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Universality of the Cold Dark Matter Power Spectrum

Co-workers:

Matthias Bartelmann, Elena Kozlikin, Carsten Littek, et al.

Result*: $P_\delta(k) \sim k^{-3}$ for $k \to \infty$

* paper in prep.

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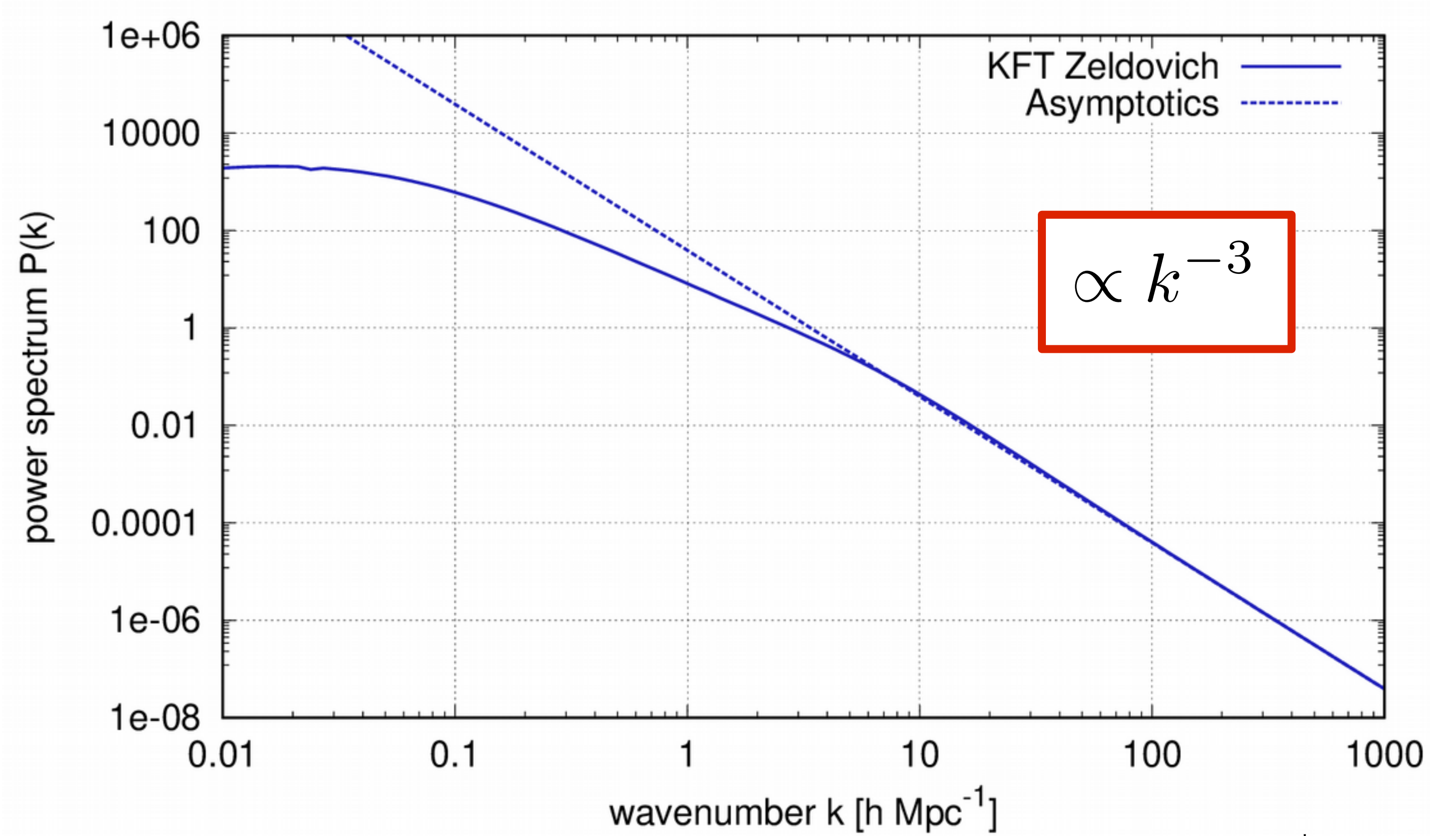

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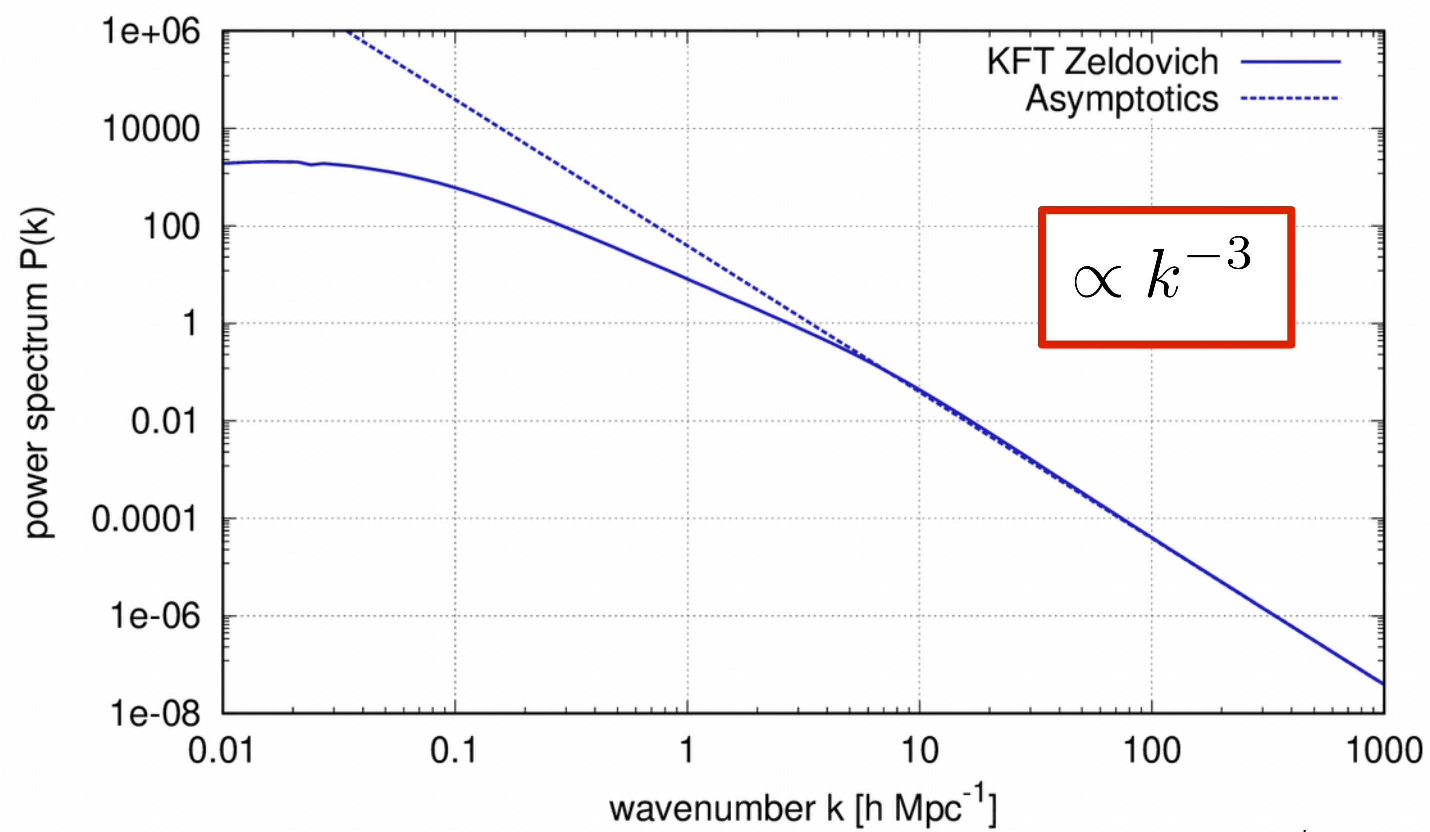

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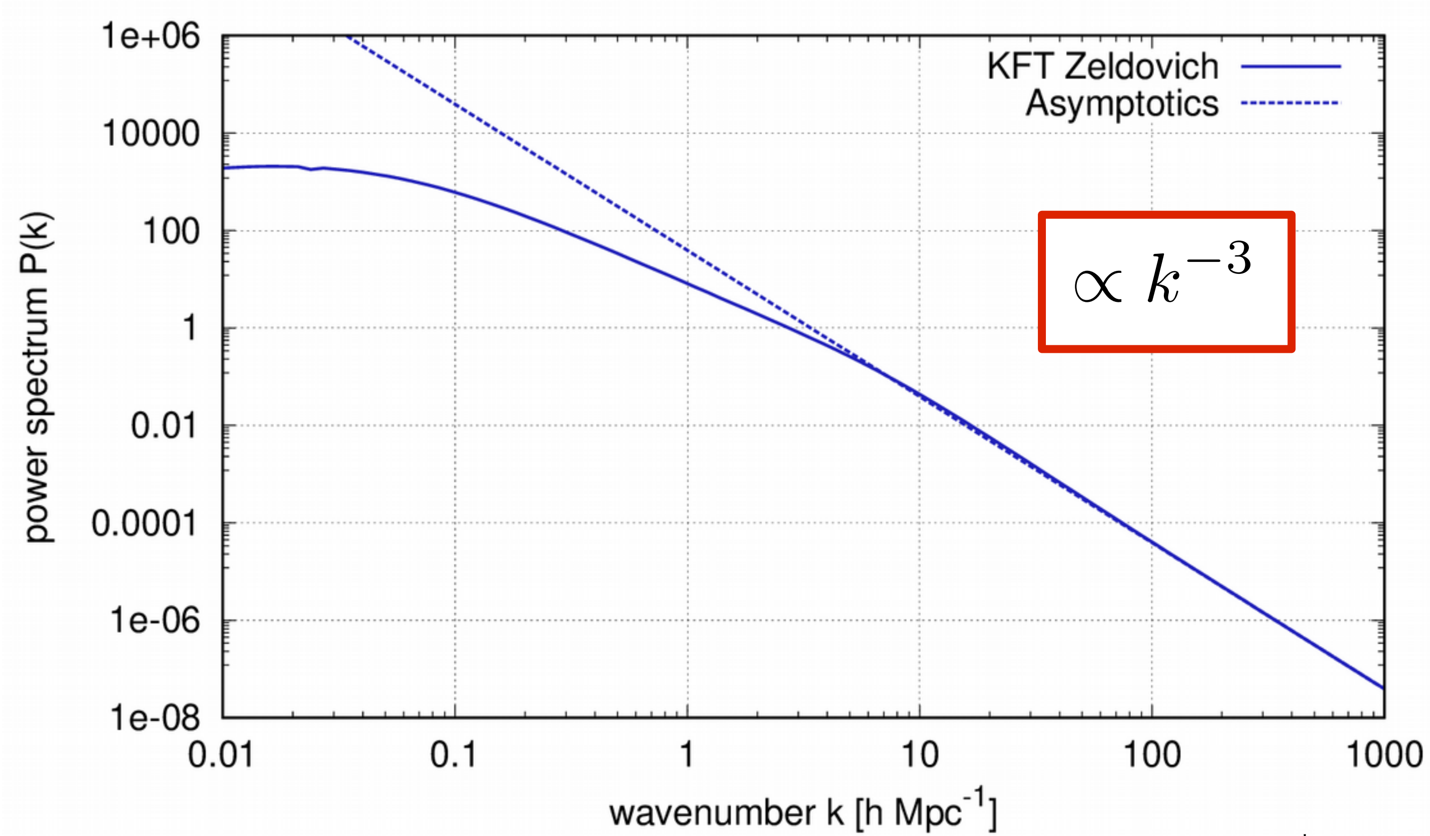
Result*: $P_\delta(k) \sim k^{-3}$ for $k \to \infty$

- Arises **before** re-expansion of structures
- Persists **after** onset of re-expansion
- Independent on the **shape** of the initial power spectrum



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If you have not seen our talk
Kinetic Field Theory: An Introduction
I recommend to watch this first,
before continuing

Zel'dovich trajectories

- Density fluctuation power spectrum in KFT

$$\mathcal{P}_\delta(k,t) = \hat{\rho}\hat{\rho}\left(1 + \mathrm{i}\hat{S}_I + ...\right) Z_0\left[\mathbf{J},\mathbf{K}\right]_{\mathbf{J}=0=\mathbf{K}}$$

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$$\vec{q}(t) = \vec{q}^{(\mathrm{i})} + g_{qp}(t,0)\vec{p}^{(\mathrm{i})} + \int_0^t \mathrm{d}t' g_{qp}(t,t')\vec{f}(t')$$

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- Linear growth on large scales

$$g_{qp}(t,t') = D_+(t) - D_+(t')$$

linear growth factor

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- Gaussian initial momentum correlations

$$\hat{C}_{pp}(\Delta\vec{q}) := \langle \vec{p}(\vec{q}) \otimes \vec{p}(\vec{q}+\Delta\vec{q})\rangle = \int_k \frac{\vec{k}\otimes\vec{k}}{k^4} P_\delta^{(\mathrm{i})}(k)\mathrm{e}^{\mathrm{i}\vec{k}\Delta\vec{q}}$$

initial density fluctuation power spectrum

CDM Power spectrum

General expression

$$\mathcal{P}_\delta(k,t) = \left\langle \mathrm{e}^{-\mathrm{i}\vec{k}\cdot[\vec{q}_1(t)-\vec{q}_2(t)]} \right\rangle$$

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For particles on straight Zel'dovich trajectories

$$\mathcal{P}_\delta(k,t) = \mathrm{e}^{-\frac{\sigma_1^2}{3} g_{qp}^2(t,0) k^2} \int \mathrm{d}^3 q \, \left(\mathrm{e}^{g_{qp}^2(t,0)\vec{k}^\intercal \hat{C}_{pp}(\vec{q})\vec{k}} - 1 \right) \mathrm{e}^{\mathrm{i}\vec{k}\cdot\vec{q}}$$

initial momentum correlations

Large scale limit $k \to 0$

$$\mathcal{P}_\delta(k,t) = \mathrm{e}^{-\frac{\sigma_1^2}{3} g_{qp}^2(t,0) k^2} \int \mathrm{d}^3 q \ \left(\mathrm{e}^{g_{qp}^2(t,0) \vec{k}^\intercal \hat{C}_{pp}(\vec{q}) \vec{k}} - 1 \right) \mathrm{e}^{\mathrm{i}\vec{k}\cdot\vec{q}}$$

Expansion for small modes → linear power spectrum for large scales

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Expansion for small modes → linear power spectrum for large scales

$$\mathcal{P}_\delta(k,t) \sim g_{qp}^2(t,0) \int \mathrm{d}^3q \; \vec{k}^\intercal \hat{C}_{pp}(\vec{q})\vec{k} \; \mathrm{e}^{\mathrm{i}\vec{k}\cdot\vec{q}}$$

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$$= g_{qp}^2(t,0) P_\delta^{(\mathrm{i})}(k), \qquad \text{for } k \to 0$$

Small scale limit $k \to \infty$

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Expansion of the exponent for small scales *

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$$g_{qp}^2(t,0)\vec{k}^\intercal \hat{C}_{pp}(\vec{q})\vec{k} \;\sim\; g_{qp}^2(t,0)k^2 \left[\frac{\sigma_1^2}{3} + \vec{q}^\intercal \hat{A}\vec{q} + O\left(q^4\right)\right]$$

* we gave a mathematical proof that this expansion correct (paper in prep.)

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and the moments of the initial power spectrum

$$\sigma_n^2 := \int \mathrm{d}k \; k^{2n-2} P_\delta^{(\mathrm{i})}(k)$$

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Expansion for large modes → power law for small scales

$$\mathcal{P}_\delta(k,t) \sim \int \mathrm{d}^3 q \; \mathrm{e}^{-g_{qp}^2(t,0) k^2 \vec{q}^\intercal \hat{A} \vec{q} + \mathrm{i}\vec{k}\cdot\vec{q}}$$

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$$= \frac{1}{k^3} \frac{(2\pi)^{3/2}}{\sqrt{3}} \left(\frac{5}{2 g_{qp}^2(t,0) \sigma_2^2} \right)^{3/2} \mathrm{e}^{-\frac{5}{2 g_{qp}^2(t,0) \sigma_2^2}}, \quad \text{for } k \to \infty$$

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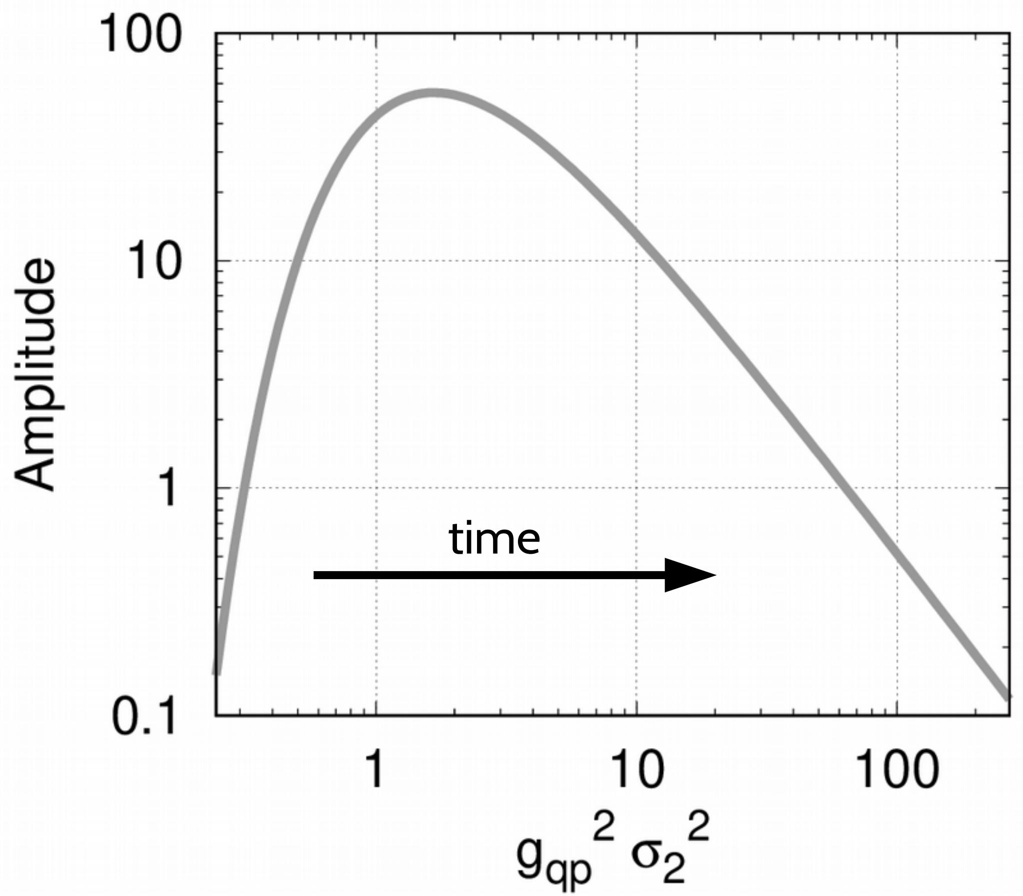
This result has also been found recently by **Chen and Pietroni** in the Lagrangian formulation of fluid mechanics arXiv:2002.11357

Amplitude Time Evolution

$$k^3 \mathcal{P}_\delta(k,t) \sim \frac{(2\pi)^{3/2}}{\sqrt{3}} \left(\frac{5}{2 g_{qp}^2(t,0)\sigma_2^2} \right)^{3/2} \mathrm{e}^{-\frac{5}{2 g_{qp}^2(t,0)\sigma_2^2}}$$



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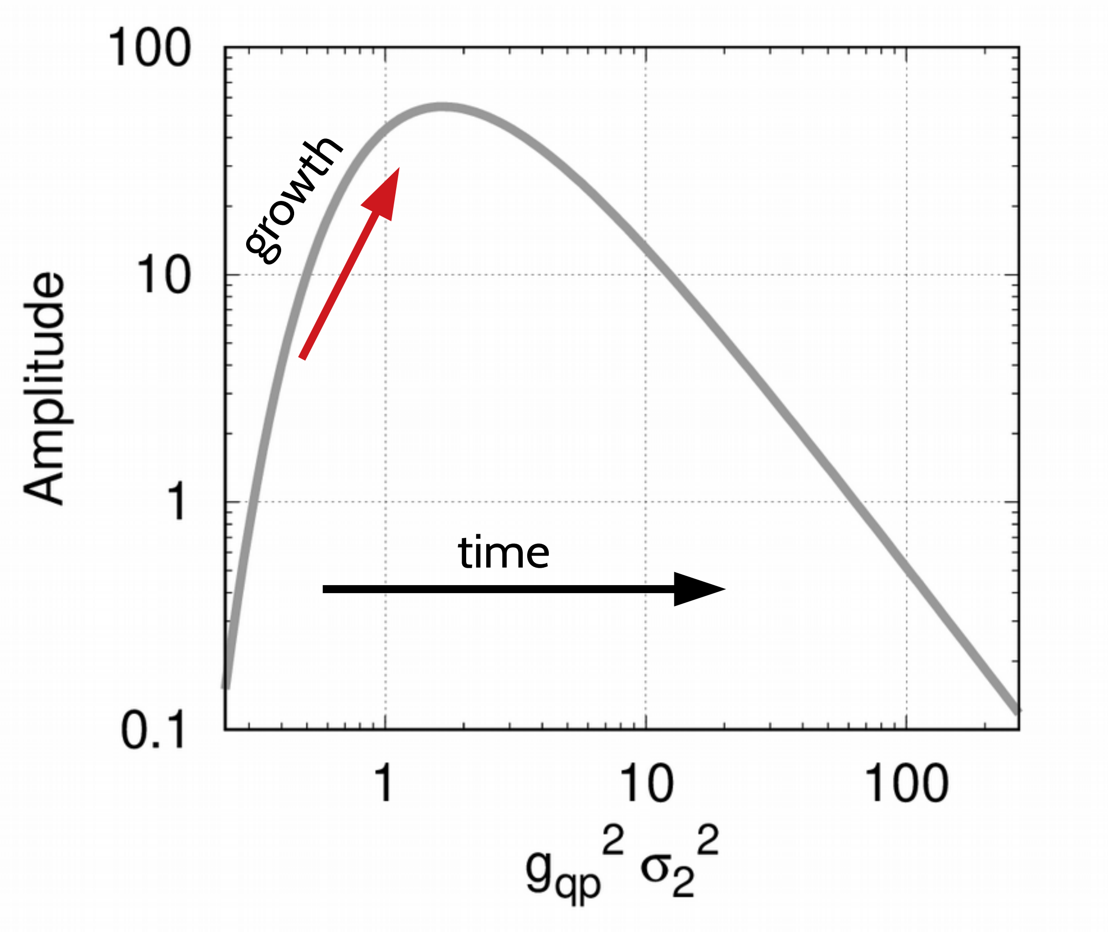

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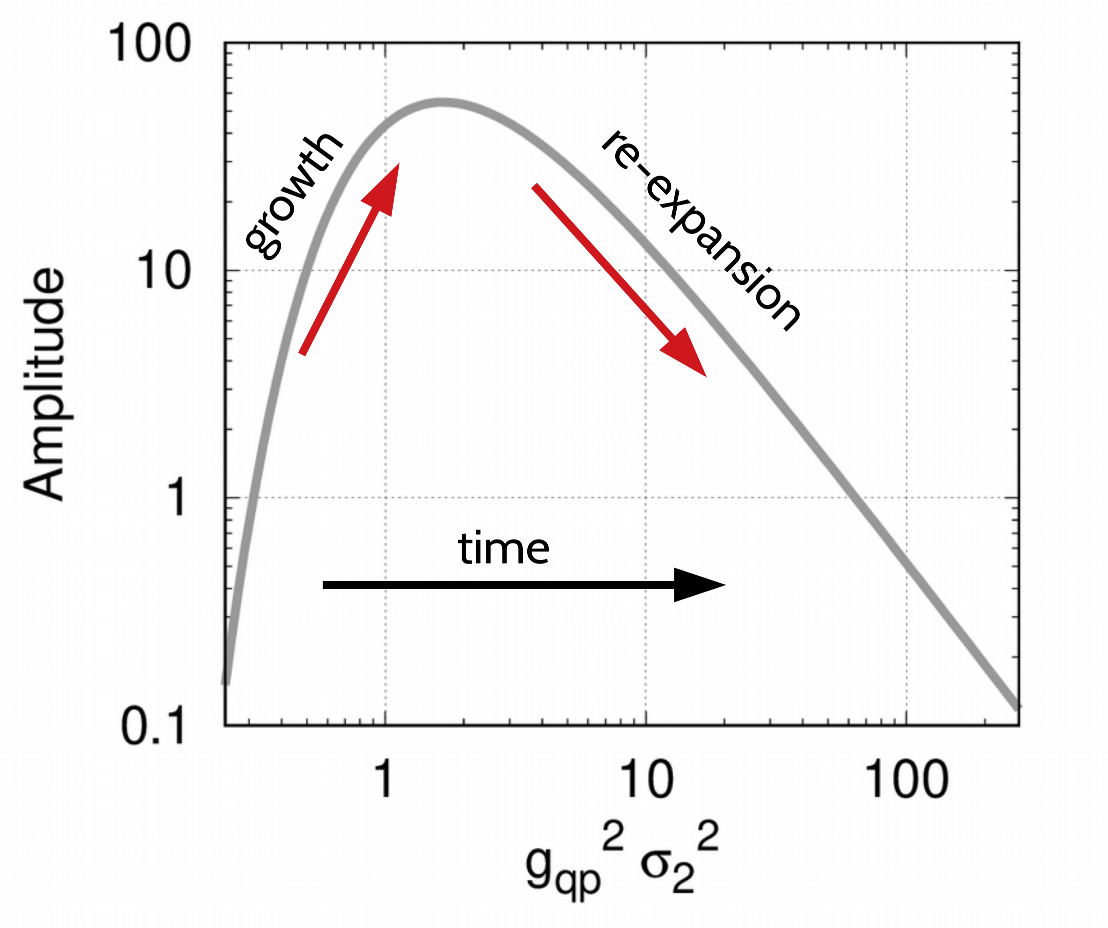
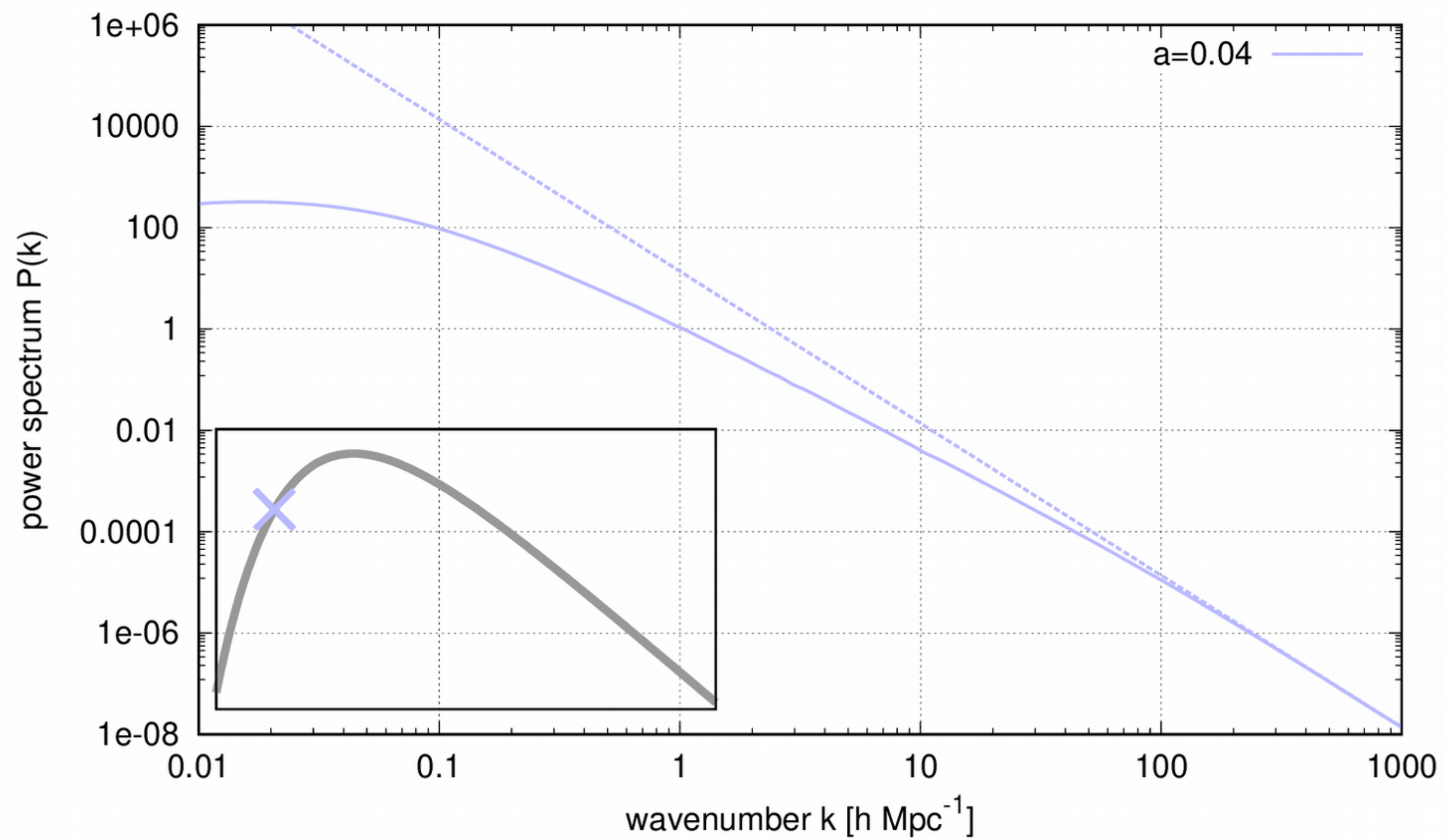


Asymptotics a = 0.04

- Arises **before** re-expansion of structures



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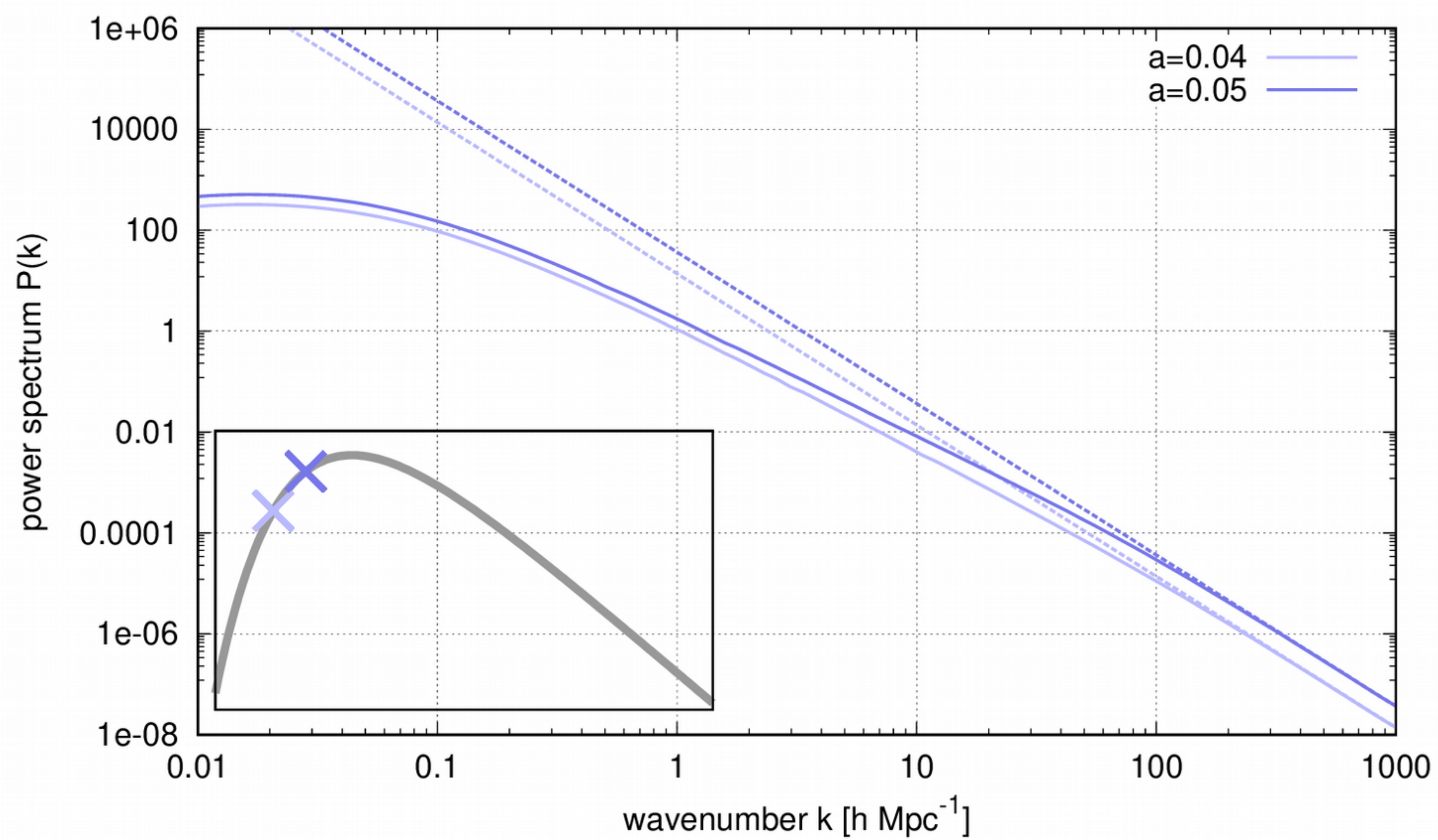

Asymptotics a = 0.065

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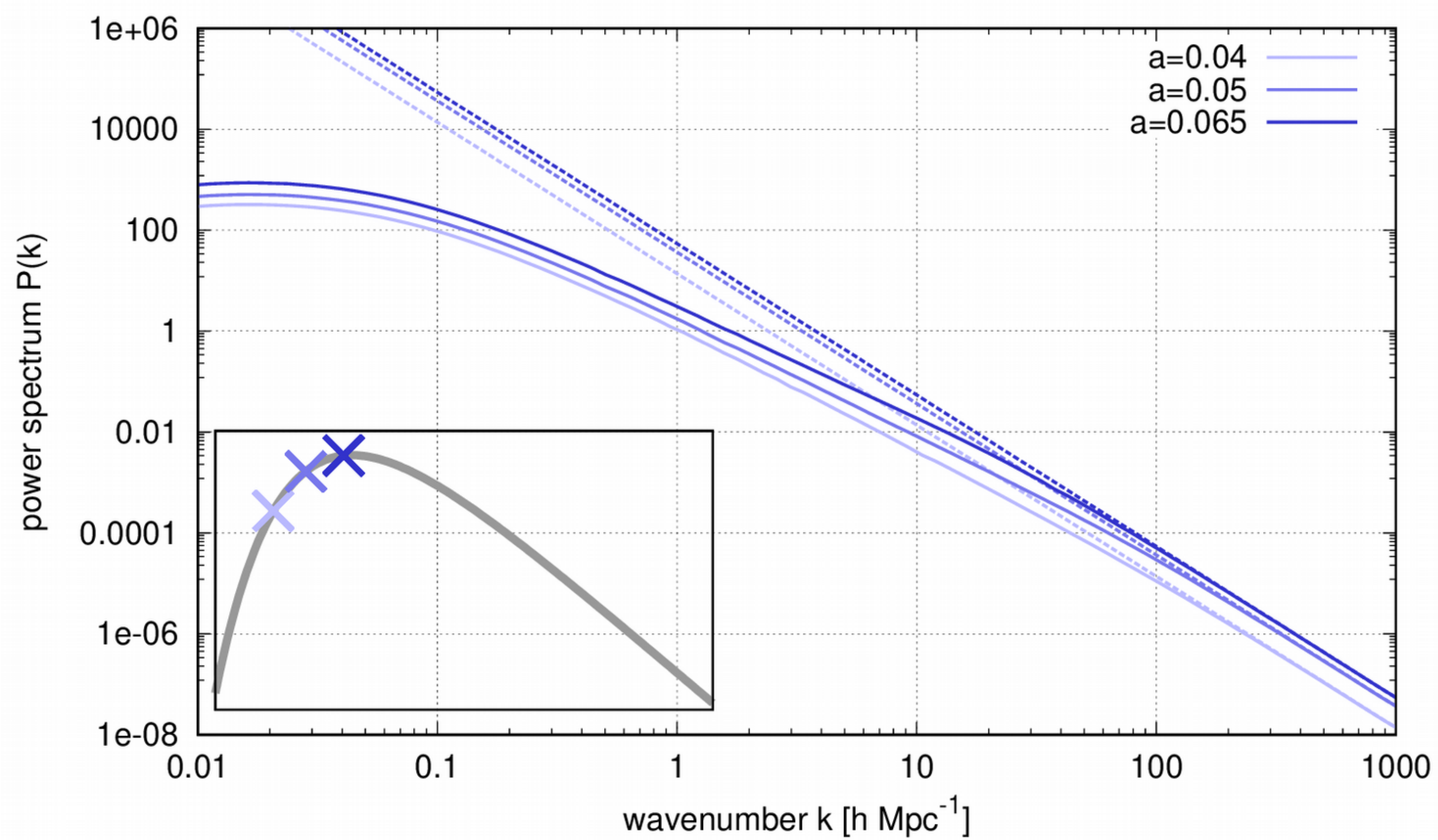
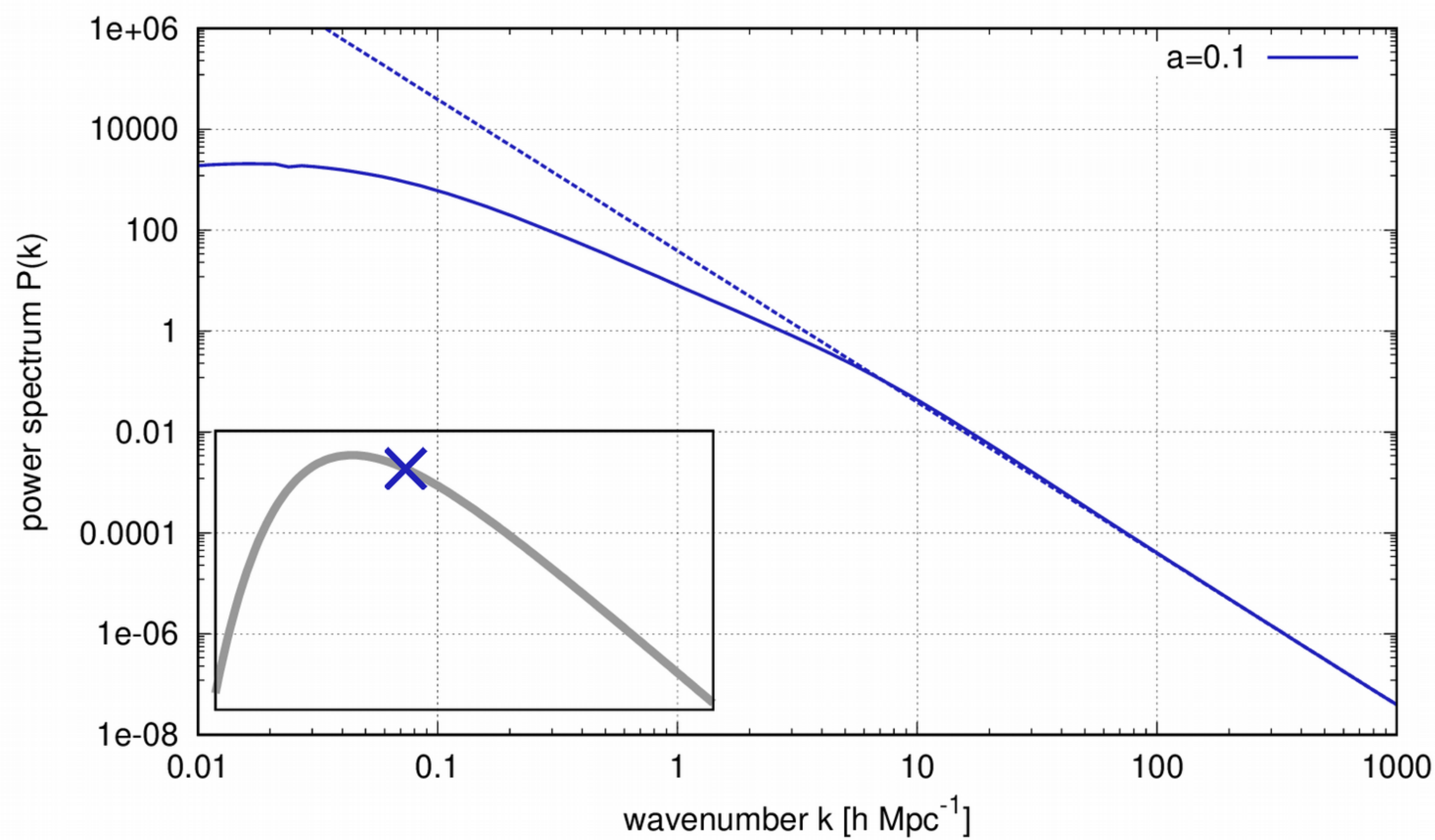


Asymptotics a = 0.1

- Persists **after** onset of re-expansion



Asymptotics a = 0.5

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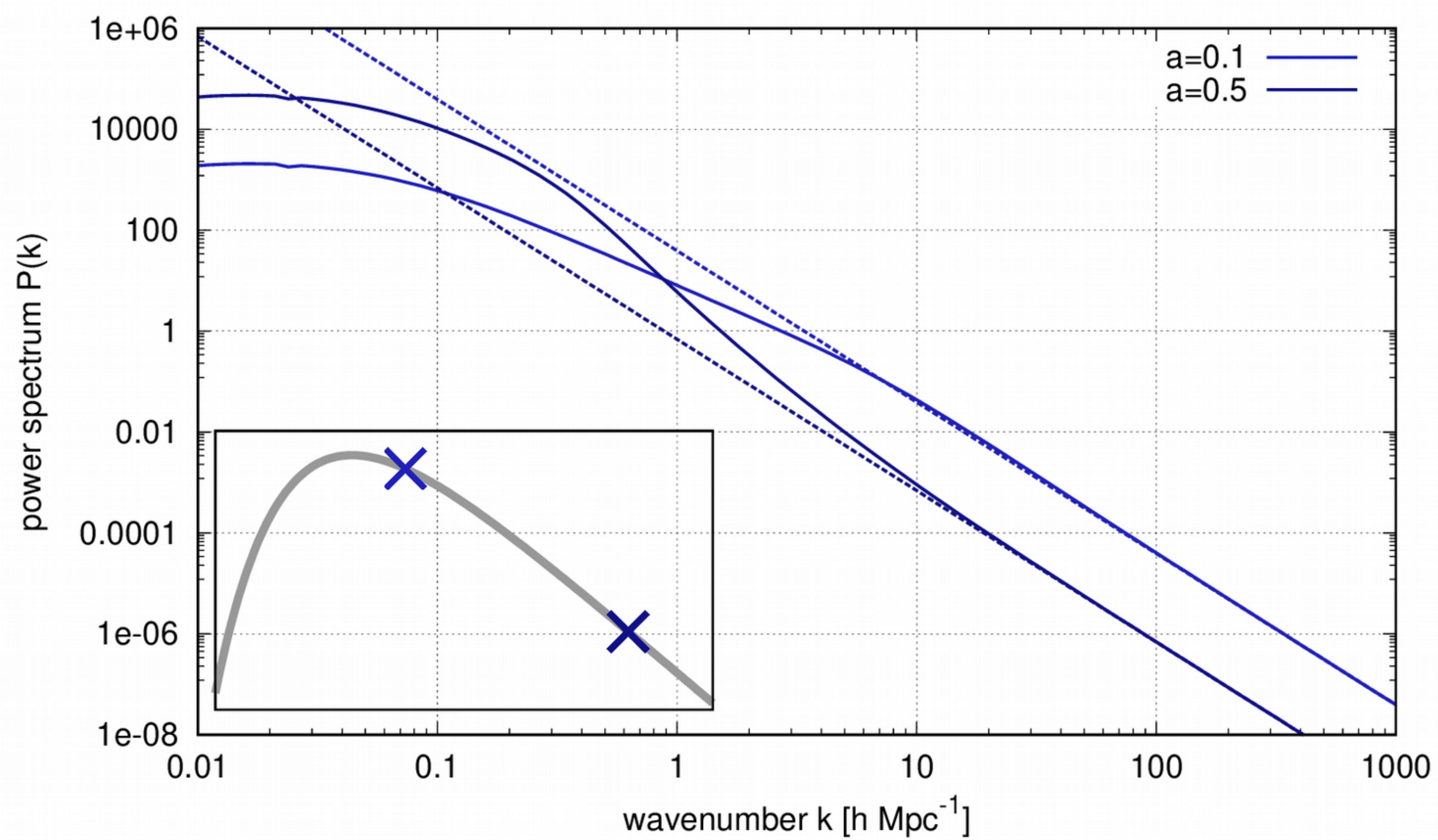

Asymptotics a = 1.0

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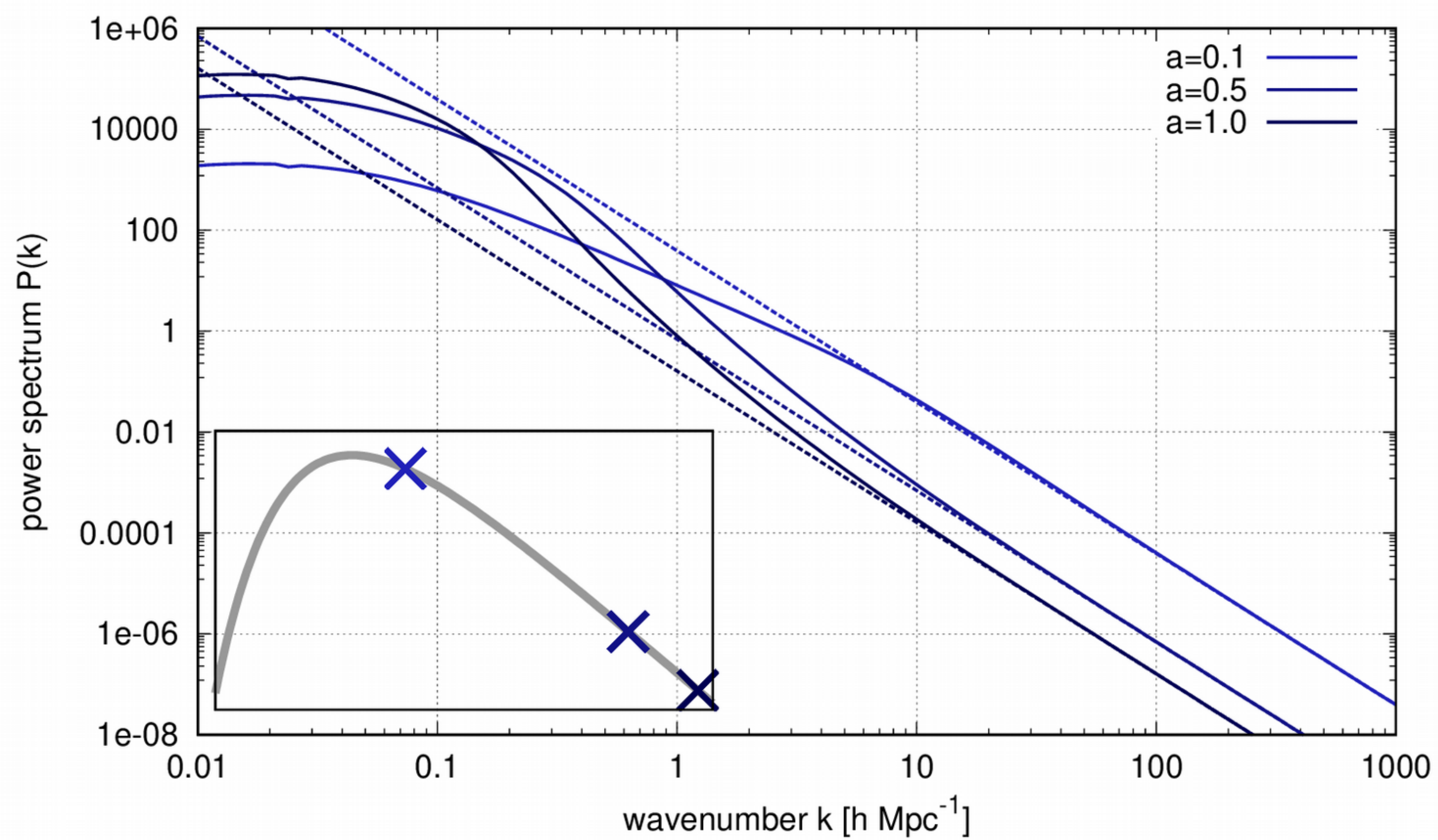

Initial Hot Dark Matter $P_\delta^{(\mathrm{i})}(k) \propto k \cdot \mathrm{e}^{-k}$

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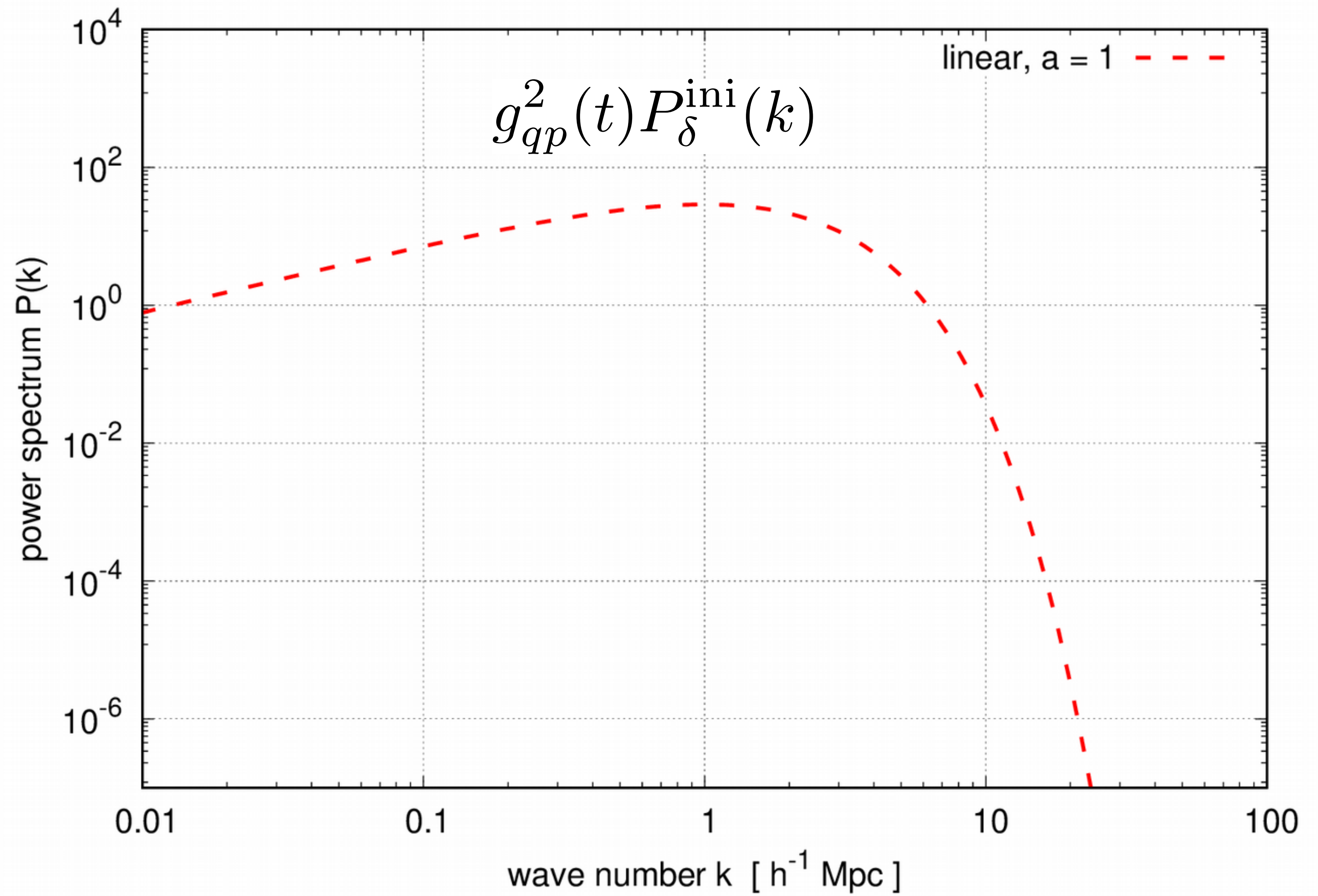

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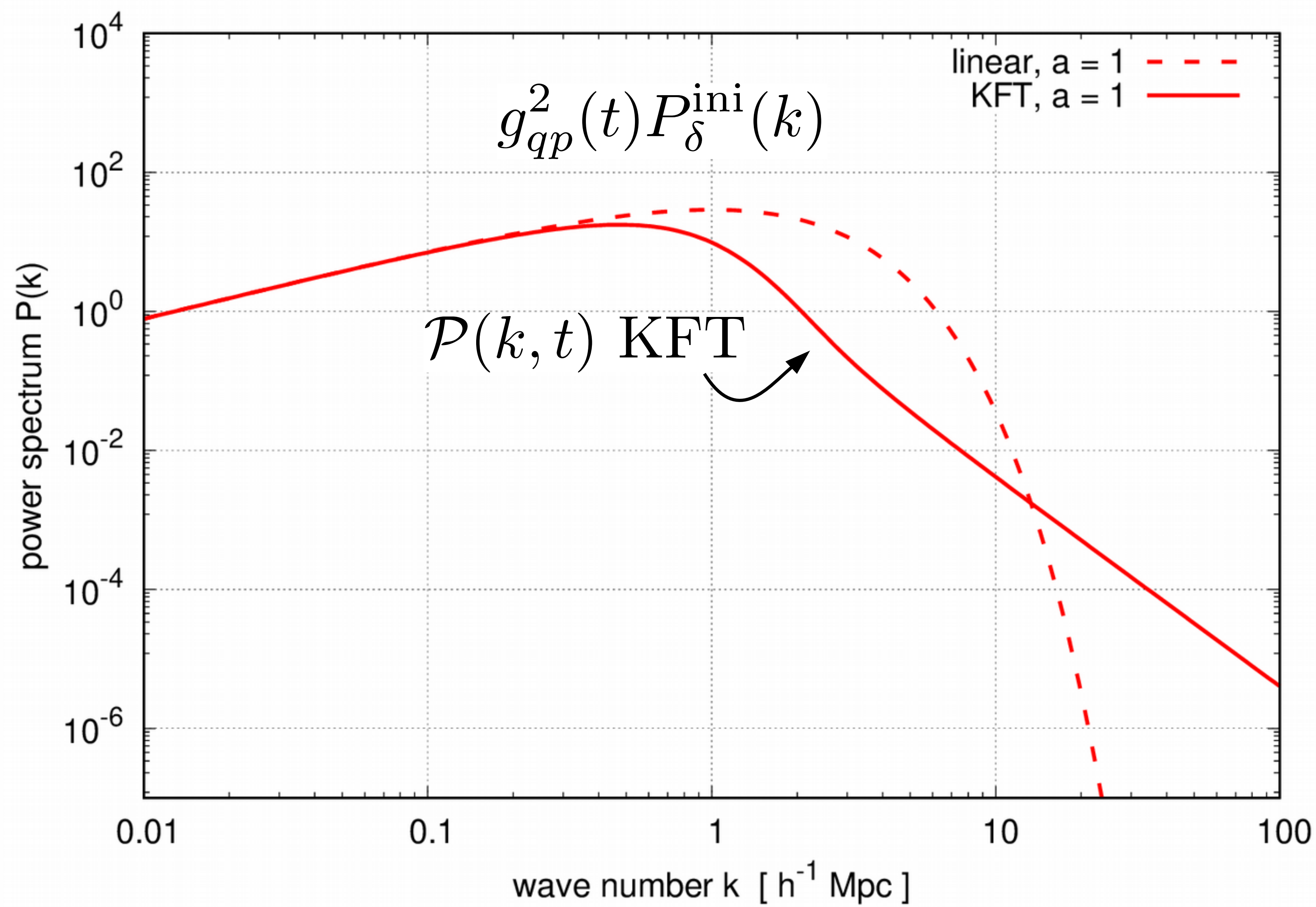

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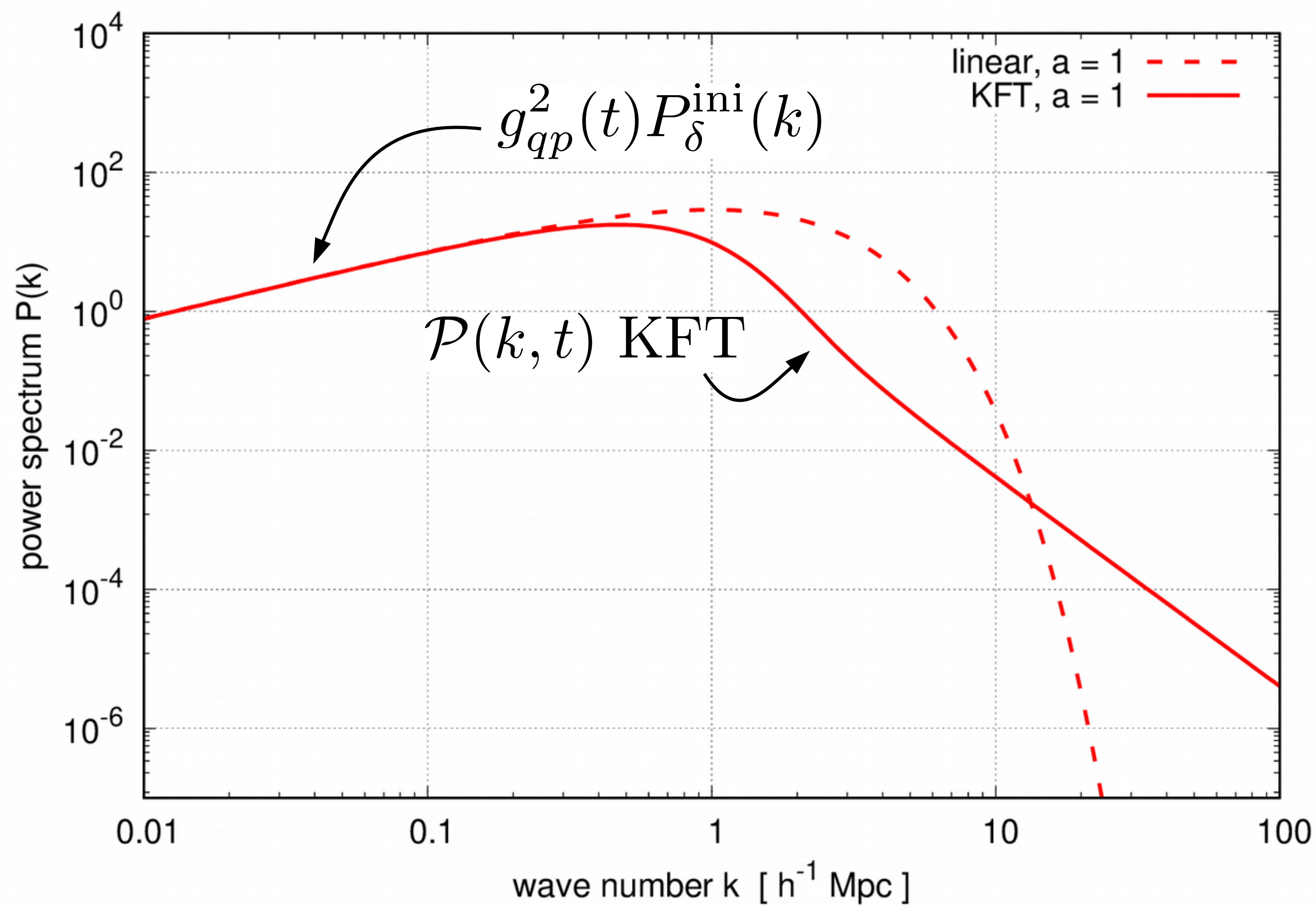

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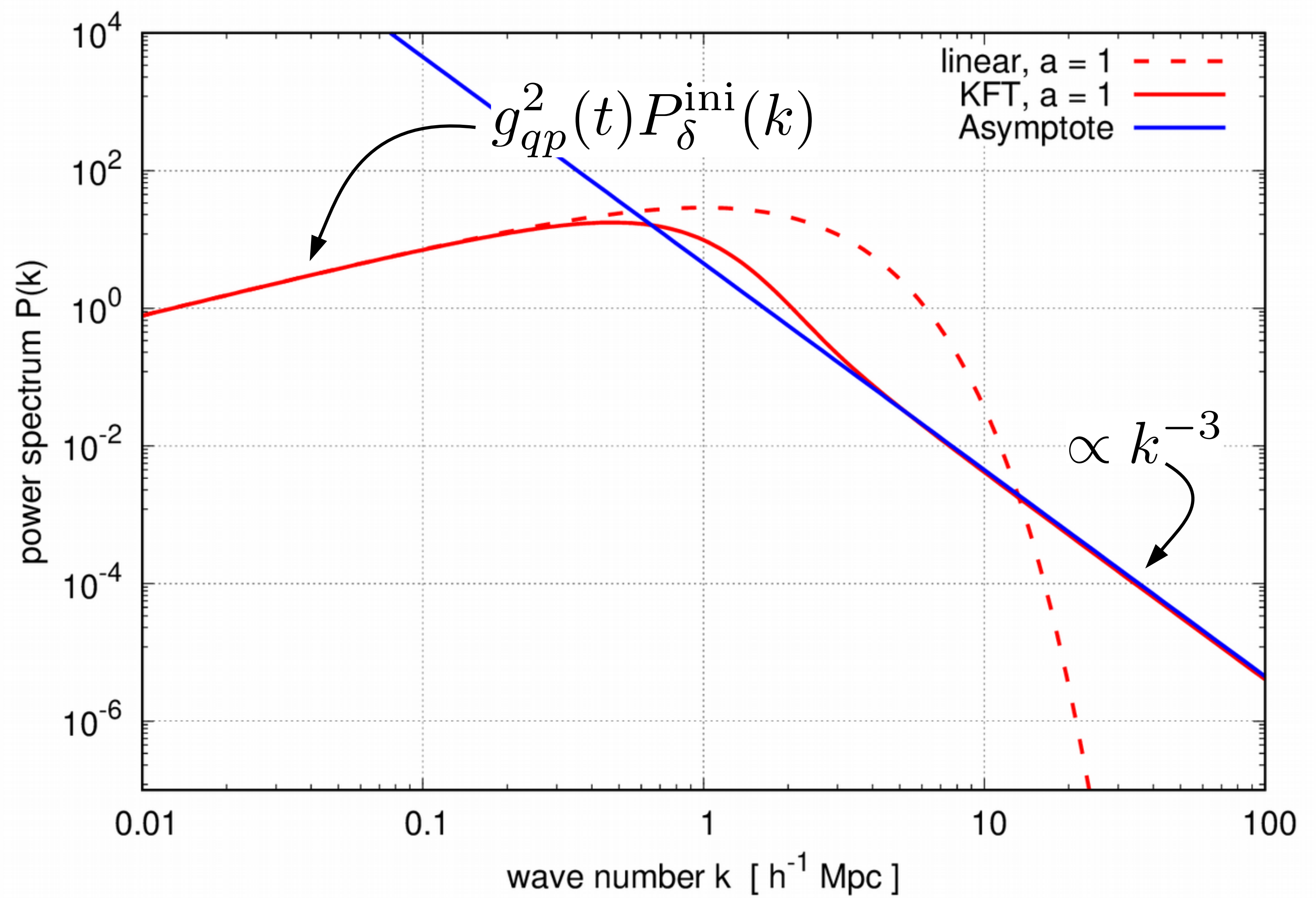

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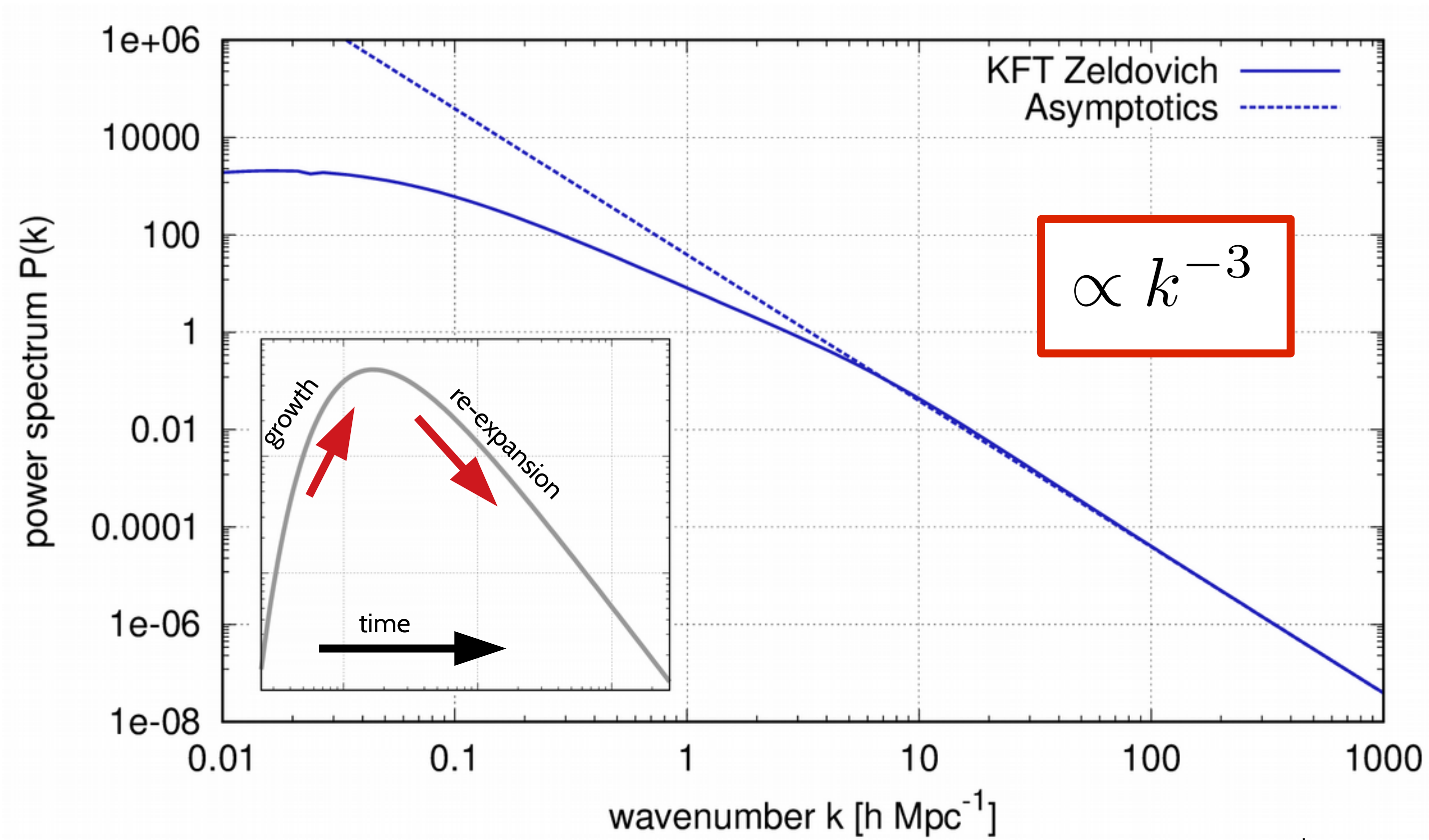

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Thank you for watching my talk!

- Please visit also the talks by **Elena Kozlikin** and **Carsten Littek**
- I am looking forward to meet you in the discussion session!

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Email: Sara.Konrad@stud.uni-heidelberg.de

LinkedIn: linkedin.com/in/sara-konrad-00a9551b3

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Zel'dovich trajectories

- Lagrangian Dynamics: $\vec{r}(\vec{x}, t)$

initial coordinate

coordinate at time t

- Ansatz for intially rotation free velocity field

$$\vec{r}(\vec{x}, t) = a(t)\vec{x} + b(t)\vec{p}(\vec{x})$$

initial momentum

- Being compatible with linear growth on large scales

$$\vec{r}(\vec{x}, t) = a(t)\vec{x} + a(t)D_+(t)\vec{p}(\vec{x})$$

linear growth factor

- Gaussian initial momentum correlations

$$\hat{C}_{pp}(\vec{x} - \vec{y}) := \langle \vec{p}(\vec{x}) \otimes \vec{p}(\vec{y}) \rangle = \int_k \frac{\vec{k} \otimes \vec{k}}{k^4} P^{ini}(k) \mathrm{e}^{\mathrm{i}\vec{k}(\vec{x}-\vec{y})}$$

initial density fluctuation power spectrum