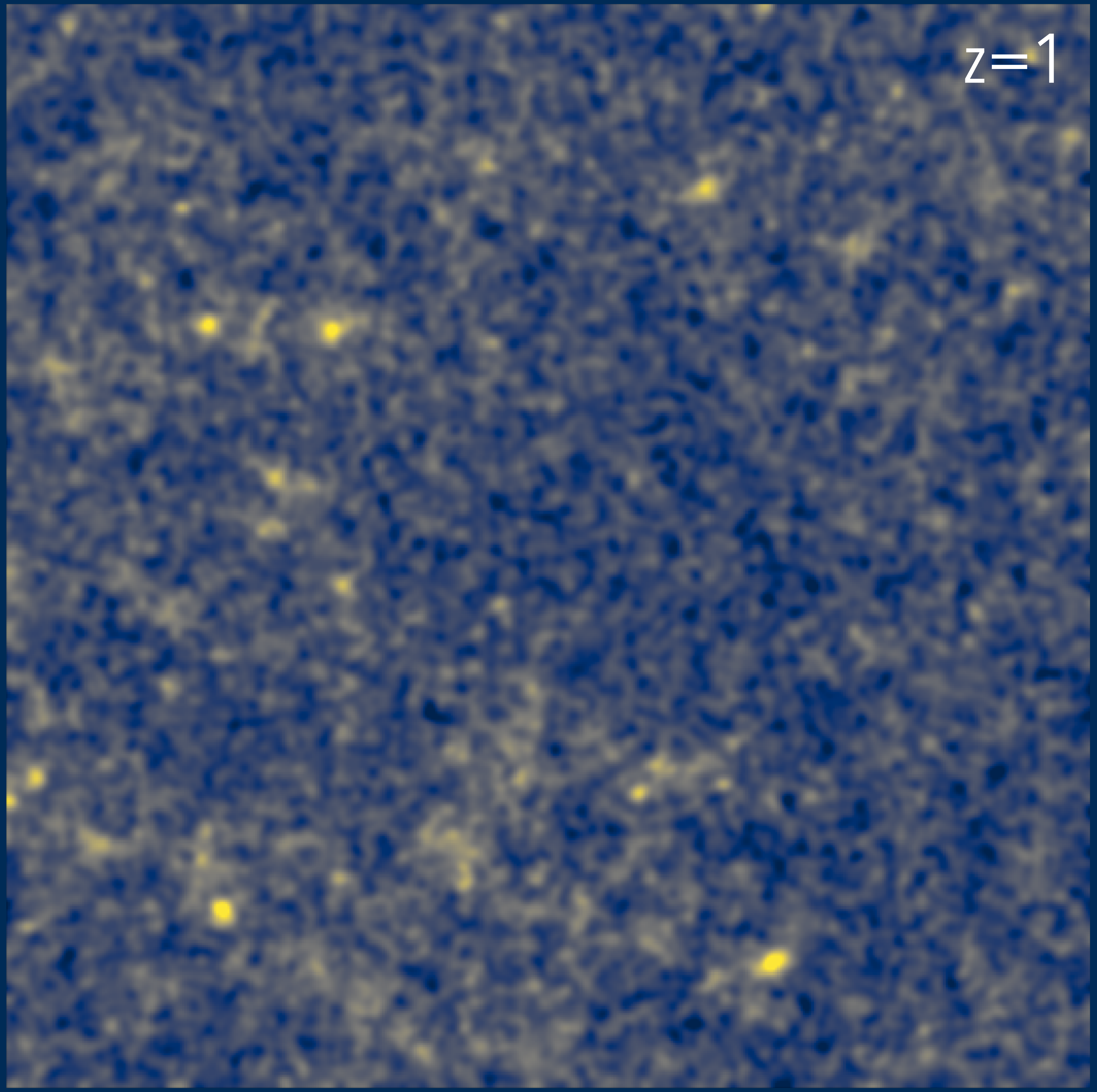
from Matilla et al. 2016 & Gupta et al. 2018

with a 3.5x3.5 deg² weak lensing map



weak lensing application (z=1)

simulated lensing κ map, noisy and smoothed



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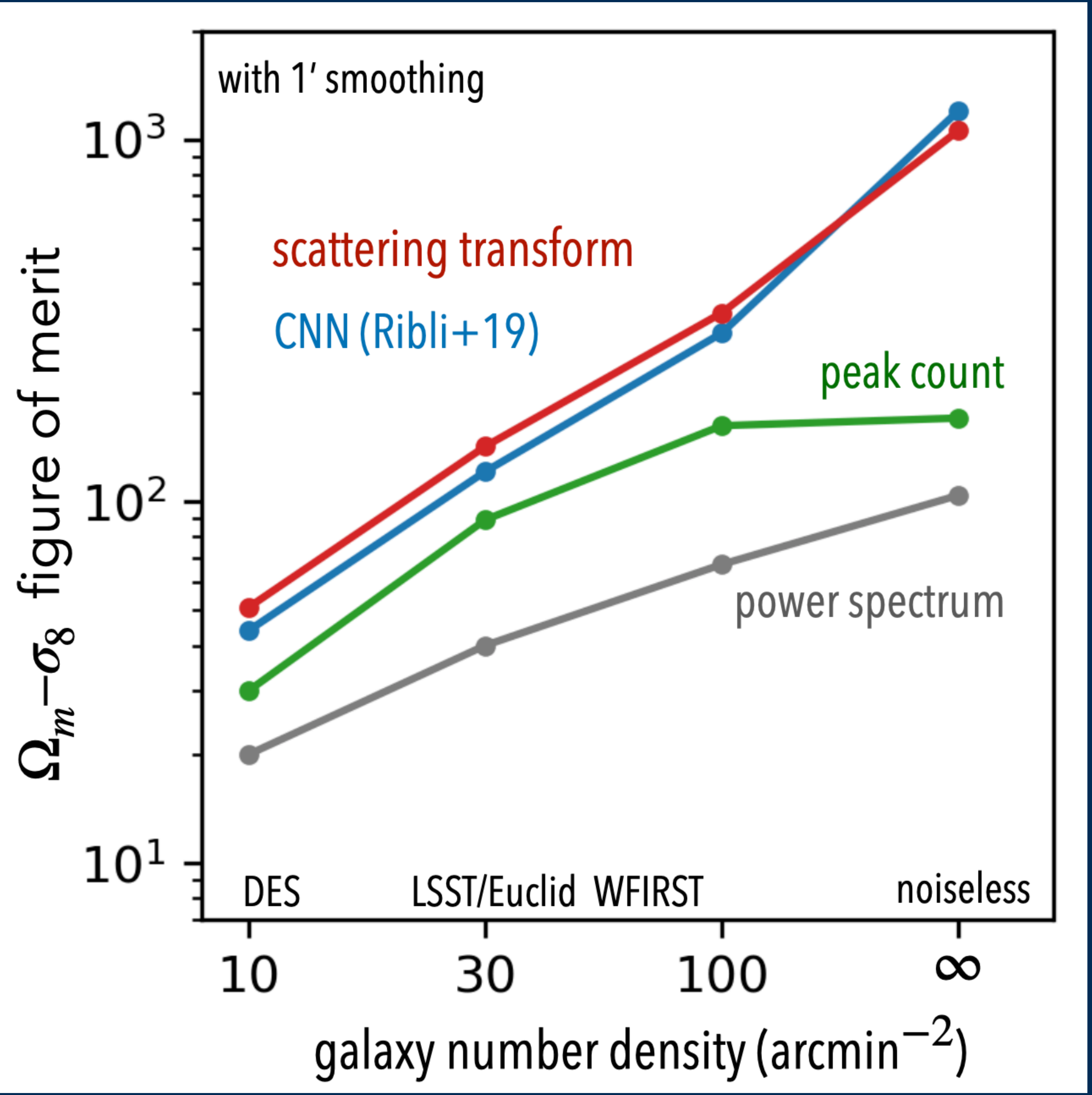

summary

scattering transform =

(wavelet convolution + modulus) × n

+ mean

- efficient (two binning strategies)
- robust ("first-order" property)
- interpretable (clustering of structures)

- weak lensing application:

CNN-level performance

arxiv:2006.08561