Model Independent Methods: Gaussian Processes and Inflation Wars

Ryan Keeley

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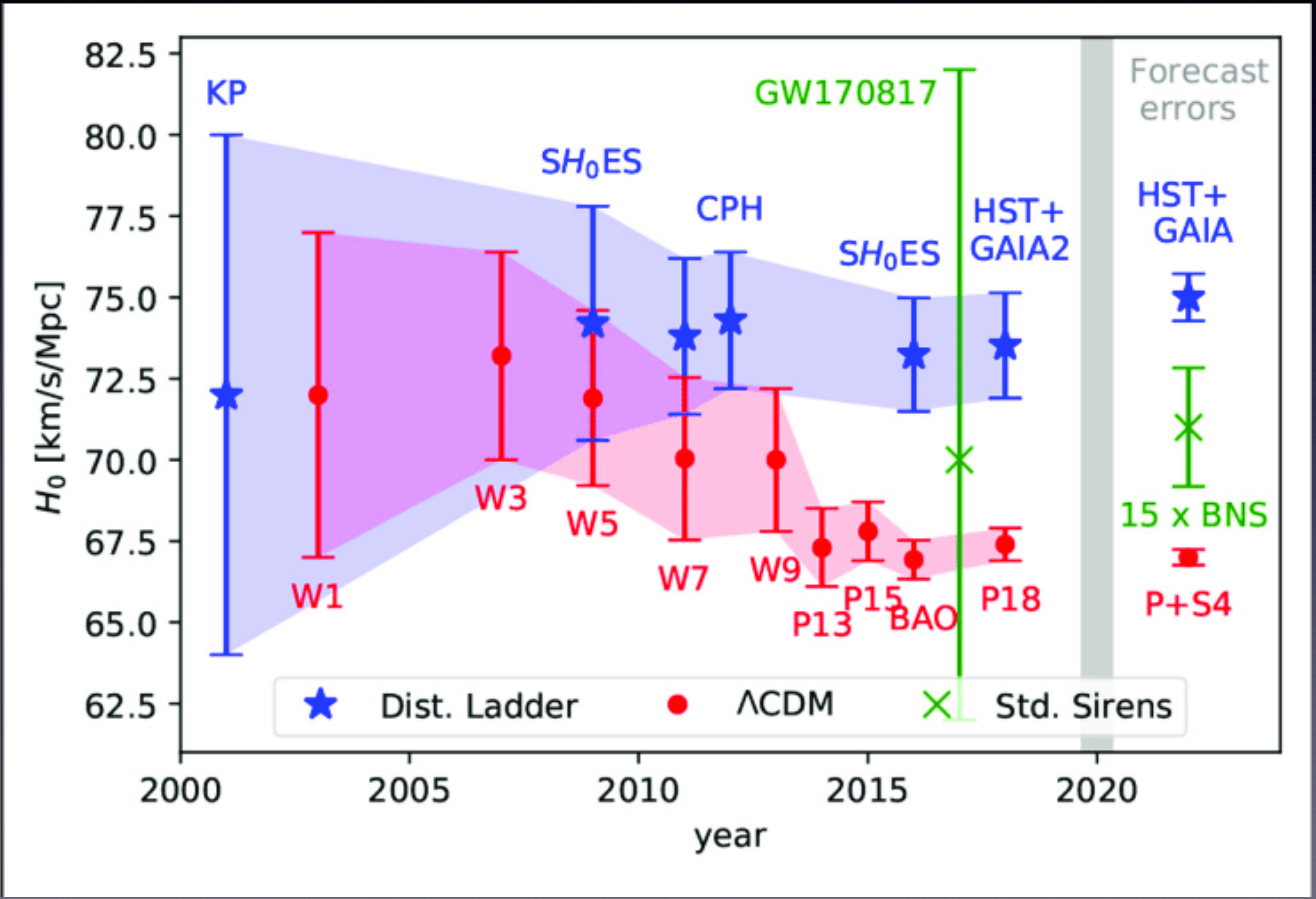
Cosmology From Home

Cosmology Group at
Korea Astronomy and Space Science Institute

Testing the Concordance Model

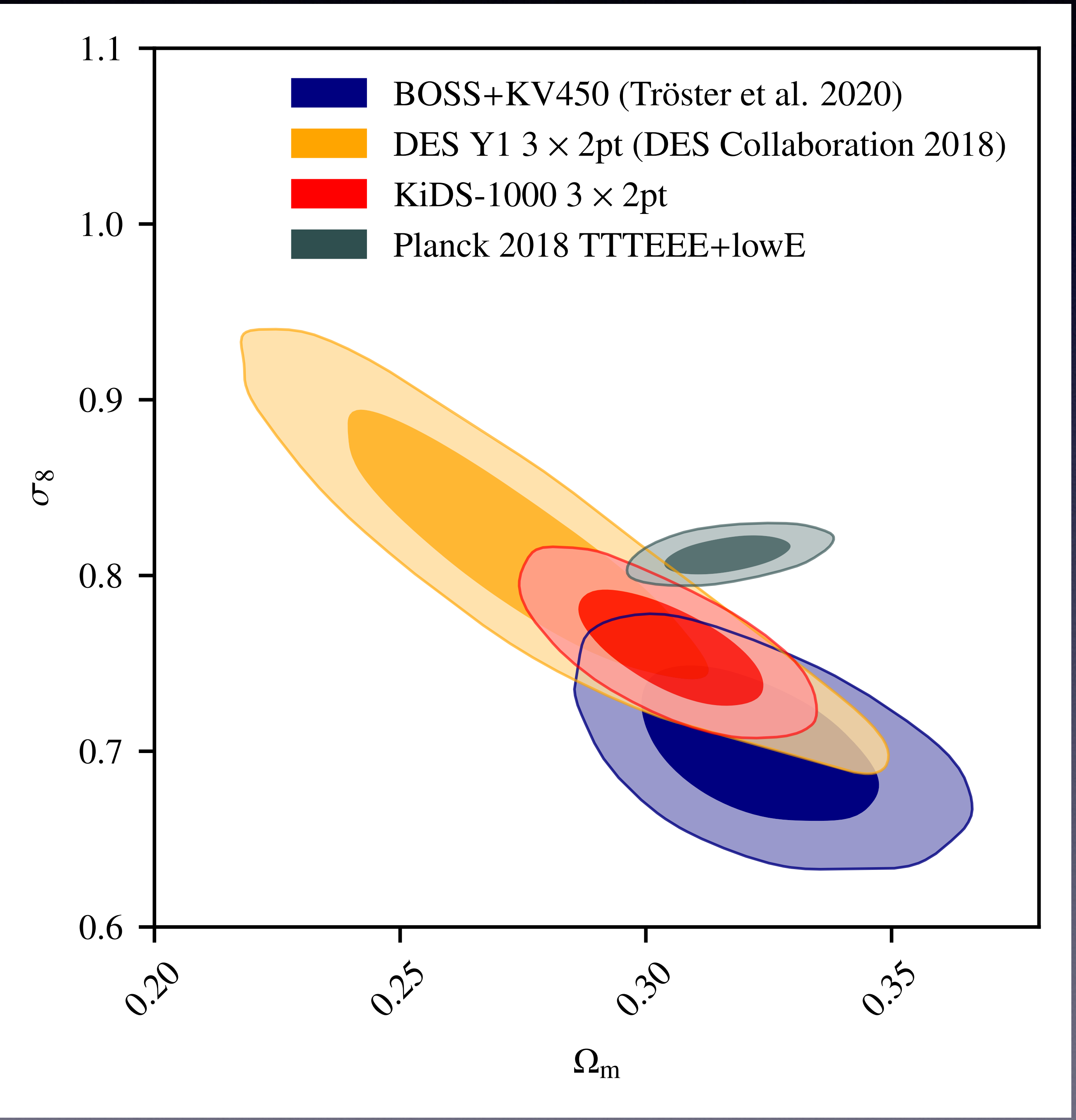
- ΛCDM + GR
- Λ - test via low-redshift distances
- CDM - test via small scale structure
- GR - test via growth rate measurements

H0 Tension



- Inferences from the CMB predict H(z=0) = 67.36 +/- 0.54 km/s/Mpc
- Measuring H0 directly gives 74.03 +/- 1.42 km/s/Mpc
- Difference is now at 4.4-σ.
- No obvious systematics
- Potentially a challenge for LCDM

Sigma_8 Tension



- KiDS-1000

$$S_8 = \sigma_8\sqrt{\Omega_m/0.3} = 0.766^{+0.020}_{-0.014}$$

- 2-3 σ significance

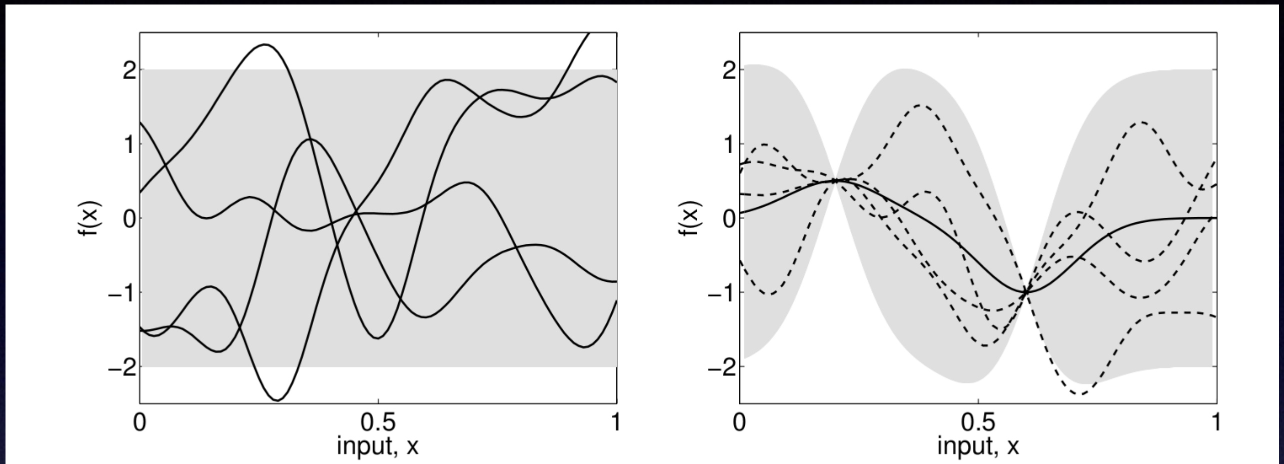
Models

- Low-z explanations
- TDE
- PEDE/GEDE
- Interacting DM/DE
- Others

- High-z explanations
- EDE
- Rock'n'Roll / Jazzy
- Neff
- Others

Instead of iterating over a potentially infinite number of models it can be better to use model independent methods

Gaussian Process



(a), prior (b), posterior

- An instance or a sample of a GP, $\gamma(z)$, is a hyperfunction that randomly varies around the "mean function", $H_{\rm mf}$, => $H_i(z) = \exp(\gamma_i(z))H_{\rm mf}(z)$

- GP *regression* then involves training these samples based on how well they fit the data

$$P(H(z)\,|\,D) = \int d\sigma_f d\ell\, d\phi\ \ \mathscr{L}(D\,|\,H(z)(\sigma_f, \ell, \phi))\ P(\sigma_f, \ell, \phi)/P(D)$$

GP

- Gaussian process - a distribution of functions characterized by a covariance function

$$\langle \phi(s_1)\phi(s_2)\rangle = \sigma_f^2\, e^{-(s_1-s_2)^2/(2\ell^2)}$$

- hyperparameters σ_f and ℓ control heights and lengths of the random fluctuations respectively.

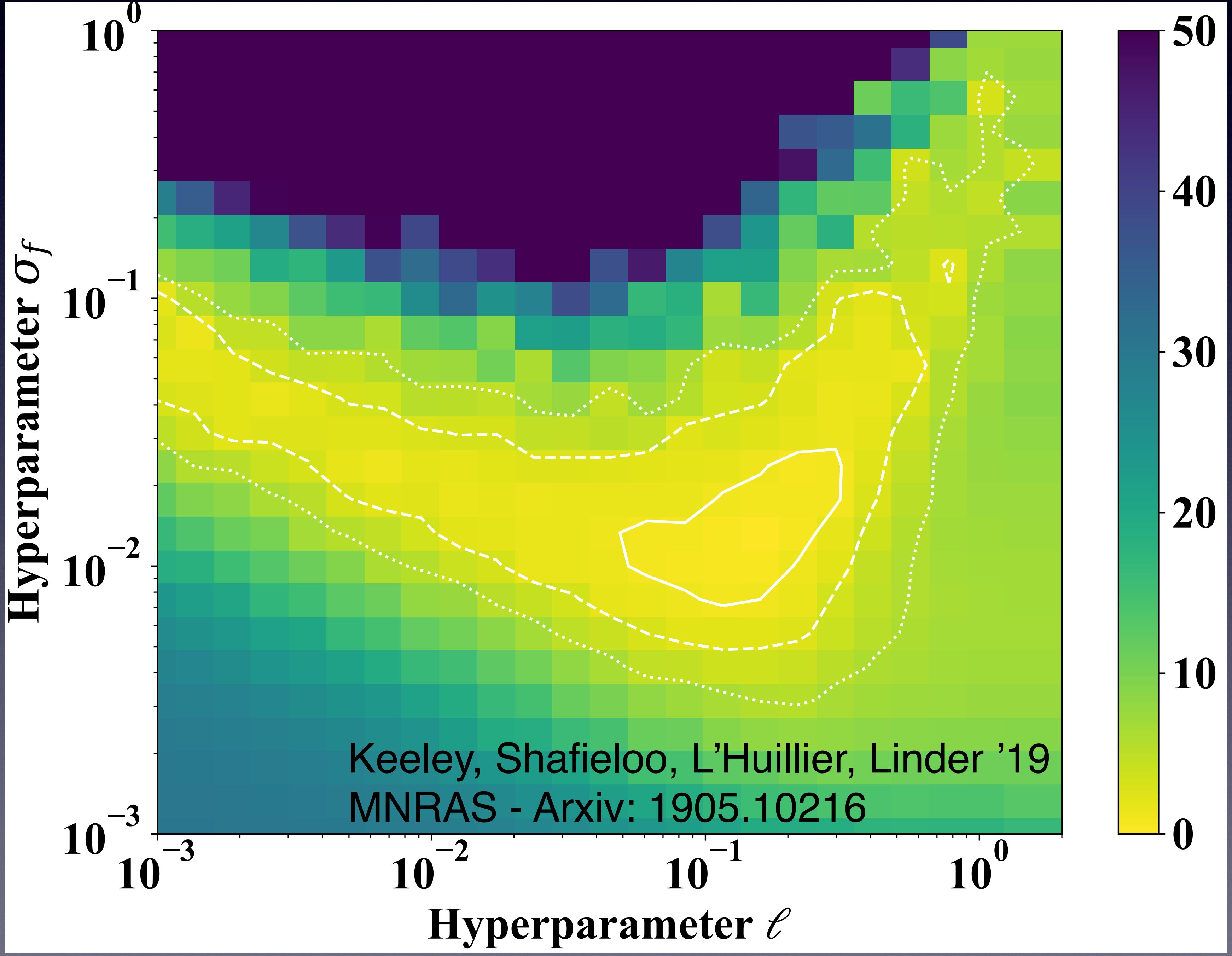
$$s(z) = \log(1+z)/\log(1+z_{\max})\ ,\ \mathrm{z}_{\max} = 3$$

Testing LCDM via GP Hyperparameters

- These sort of tests can be performed because the hyperparameters of the GP regression encode information about whether the mean function is a good fit to the data
- i.e. how much information beyond the mean function is required to fit the data
- This test is performed by calculating the posterior of the hyperparameters to see if sigma_f, the parameter that describes the heights of the fluctuations in the GP, is consistent with 0 or not
- If sigma_f > 0 then, data need more flexibility than the given mean function
- If mean function standard model, the GP can test if the standard model is sufficient
- Shafieloo A., Kim A. G., Linder E. V., 2012, Phys. Rev. D

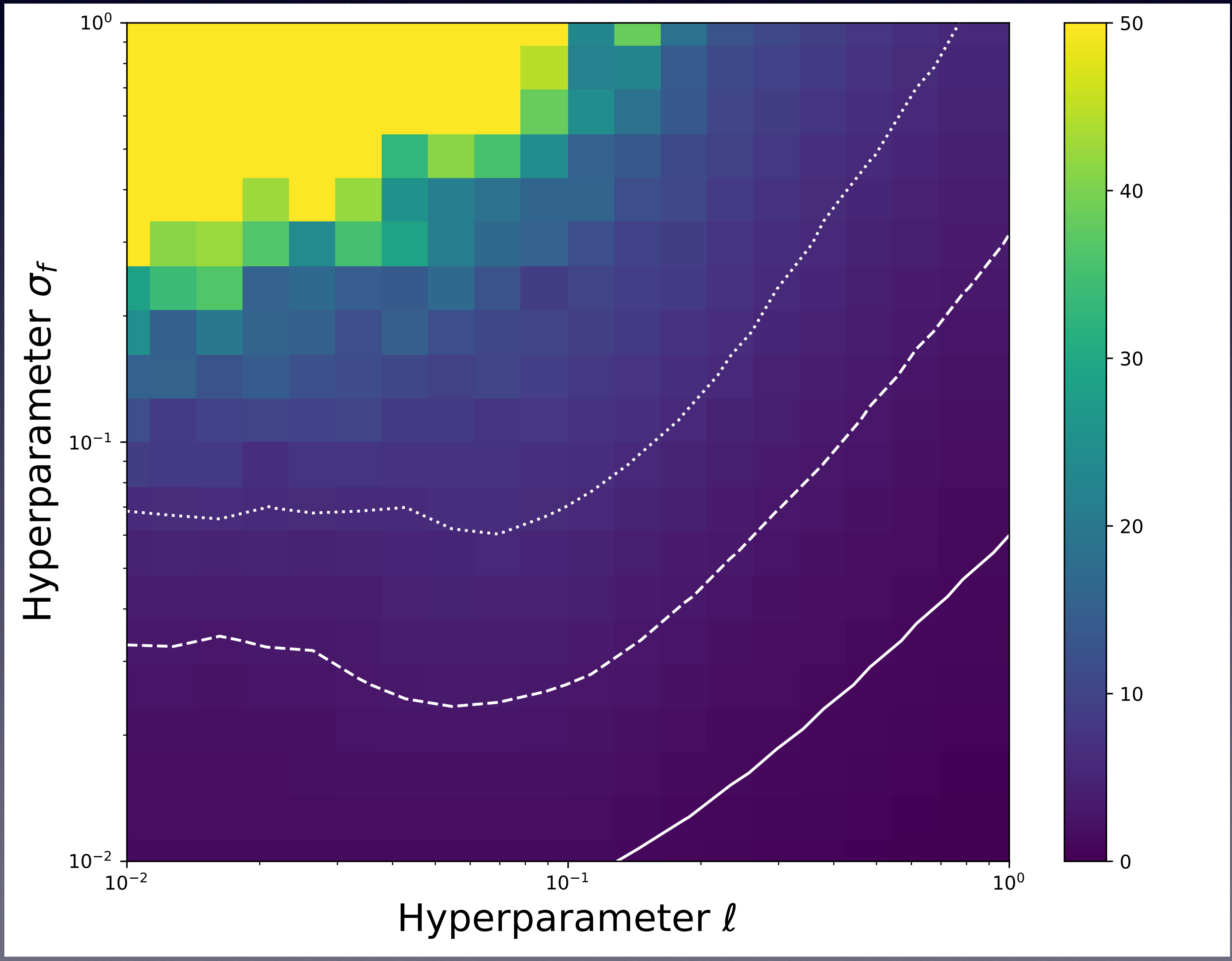
GP Hyperparameters

- Data: Mock Pantheon and GW datasets from CPL cosmology
- Mean function: best-fit LCDM model fit to the mock datasets
- Posterior prefers sigma_f > 0
- Evidence that there exists information in the data beyond the mean function

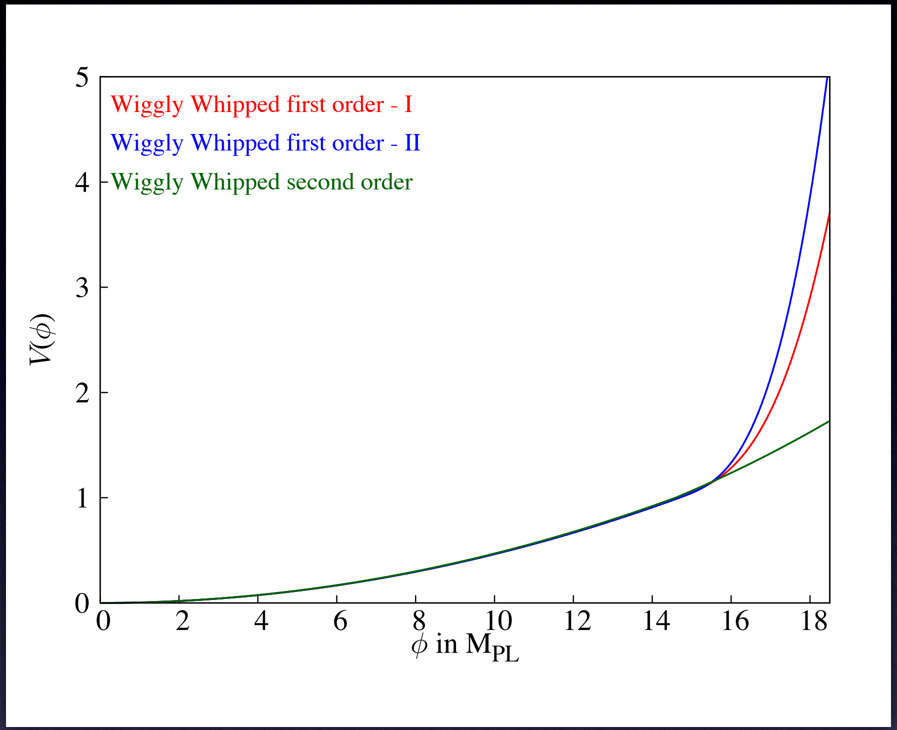


Consistency between SN and SDSS BAO

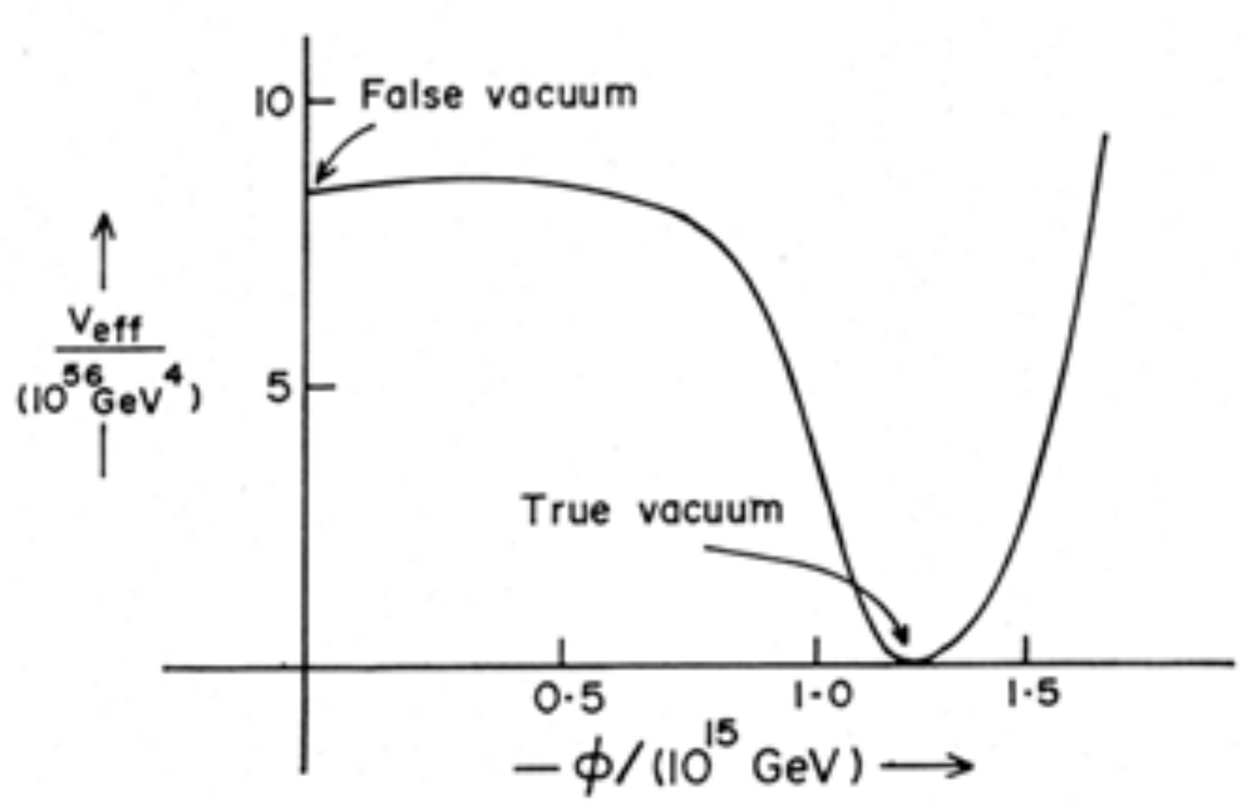
- Hyperparameters of GP reconstruction
- Data : Pantheon SN
- Mean function : GP reconstruction of the SDSS data
- Thus SN and SDSS are consistent, made no assumptions about a model



Inflation



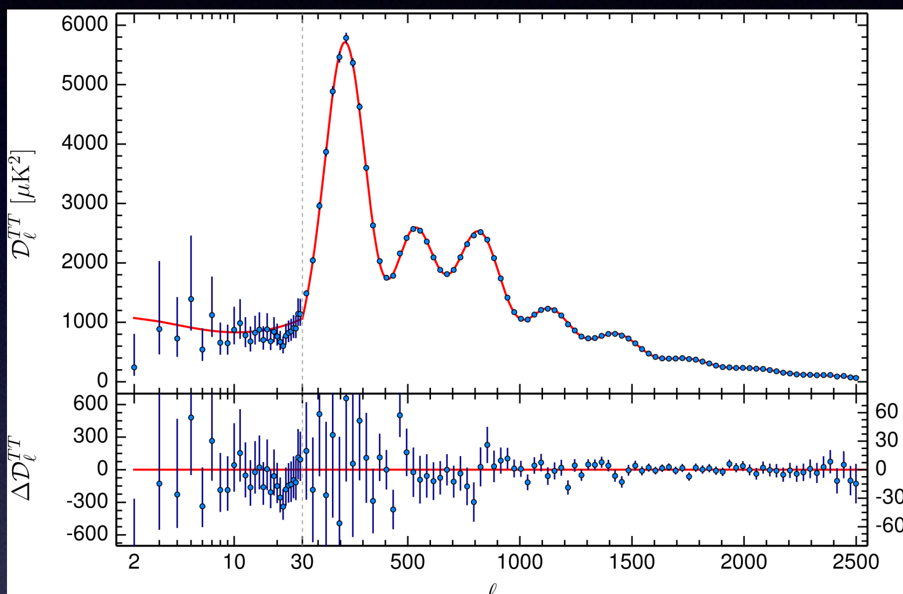
- Single-field slow-roll inflation typically predicts a featureless power-law primordial power spectra

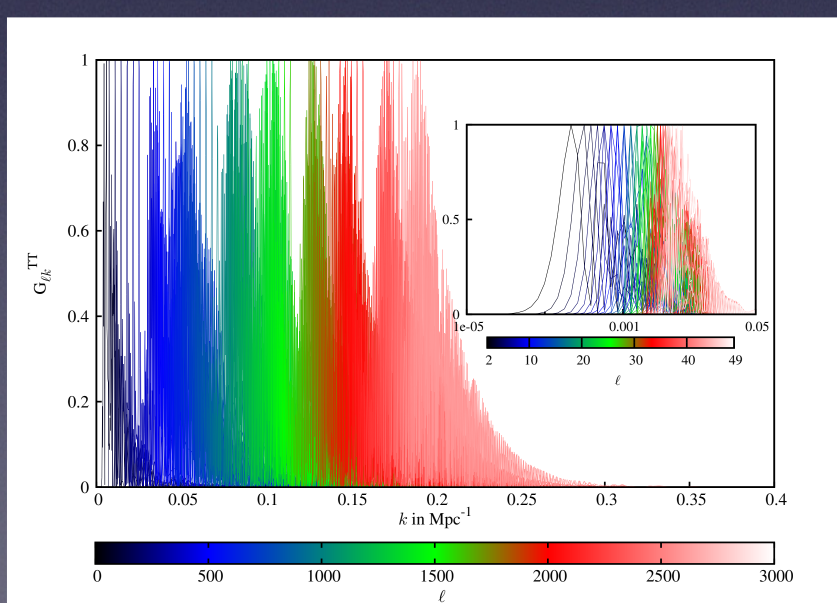


- More complicated inflationary physics can yield complicated PPS with features

Deconvolution

$$C_\ell = \sum_{k_i} G_{\ell,k_i} P(k) \rightarrow P(k) = \mathrm{MRL}(\mathrm{C}_\ell/\mathrm{G}_{\ell,\mathrm{k_i}})$$





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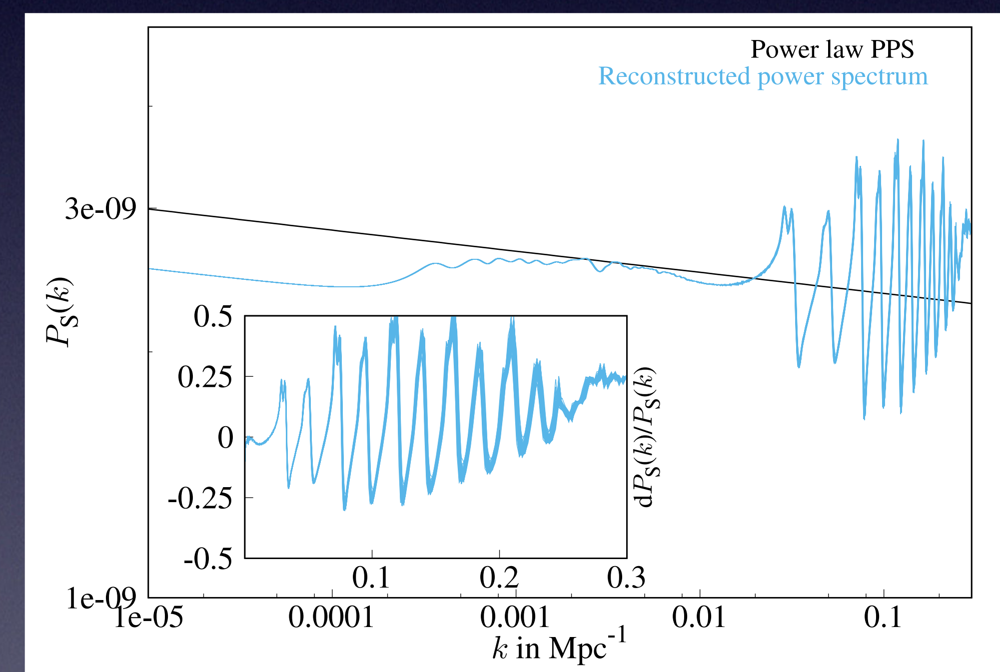


- Hazra et al 2014 JCAP
- Hazra et al 2019 JCAP

Modified Richardson-Lucy Deconvolution

- ➔ Iterative algorithm.
- ➔ Not sensitive to the initial guess.
- ➔ Enforce positivity of P(k).

[$G(l,k)$ is positive definite and C_l is positive]

$$C_\ell = \sum_i G_{\ell k_i} P_{k_i}$$

$$P_k^{(i+1)} - P_k^{(i)} = P_k^{(i)} \times \left[\sum_{\ell=2}^{\ell=900} \tilde{G}_{\ell k}^{\mathrm{un-binned}} \left\{ \left(\frac{C_\ell^{\mathrm{D}} - C_\ell^{\mathrm{T}(i)}}{C_\ell^{\mathrm{T}(i)}} \right) \tanh^2 \left[Q_\ell (C_\ell^{\mathrm{D}} - C_\ell^{\mathrm{T}(i)}) \right] \right\}_{\mathrm{un-binned}} + \sum_{\ell_{\mathrm{binned}} > 900} \tilde{G}_{\ell k}^{\mathrm{binned}} \left\{ \left(\frac{C_\ell^{\mathrm{D}} - C_\ell^{\mathrm{T}(i)}}{C_\ell^{\mathrm{T}(i)}} \right) \tanh^2 \left[\frac{C_\ell^{\mathrm{D}} - C_\ell^{\mathrm{T}(i)}}{\sigma_\ell^{\mathrm{D}}} \right]^2 \right\}_{\mathrm{binned}} \right], \quad (1)$$

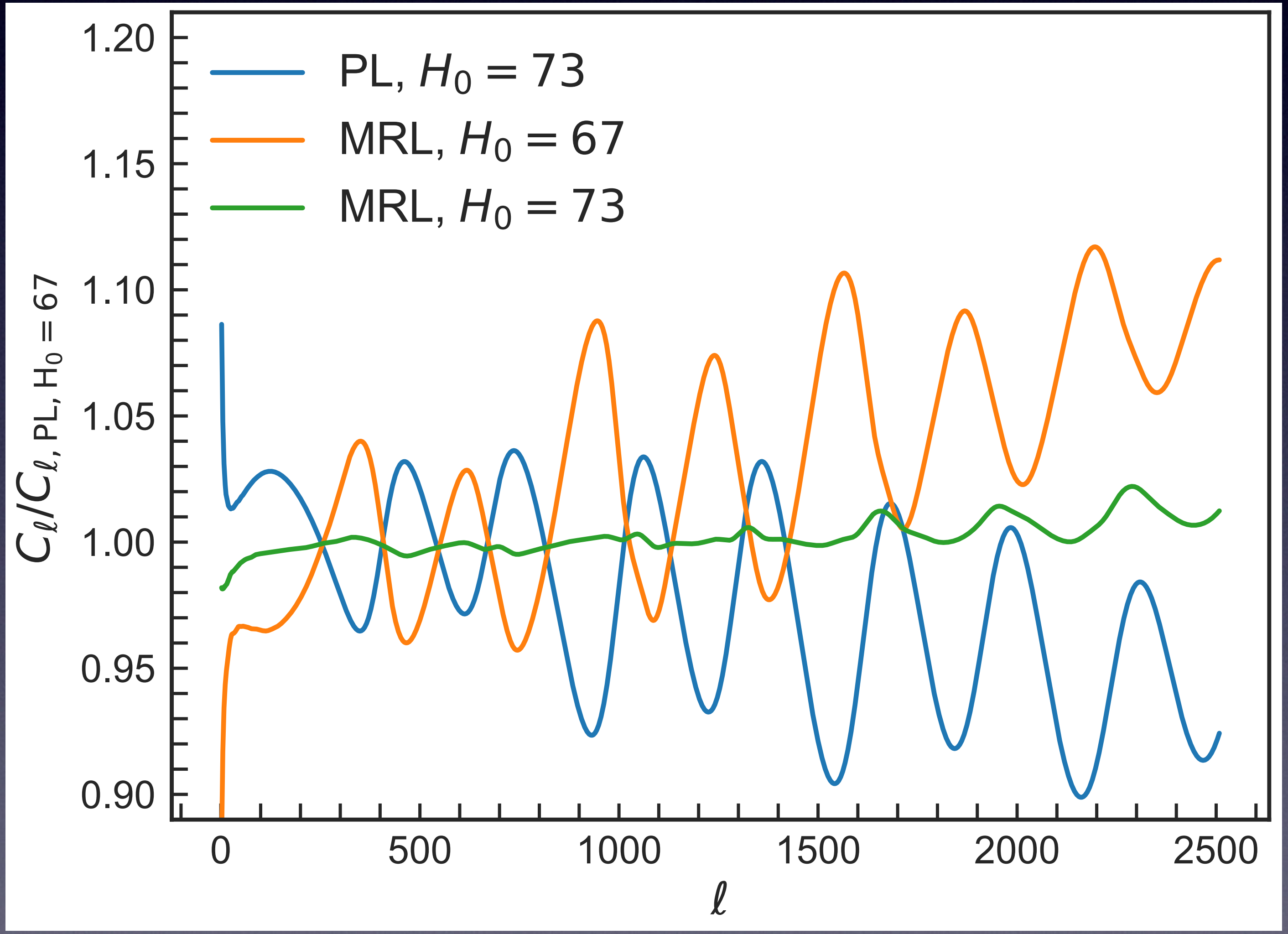
$$Q_\ell = \sum_{\ell'} (C_{\ell'}^{\mathrm{D}} - C_{\ell'}^{\mathrm{T}(i)}) COV^{-1}(\ell, \ell'),$$

Shafieloo & Souradeep PRD 2004 ;
Shafieloo et al, PRD 2007;
Shafieloo & Souradeep, PRD 2008;
Nicholson & Contaldi JCAP 2009
Hamann, Shafieloo & Souradeep JCAP 2010
Hazra, Shafieloo & Souradeep PRD 2013
Hazra, Shafieloo & Souradeep JCAP 2013
Hazra, Shafieloo & Souradeep JCAP 2014
Hazra, Shafieloo & Souradeep JCAP 2015
Hazra, Shafieloo, Souradeep, JCAP 2019

Sound Cancellation

- High value of H0 changes the angular diameter distance to the CMB and hence shifts all of the acoustic peaks

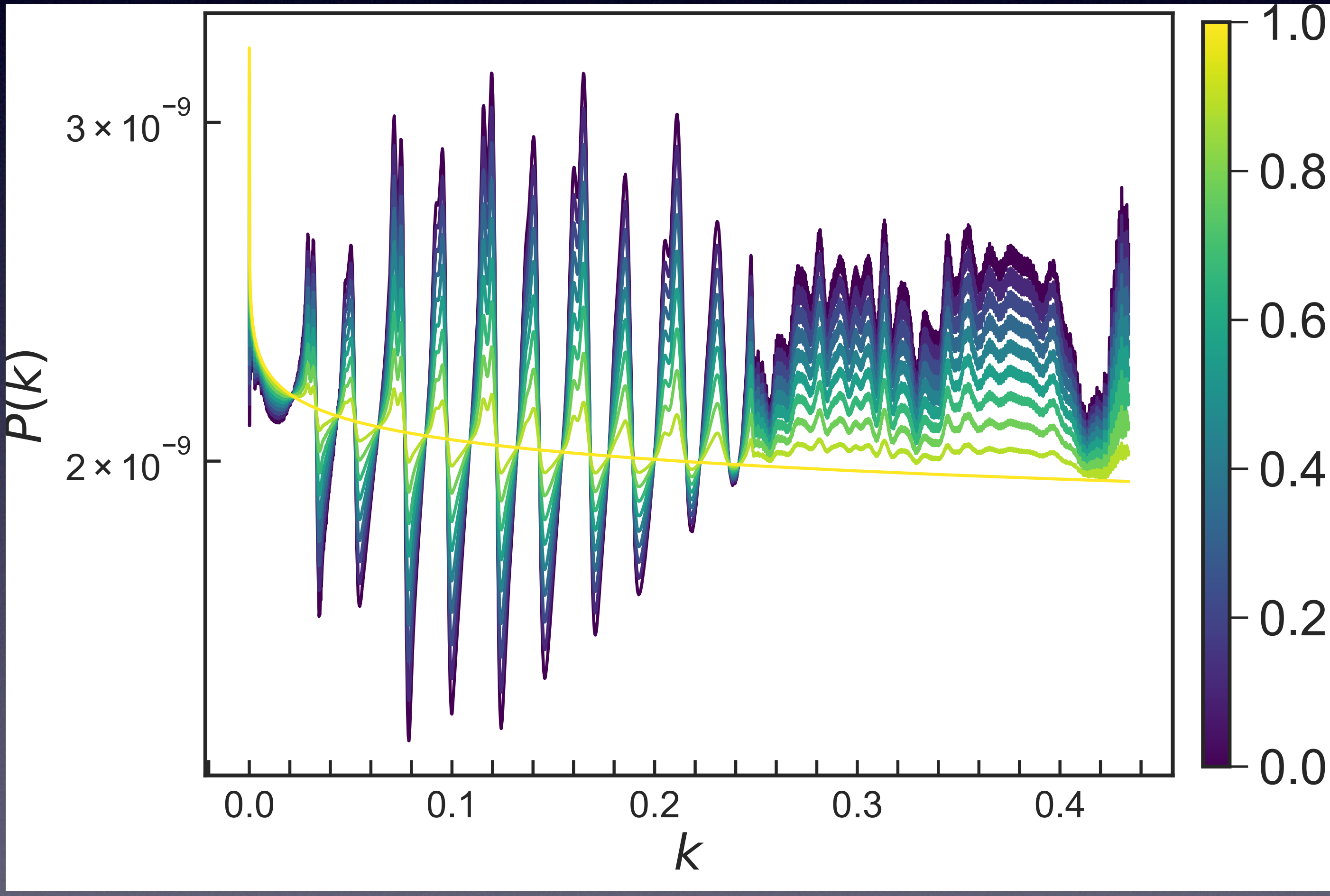
- The MRL-PPS shifts power around in the PPS to cancel these "induced sound waves"



Features

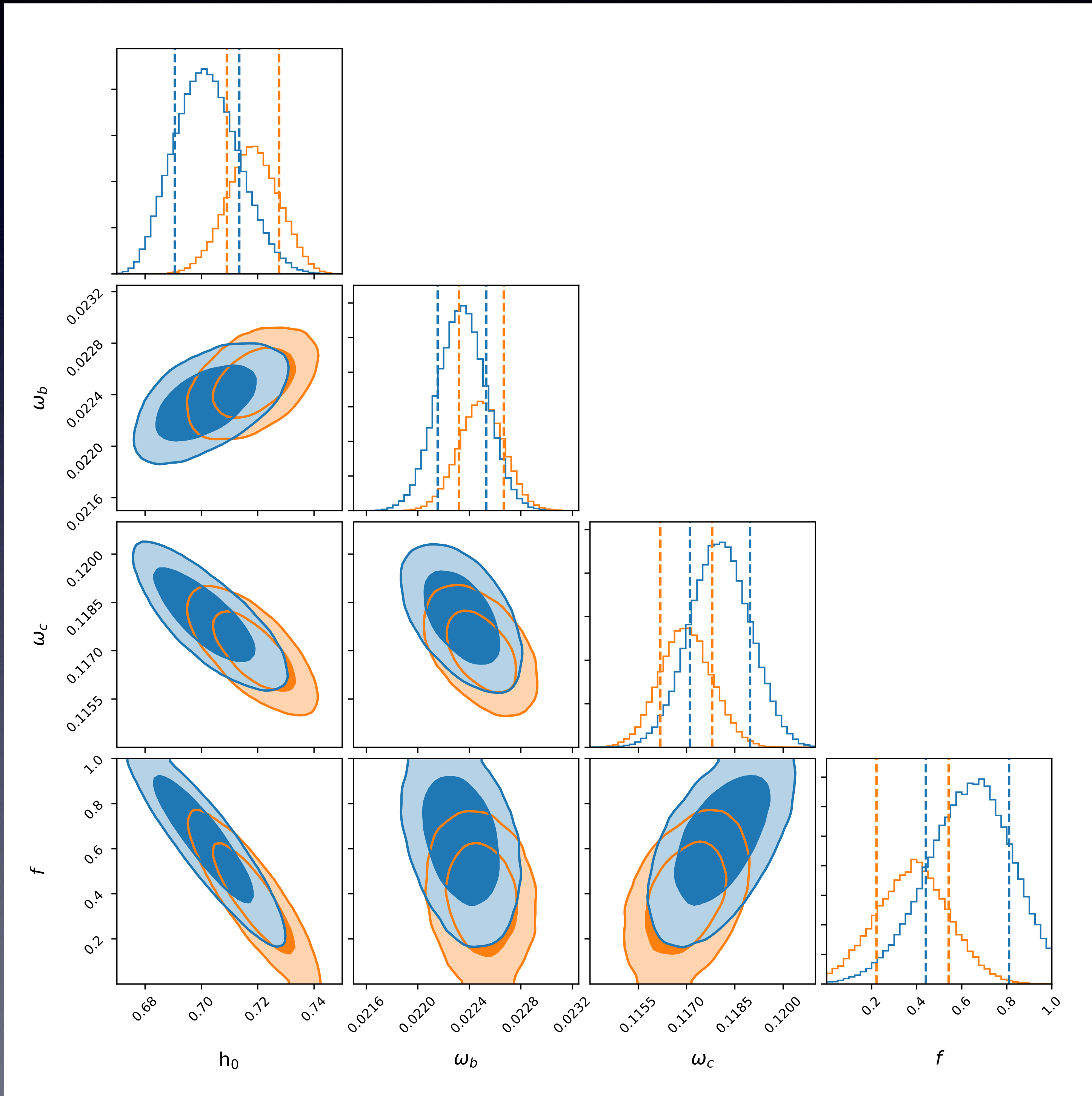
$$P(k,f) = P_{\mathrm{MRL}}(k) + f(P_{\mathrm{PL}}(k) - P_{\mathrm{MRL}}(k))$$

- P_{PL}: best-fit LCDM Power Law
- f= 0: MRL
- f= 1: Power Law
- Bayesian analysis where the base 6 LCDM parameters and f are varied



Posteriors

- In this “deformation model”, f is degenerate with H0
- With just the TT dataset, there is a marginal improvement to the likelihood
- With H0 constraint, calculate a Bayes factor of logK= 5.7

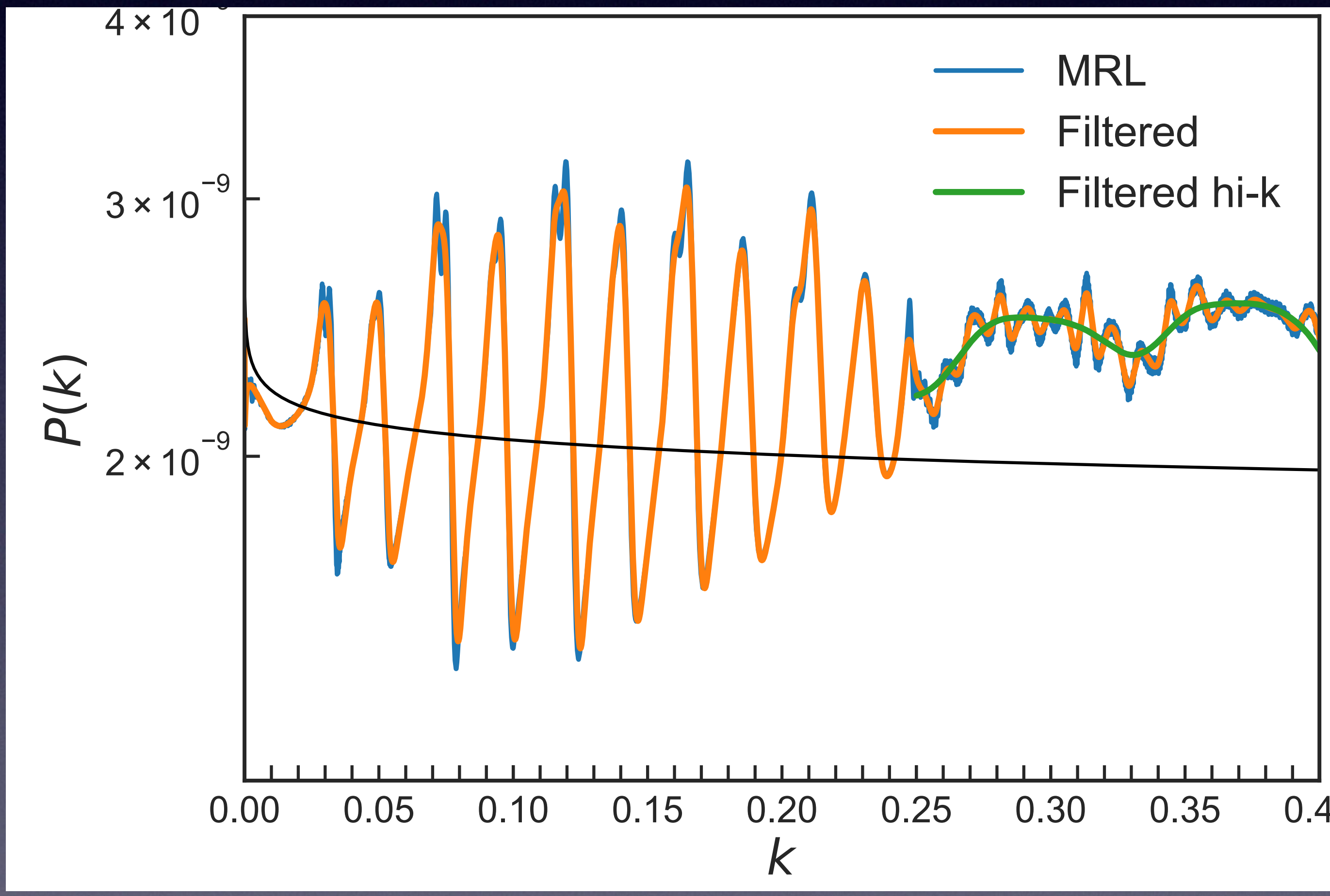


Priors

- MRL-reconstructed PPS is non-trivial
- Potentially over-fit
- or a priori unlikely.
- Maybe just fitting noise in the Planck 2015 data.
- However, this PPS survived new additions to the 2018 dataset
- That this PPS has a well-defined observable effect on the C_ℓs further contradicts the idea that the result is just noise.
- Maybe the numerous, non-trivial features in this PPS are a priori unlikely.
- Subjective prior belief is nothing to build firm conclusions on.
- Such a prior preference for a featureless PPS lasts until someone writes down an inflationary potential that predicts the features derived in the deconvolution.
- We do not seek to rule out ideas solely on a priori arguments.

Filtering

- Find which features are important
- Applying a low-pass filter, with a cutoff frequency that only mildly degrades the likelihood, yields the orange PPS
- Can filter all the features above k~0.25 away and yield the same likelihood



Conclusion

- Model independent methods are necessary to explain the H0 tension
- GP can test consistency between datasets and between LCDM - its a powerful systematics finder
- Model independent methods can give us surprising results such as the MRL PPS solution to the H0 tension
- A power-law PPS is not the sole form of PPS that fits the Planck data well.
- Single-field slow-roll inflation is not the only game in town