On the evolution of cosmological correlation functions

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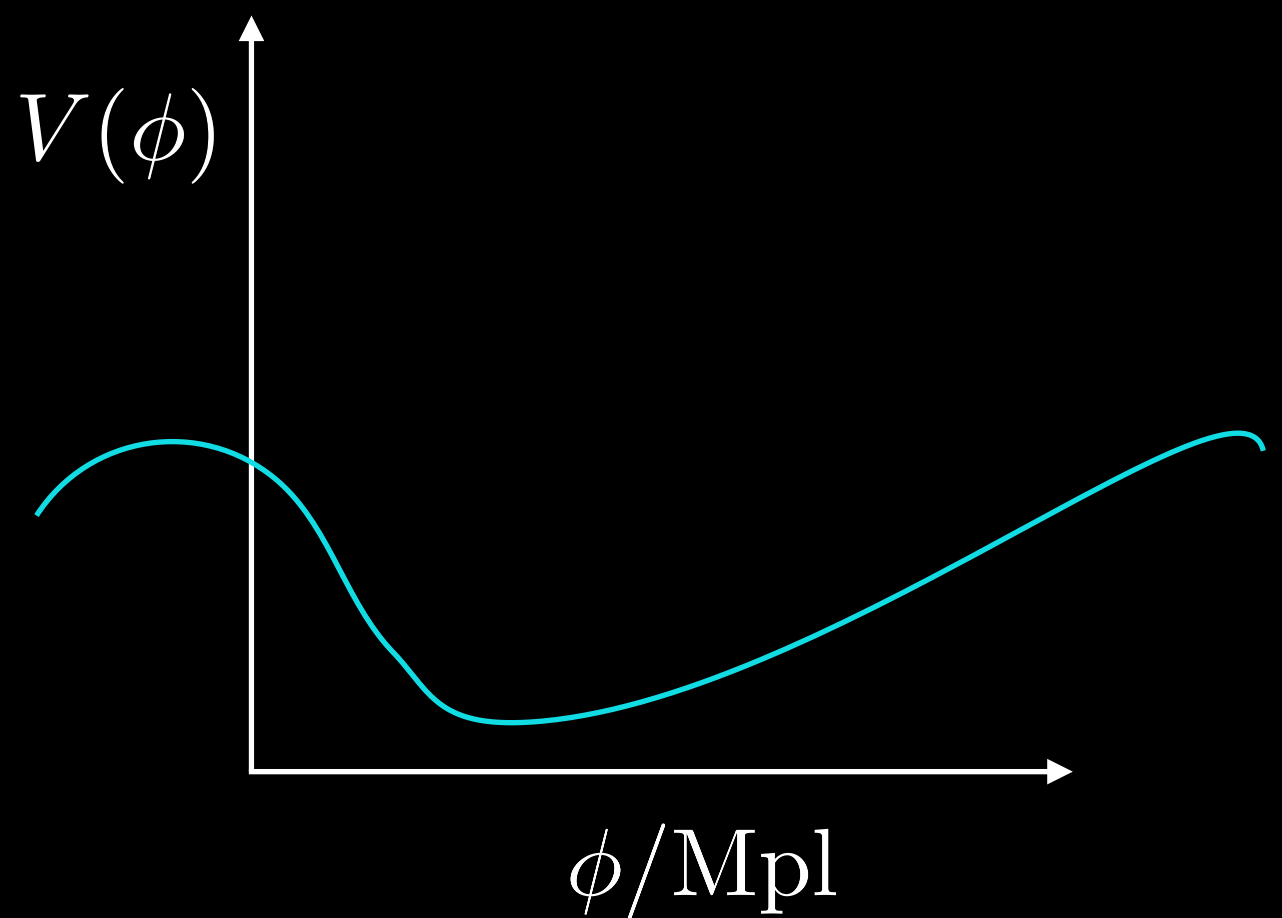
Inflation



$$ds^2 = -dt^2 + e^{2Ht} d\vec{x}^2$$

Quasi de Sitter epoch parametrised by

$$\epsilon \equiv -\frac{\dot{H}}{H^2} = \frac{\dot{\phi}^2}{2H^2} < 1$$

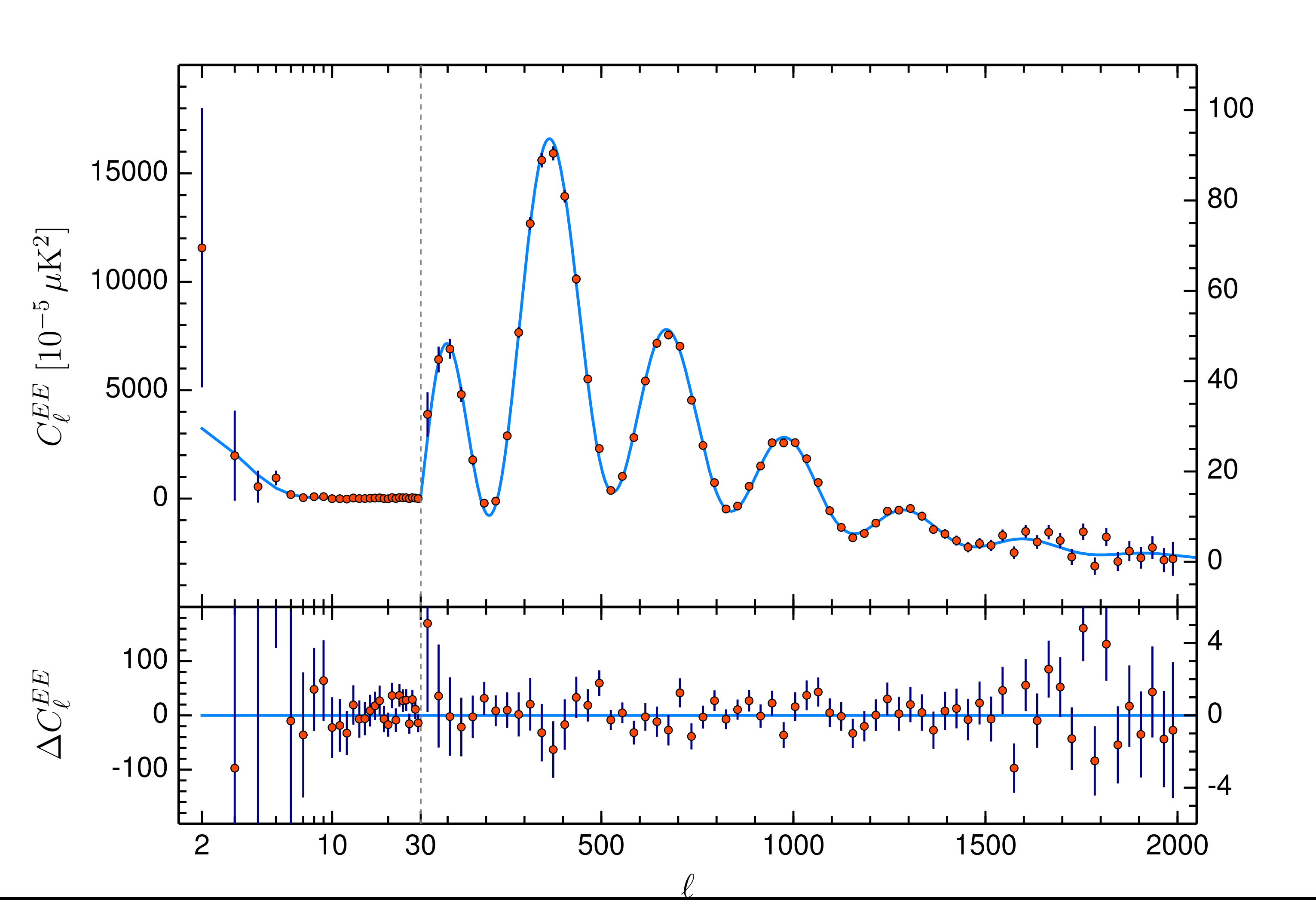
Energy scale of inflation

$$H \ll M_{\mathrm{pl}}$$

Correlation functions

- (Quantum) fluctuations are parametrised by scalar field ζ
- Modes freeze after leaving the horizon
- Correlation functions are highly constrained by de Sitter symmetries.
- 2-points function almost scale invariant.
- Soft limits on higher order correlation functions

Creminelli et al (2012), Hui et al (2012)



Planck (2019)

$$\Delta_\zeta^2 \sim 10^{-9}$$

$$n_s \sim 0.966$$

$$r \equiv \frac{\Delta_t}{\Delta_s} < 0.04$$

$$f_{\rm NL} \propto \mathcal{O}(10)$$

Generalities

- We can only observe modes after they leave the horizon.
- In this talk we will try to understand how much can we learn about sub horizon physics.
- For example what are the consequences of unitarity and analyticity for bulk dynamics?

Wavefunction

- Given a quantum state $\Psi[\phi]$ defined on a given time slice

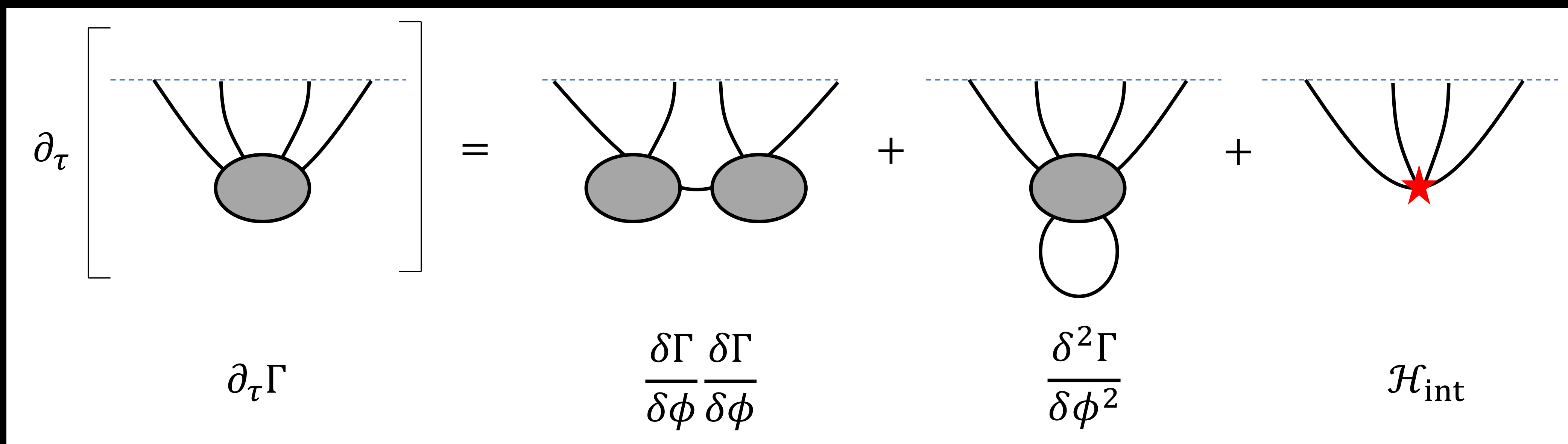
- Its evolution is given by Schroedinger equation
$$i\partial_\eta \Psi = \mathcal{H}[\phi]$$

- Considering isotropic states. $\Psi[\phi] = e^{i\Gamma[\phi]}$
Schroedinger equation reduces to Hamilton Jacobi

- For a weakly coupled theory we can consider states such that,

$$\Gamma[\phi] = \frac{i}{2}\int_{\mathbf{k}_1\mathbf{k}_2} c_{\mathbf{k}_1\mathbf{k}_2}(\eta)\phi_{\mathbf{k}_1}\phi_{\mathbf{k}_2} + \frac{i}{3!}\int_{\mathbf{k}_1\mathbf{k}_2\mathbf{k}_3} c_{\mathbf{k}_1\mathbf{k}_2\mathbf{k}_3}(\eta)\phi_{\mathbf{k}_1}\phi_{\mathbf{k}_2}\phi_{\mathbf{k}_3} + ..$$

- Hamilton-Jacobi equation implies an equation for each coefficient depending on lower order ones.



∂_τ
=
+
+
$\partial_\tau \Gamma$
$\frac{\delta\Gamma}{\delta\phi}\frac{\delta\Gamma}{\delta\phi}$
$\frac{\delta^2\Gamma}{\delta\phi^2}$
$\mathcal{H}_{\text{int}}$

Correlation functions

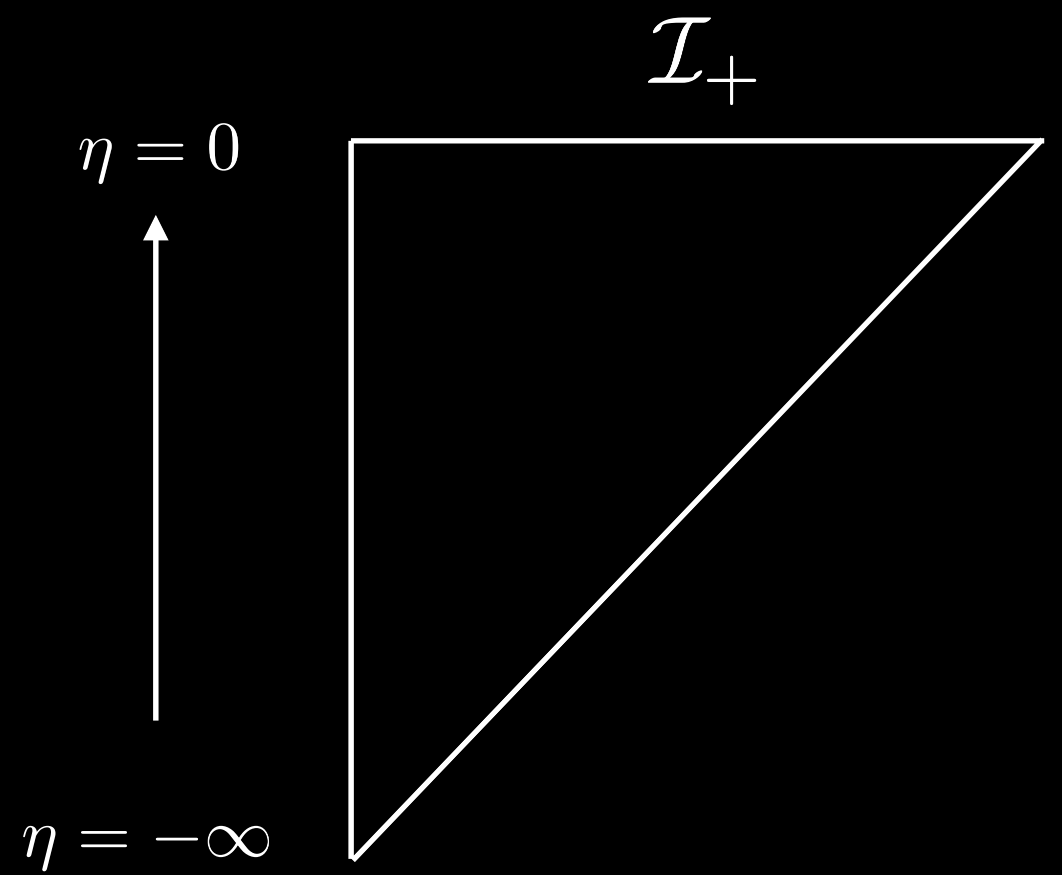
- Coefficients are computed by solving a 1st order ODE
  - It can be expressed as integral.
  - Initial conditions are given by defining a slice a some particular time.
  - It is also necessary to define the vacuum of the theory. This might be related to the initial conditions but not necessarily
- Correlation functions are given by

$$\langle \phi_{\mathbf{k}_1} \phi_{\mathbf{k}_2} \rangle = \frac{1}{2\,\mathrm{Im}\, c_{\mathbf{k}_1 \mathbf{k}_2}} \delta^3(\mathbf{k}_1 + \mathbf{k}_2)$$

- $$\langle \phi_{\mathbf{k}_1} \phi_{\mathbf{k}_2} \phi_{\mathbf{k}_3} \rangle = -\frac{\mathrm{Im}\, c_{\mathbf{k}_1 \mathbf{k}_2 \mathbf{k}_3}}{4\,\mathrm{Im}\, c_{\mathbf{k}_1,-\mathbf{k}_1} \mathrm{Im}\, c_{\mathbf{k}_2,-\mathbf{k}_2} \mathrm{Im}\, c_{\mathbf{k}_3,-\mathbf{k}_3}} \, \delta^3(\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3)\,.$$

de Sitter spacetime

$$ds^2 = \frac{1}{H^2\eta^2}(d\eta^2 - d\vec{x}^2)$$



Isometries

- Translations
- Rotations
- Dilatations $-\eta\partial_\eta - x^i\partial_{x^i}$
- $-2(\vec{b}\cdot\vec{x})\eta\partial_\eta - 2(\vec{b}\cdot\vec{x})\vec{x}\partial_{\vec{x}} - (\eta^2 - |\vec{x}|^2)\vec{b}\cdot\partial_{\vec{x}}$

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$\eta \to 0$ ⟶

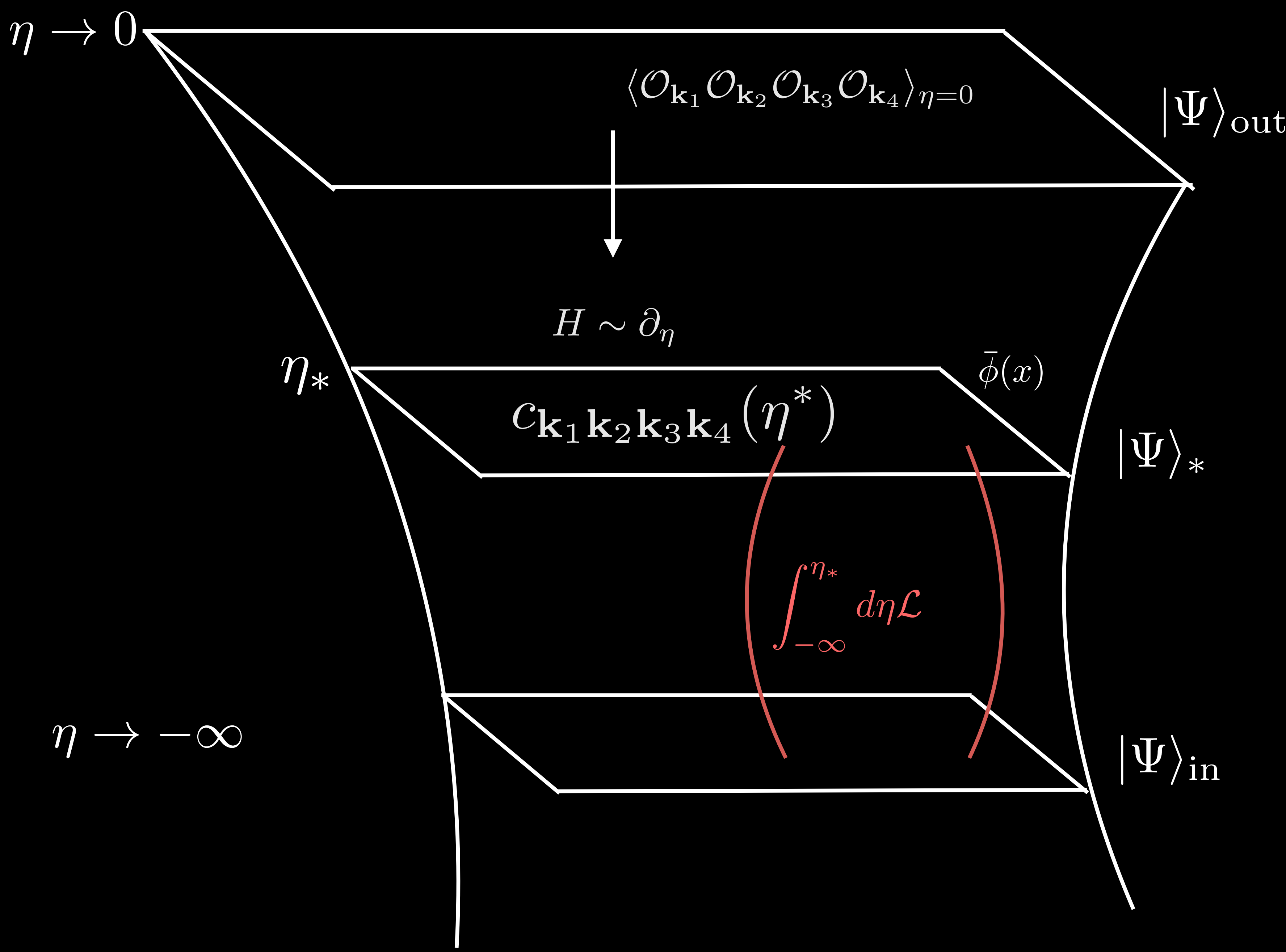
Conformal symmetry

$-\Delta - \vec{x}\cdot\partial_{\vec{x}}$

SCT $-2\Delta(\vec{b}\cdot\vec{x}) - 2(\vec{b}\cdot\vec{x})\vec{x}\partial_{\vec{x}} + |\vec{x}|^2\vec{b}\cdot\partial_{\vec{x}}$

Conformal bootstrap

Arkani-Hamed et al. (2016,2018), Baumann et al (2019,2020)



Conformal invariance

$f(k) \sim \eta^{\pm\Delta}$

$f(k) \sim e^{\pm ik\eta}$

Bunch-Davies

Although other vacua are also allowed

Inside the bulk

- De Sitter isometries need to be respected
  - Dilatation $\hat{D} = -x^\mu \partial_\mu$
  - Boosts $\hat{K} = 2x_i x^\nu \partial_\nu - x^2 \partial_i$
- Interactions term can be neglected at $\eta \to 0$
  - Universality of boundary terms.
- Initial vacuum determines the value of the wave function coefficients.

Superhorizon scales

- Cosmological bootstrap fixes the value at $\eta \to 0$

$$\alpha(\mathbf{k}_1, ..\mathbf{k}_n)$$

Coefficients can depend on ratios and be non analytical.

- We can make an expansion to small η

$$c_{\mathbf{k}-\mathbf{k}}(\eta) = \frac{c^{\text{series}}_{\mathbf{k}-\mathbf{k}}(\eta)}{(-\eta)^d} + \frac{\alpha_k}{(-\eta)^{2\Delta_-}} c^{\text{initial}}_{\mathbf{k}-\mathbf{k}}(\eta)$$

- Both. $c^{\text{initial}}_{\mathbf{k}-\mathbf{k}}$ and $c^{\text{series}}_{\mathbf{k}-\mathbf{k}}$ are analytical in $\mathbf{k}$ and η

- Sources of non-analiticity can come only from initial conditions.
- These can be for example non Bunch-Davies initial conditions or excited initial states.
- Coefficients can diverge at $\eta \to 0$, in which case it is necessary to add counterterms to renormalise

Conformally coupled field

- Let us study the case of $m^2 = 2H^2$ in more detail

$$\langle \sigma_{\mathbf{k}} \sigma_{-\mathbf{k}} \rangle'(\eta) = -\frac{H^2\eta^2}{2k} \frac{1+\alpha_{\mathbf{k}}^2 + 2\alpha_{\mathbf{k}} \cos(2k\eta)}{1-\alpha_{\mathbf{k}}^2}$$

$$\langle \Sigma_{\mathbf{k}} \sigma_{-\mathbf{k}} \rangle'(\eta) = -\frac{1}{2k\eta} \frac{1+\alpha_{\mathbf{k}}^2 + 2\alpha_{\mathbf{k}} \cos(2k\eta) - 2\alpha_{\mathbf{k}} k\eta \sin(2k\eta)}{1-\alpha_{\mathbf{k}}^2}$$

- Requiring dilation invariance fix α_k to be scale independent

- Cubic coefficient.

$$c_{\mathbf{k}_1\mathbf{k}_2\mathbf{k}_3}(\eta) = \frac{\lambda e^{i\sum_J k_J\eta}}{H^3\eta^3}\left(\mathrm{Ei}\left(-i\sum_J k_J\eta\right) + \alpha_3(k_J)\right)$$

  - Requiring boost invariance implies α_3 is scale independent.
  - Coefficient diverges in the limit
  - It can be renormalised by adding local counterterms to the boundary action

$$\langle\mathcal{O}_{\mathbf{k}_1}\mathcal{O}_{\mathbf{k}_2}\mathcal{O}_{\mathbf{k}_3}\rangle_{\mathrm{ren}}(\mu) = \lambda z_3 + \frac{\lambda}{2}\left[\log\left(\frac{k_{123}}{\mu}\right)\right],$$

  - Providing only the value at $\eta \to 0$ implies

$$\alpha_3(\mu) = \frac{\langle\mathcal{O}_{\mathbf{k}_1}\mathcal{O}_{\mathbf{k}_2}\mathcal{O}_{\mathbf{k}_3}\rangle'_{\mathrm{ren}}}{\lambda} - \log\left(\frac{k_{123}}{\mu}\right).$$

Bwzowski et al. (2016)

EFT of inflation

- We can translate this into a more realistic setting by using the EFT of inflation. *Creminelli et al. (2006)*

- Past results holds as long as we consider the decoupling limit.

- Broken de Sitter invariance implies less constraints over initial states

Conclusions

- We have study the wave function evolution through the Schroedinger equation.
- Through general requirement is possible to constraint its evolution at all times
- Consistency requirement fix the initial answer
- It can be generalised to more realistic cases but with less constraints.