Initial Conditions for Cosmological Simulations: The next generation

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with Cornelius Rampf, Raul Angulo, Cora Uhlemann, Michael Buehlmann, and others

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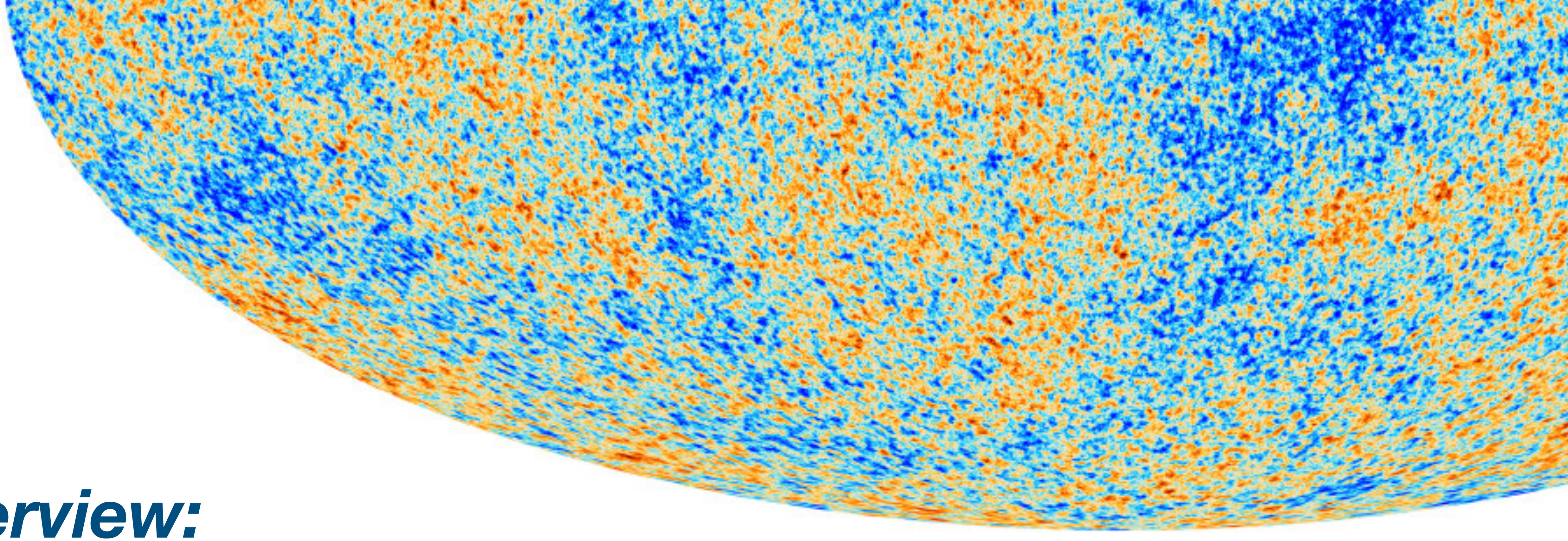


Overview:

1 **The precision challenge: high order, convergence, discreteness**

with ***Michaël Michaux, Cornelius Rampf, Raul Angulo***

Michaux, OH, Rampf, Angulo (2020)

2 **Higher order ICs for CDM+baryon two-fluid sims and Field level PT based on Semiclassical Dynamics**

with ***Cora Uhlemann and Cornelius Rampf***

OH, Rampf, Uhlemann (2020)
Rampf, Uhlemann, Hahn (2020)

https://bitbucket.org/ohahn/monofonic

Simulation workflow

IC phase

linear Einstein-Boltzmann solver (CLASS,CAMB)

isotropic and homogeneous N-body universe

Lagrangian perturbation theory (LPT) to displace particles

sim. phase

N-body simulation

analysis

summary statistics

(with sub-% accuracy requirements for PC)

background movie: Ralf Kaehler, Tom Abel & OH

The precision challenge:
high order, convergence, discreteness

with Michaël Michaux, Cornelius Rampf, Raul Angulo

Michaux, OH, Rampf, Angulo (2020, submitted)

Lagrangian Perturbation Theory

(for single fluid with cold initial data)

Lagrangian map

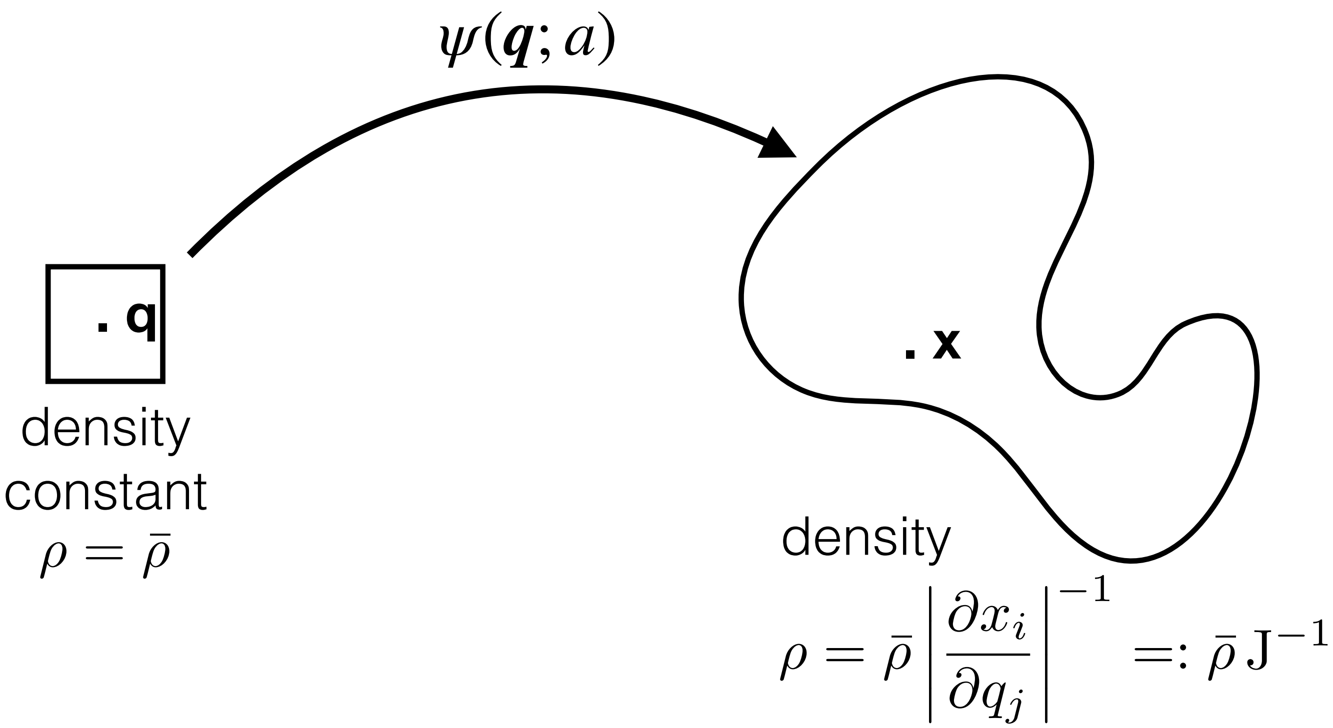
$$\boldsymbol{x}(\boldsymbol{q},t) = \boldsymbol{q} + \boldsymbol{\Psi}(\boldsymbol{q},t)$$

Density can be written as overdensity

$$\rho(\boldsymbol{x},t) = \overline{\rho}(t)\left[1+\delta(\boldsymbol{x},t)\right]$$

Overdensity given by Jacobian

$$\delta(\boldsymbol{x},t) = \frac{1}{J(\boldsymbol{q},t)} - 1$$



We want to solve this as a perturbative series (D is small parameter)

$$\boldsymbol{\Psi}(\mathbf{q},\tau) = \sum_{n=1}^{\infty} D(\tau)^n\,\boldsymbol{\Psi}^{(n)}(\mathbf{q})$$

Buchert (1994), Catelan (1995), Bouchet+(1995)

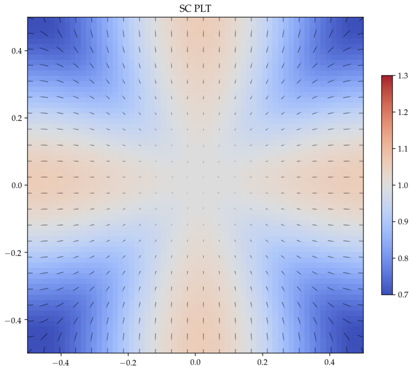
$$\boldsymbol{\psi}_{\mathrm{3LPT}}(\boldsymbol{q},t) = \boldsymbol{\psi}^{(1)}(\boldsymbol{q})\,D_+ + \boldsymbol{\psi}^{(2)}(\boldsymbol{q})\,D_+^2 + \boldsymbol{\psi}^{(3)}(\boldsymbol{q})\,D_+^3$$

Only one d.o.f. : the initial ϕ^{ini} $\quad\Rightarrow\quad$ $\boldsymbol{\psi}^{(1)} = \nabla\phi^{\mathrm{ini}}$

Helmholtz-decomposition yields series of potentials

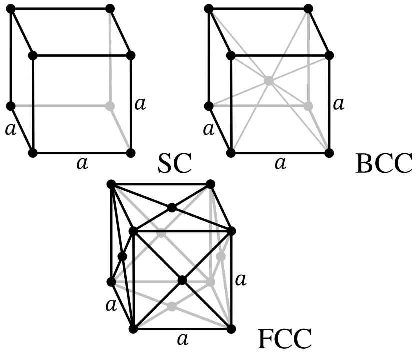
$$\begin{aligned}
\Phi^{(1)} &= \varphi_{\mathrm{ini}}\,,\\
\Phi^{(2)} &= \frac{1}{2}\nabla^{-2}\left[\Phi^{(1)}_{,ii}\Phi^{(1)}_{,jj} - \Phi^{(1)}_{,ij}\Phi^{(1)}_{,ij}\right],\\
\Phi^{(3a)} &= \nabla^{-2}\left[\det\Phi^{(1)}_{,ij}\right],\\
\Phi^{(3b)} &= \frac{1}{2}\nabla^{-2}\left[\Phi^{(2)}_{,ii}\Phi^{(1)}_{,jj} - \Phi^{(2)}_{,ij}\Phi^{(1)}_{,ij}\right],\\
\boldsymbol{A}^{(3c)} &= \nabla^{-2}\left[\nabla\Phi^{(2)}_{,i}\times\nabla\Phi^{(1)}_{,i}\right].
\end{aligned}$$

Discreteness correction



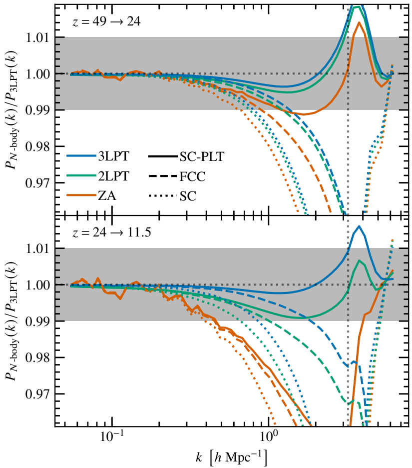
- PLT correct for the deviation of the discrete N-body system to the fluid solution (Joyce et al. 2005, Marcos 2008, Garrison et al. 2016).
- Decaying mode.
- Only valid at linear order.

Discreteness correction



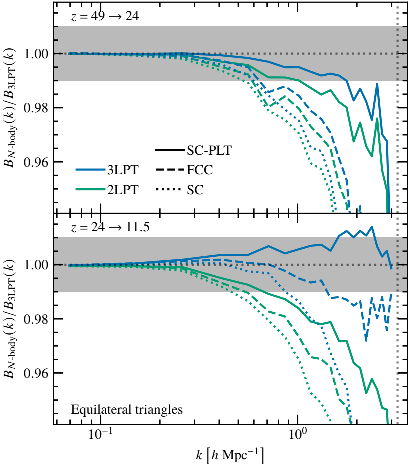
- Oversampling is another way to reduce discreteness effects.
    - Same number of Fourier modes.
    - More particles for each of these modes.
    - We use Face-Centered-Cubic (FCC) runs as reference.

Power Spectrum



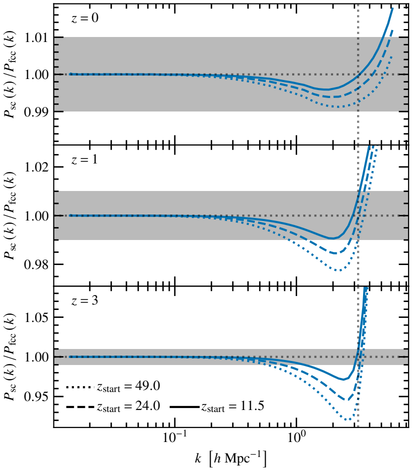
- Discreteness errors reduce power at small scales.
- 3LPT performs significantly better than 2LPT when corrected.

Bispectrum



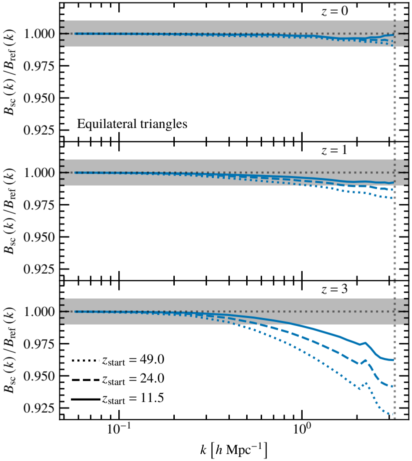
- 3LPT performs significantly better than 2LPT when corrected.
- ZA is completely wrong for the bispectrum.

Discreteness on Power Spectrum



- Loss of power at small scales.
- Errors are more important for high starting redshifts.
- Sub-percent effect at $z = 0$.
- Independent of the LPT order.

Discreteness on Bispectrum



- Loss of power at small scales.
- Errors are more important for high starting redshifts.
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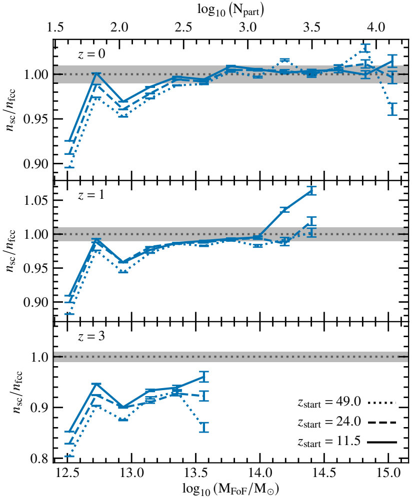
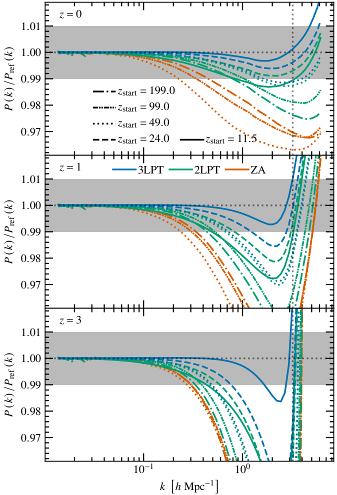
- Independent of the LPT order.

Discreteness on the Mass Function



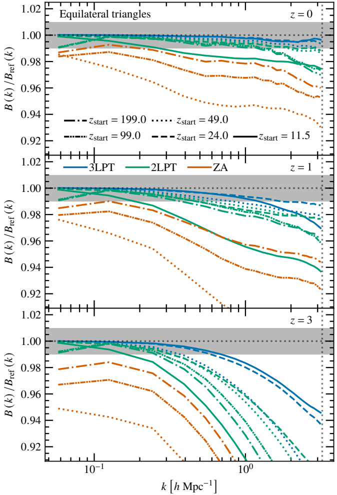
- Underestimates the number of halos, especially the smaller ones.
- Errors are more important for high starting redshifts.

Power Spectrum



- 3LPT started late agrees best at all redshifts.
- ZA started very early converges very slowly.

Bispectrum



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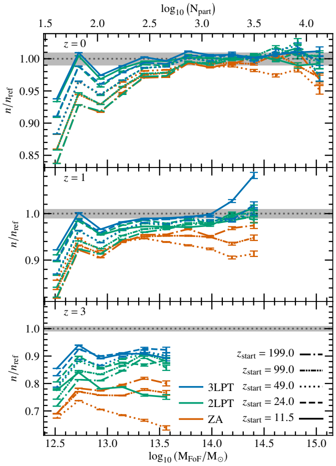
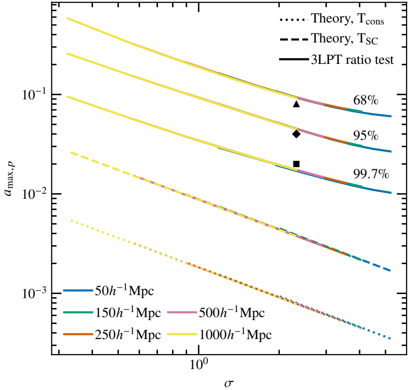
Mass Function



- The LPT order does not matter at $z = 0$, only the starting redshift does.

Starting time



Michaux et al. 2020 (submitted)

- σ: standard deviation of the density field. Higher values when resolving smaller scales.
- $a_{\max,p}$: maximum scale factor for percentile p.
- $a_{\max,n} \approx \frac{0.2}{n}\sigma^{-0.8}$
- $a_{\max,\mathrm{T}} \approx \frac{\mathrm{T}}{12.2\sigma}$

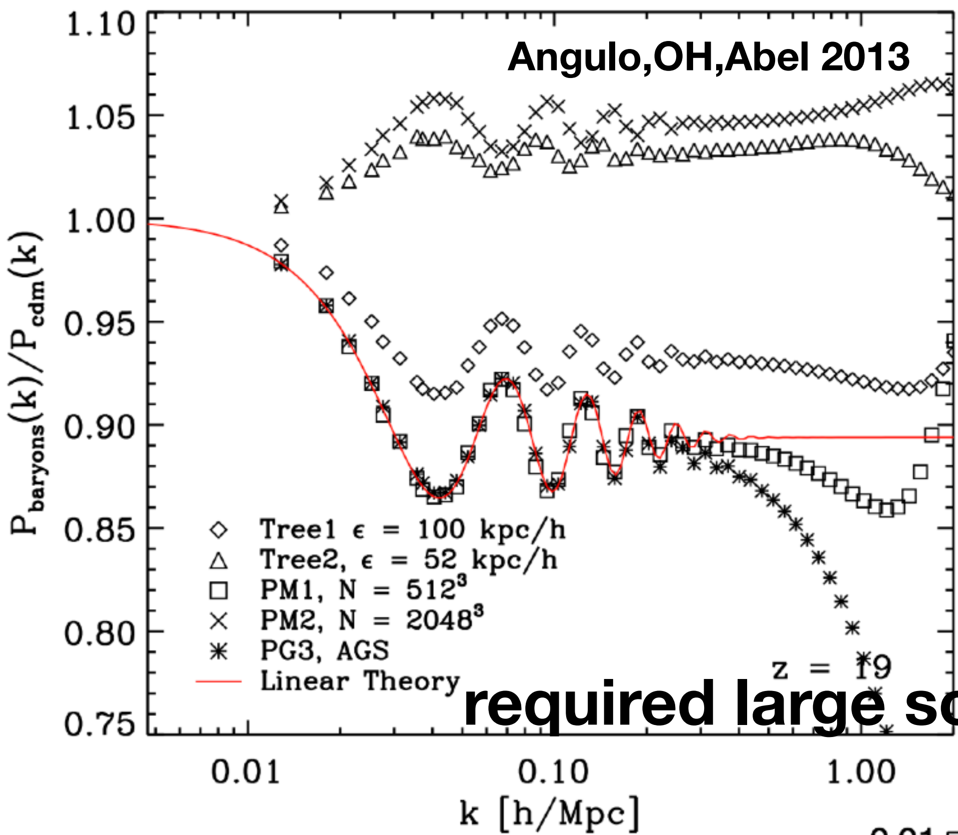
Higher order ICs for CDM+baryon two-fluid sims and Field level PT based on Semiclassical Dynamics

with Cornelius Rampf & Cora Uhlemann

Rampf,Uhlemann,OH 2020, OH, Rampf, Uhlemann 2020

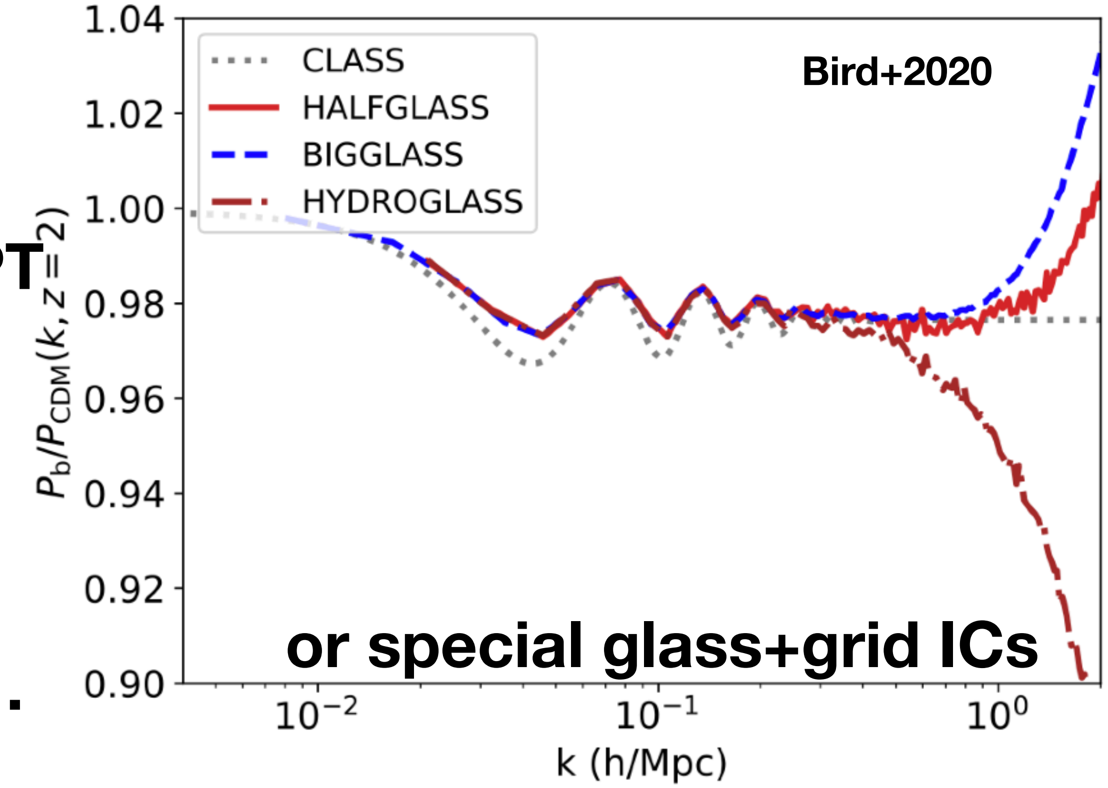
Precision CDM+baryon two-fluid simulations

N-body two-fluid sims have dominant discreteness errors



required large softening…

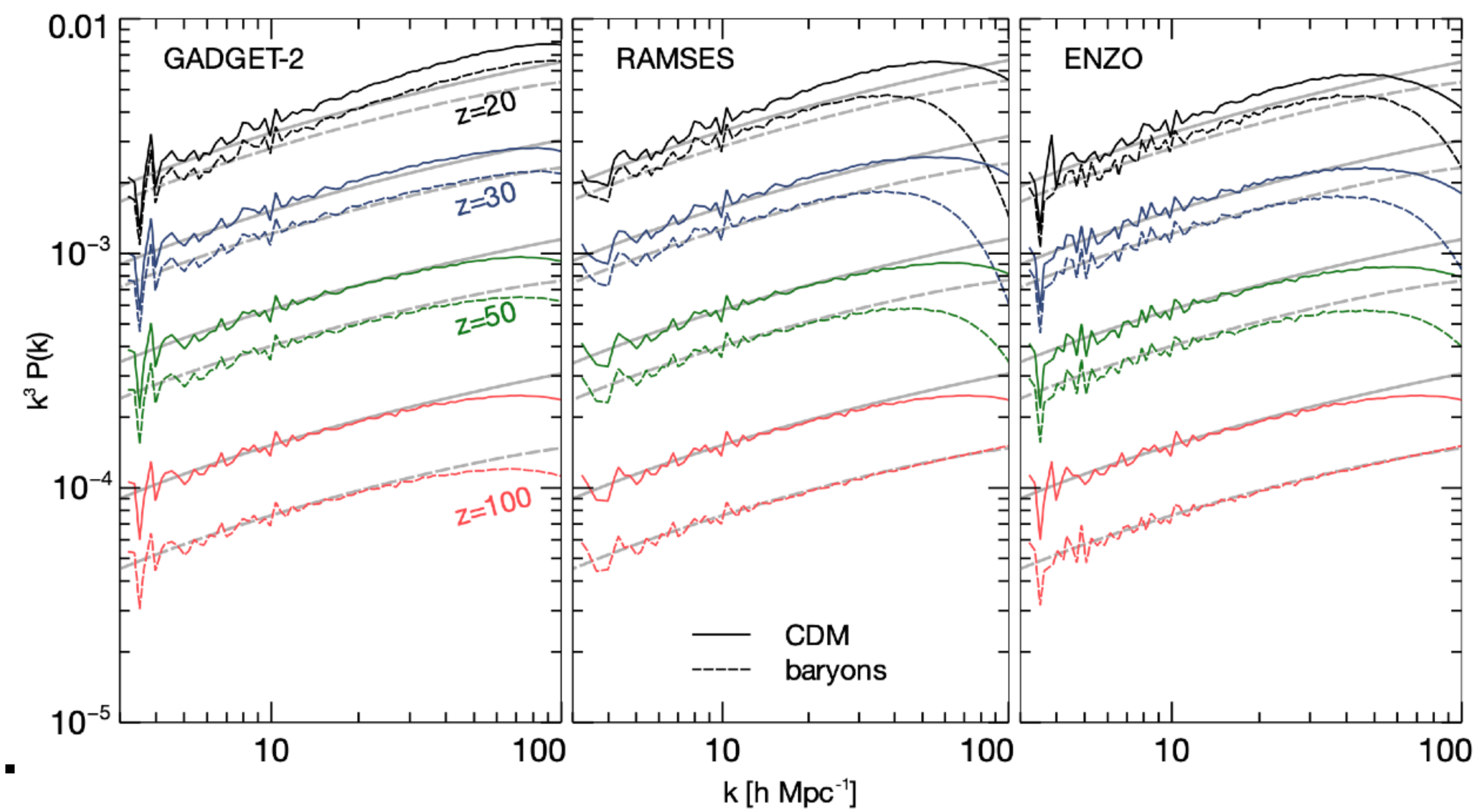
only 1LPT



or special glass+grid ICs

Finite-Volume Eulerian simulations suffer from advection errors:

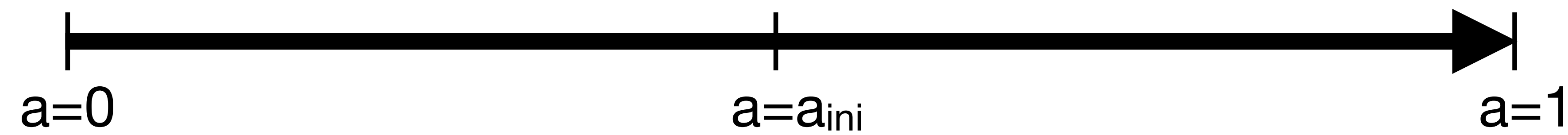
OH&Abel 2012



we solved most of this now…

Forward vs. backscaling ICs

all two-fluid ICs so far used forward approach

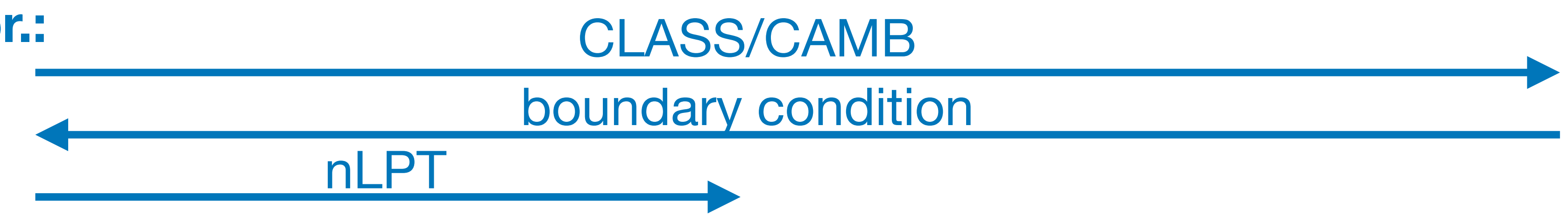


FORWARD appr.:

directly use output from CLASS/CAMB at initial time:

$$\mathbf{x}_\alpha = \mathbf{q} - \frac{\nabla}{\nabla^2}\delta_\alpha \qquad \mathbf{v}_\alpha = \frac{\nabla}{\nabla^2}\theta_\alpha$$

BACKWARD appr.:



in standard LPT, only `growing modes' are used. Do same, use only those that are regular at a=0

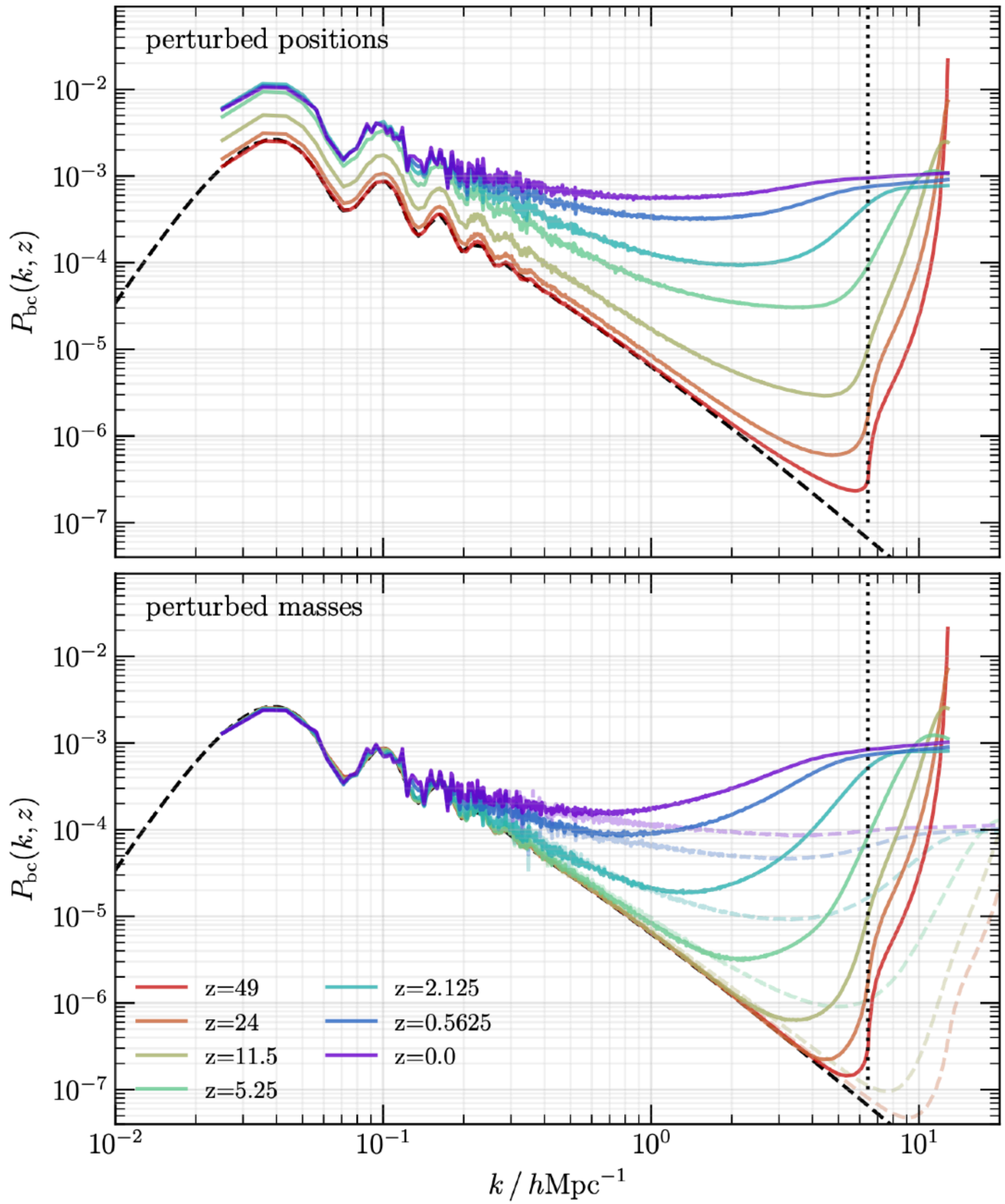
$$\varphi^{\rm ini} = \frac{\nabla^{-2}\delta_{\rm m}^{\rm code}(a_{\rm ref})}{D_+(a_{\rm ref})} \lim_{a\to 0}\frac{D_+(a)}{a} \qquad \delta_{\rm bc}^{\rm ini} = \delta_b^{\rm code}(a_{\rm ref}) - \delta_c^{\rm code}(a_{\rm ref})$$

Both are constant. Pull of PT from them! (cf. Rampf, Uhlemann & Hahn 2020)

This means, use standard LPT, but add additional mass perturbation as

$$m_\alpha(\boldsymbol{q}) = \overline{m}_\alpha\left(1 + \delta_\alpha^{\rm ini}(\boldsymbol{q})\right), \qquad \overline{m}_\alpha := \Omega_\alpha \,/\, \Omega_{\rm m} \qquad \delta_{\rm b}^{\rm ini} = f_{\rm c}\delta_{\rm bc}^{\rm ini} \text{ and } \delta_{\rm c}^{\rm ini} = -f_{\rm b}\delta_{\rm bc}^{\rm ini}$$

Preservation of compensated mode



First test: gravity only evolution…

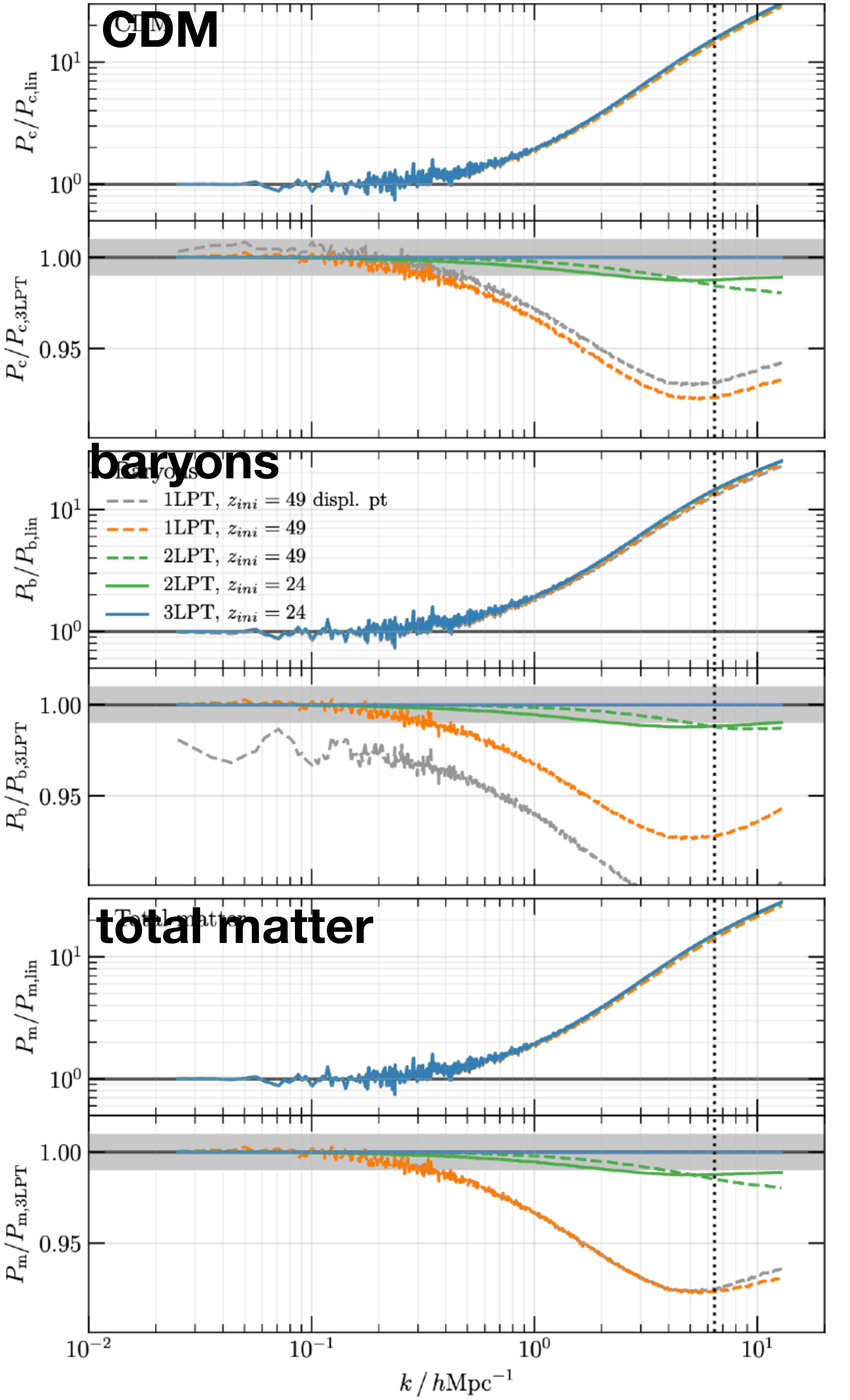
standard displacement perturbed ICs are not able to preserve the compensated mode!

cf. also Angulo,OH&Abel(2013), Bird+2020

mass perturbed `persistent mode' ICs preserve the compensated mode

Hahn, Rampf & Uhlemann 2020, submitted

Gravitional evolution of two-fluid system



With the `persistent modes' approach, we can now push up to 3LPT for two fluids

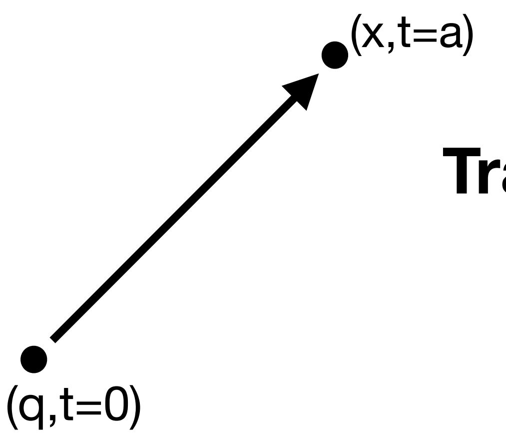
overcomes first order problem of forward IC two-fluid sims!

improvement over 1LPT at z=2 is ~10 per cent on small scales

Hahn, Rampf & Uhlemann 2020, submitted

PPT for Eulerian baryons

How to set up ICs for Eulerian codes consistent with LPT?



Zel'dovich approximation: particle moves on straight line

Transition amplitude for fluid element to go from q to x in time a

Rewrite these simple trajectories as a classical action

$$S_0(\boldsymbol{x},\boldsymbol{q};a) = \frac{1}{2}(\boldsymbol{x}-\boldsymbol{q})\cdot\frac{\boldsymbol{x}-\boldsymbol{q}}{a}$$

Apply Feynman trick to get propagator

$$K_0(\boldsymbol{x},\boldsymbol{q};a) = N\exp\left\{\frac{\mathrm{i}}{\hbar}S_0(\boldsymbol{x},\boldsymbol{q};a)\right\}$$

at NLO have also effective potential

then evolve field $\psi_0(\boldsymbol{x};a) = \int \mathrm{d}^3q\, K_0(\boldsymbol{x},\boldsymbol{q};a)\,\psi_0^{(\mathrm{ini})}(\boldsymbol{q})$

Recover moment hierarchy of evolved field by taking gradients

$$\rho = \psi\psi^* \qquad \mathbf{j} = \frac{i\hbar}{2}\left(\psi\nabla\psi^* - \psi^*\nabla\psi\right) \qquad \ldots$$

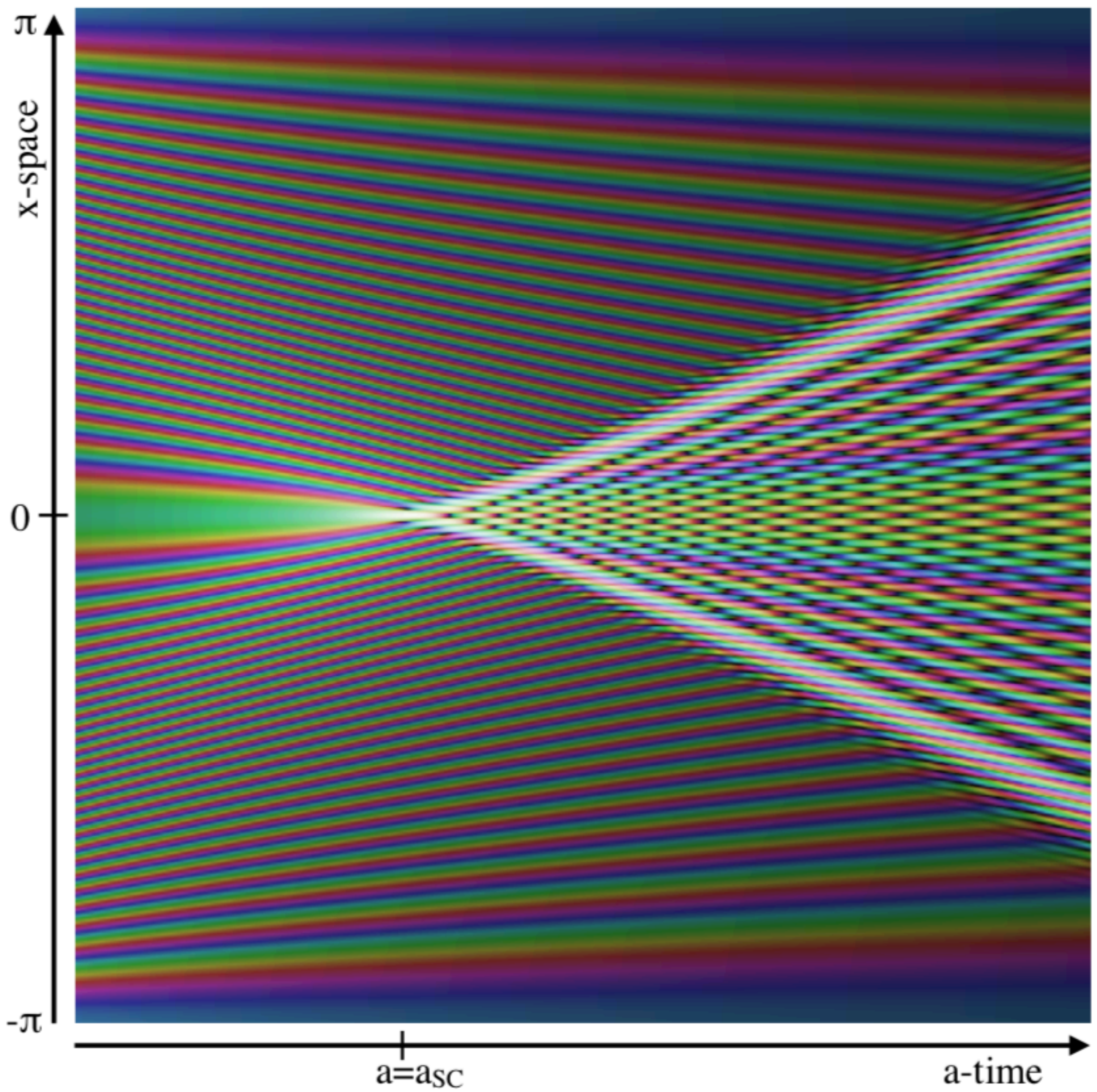
Simple to extend to two fluids (Rampf+2020)

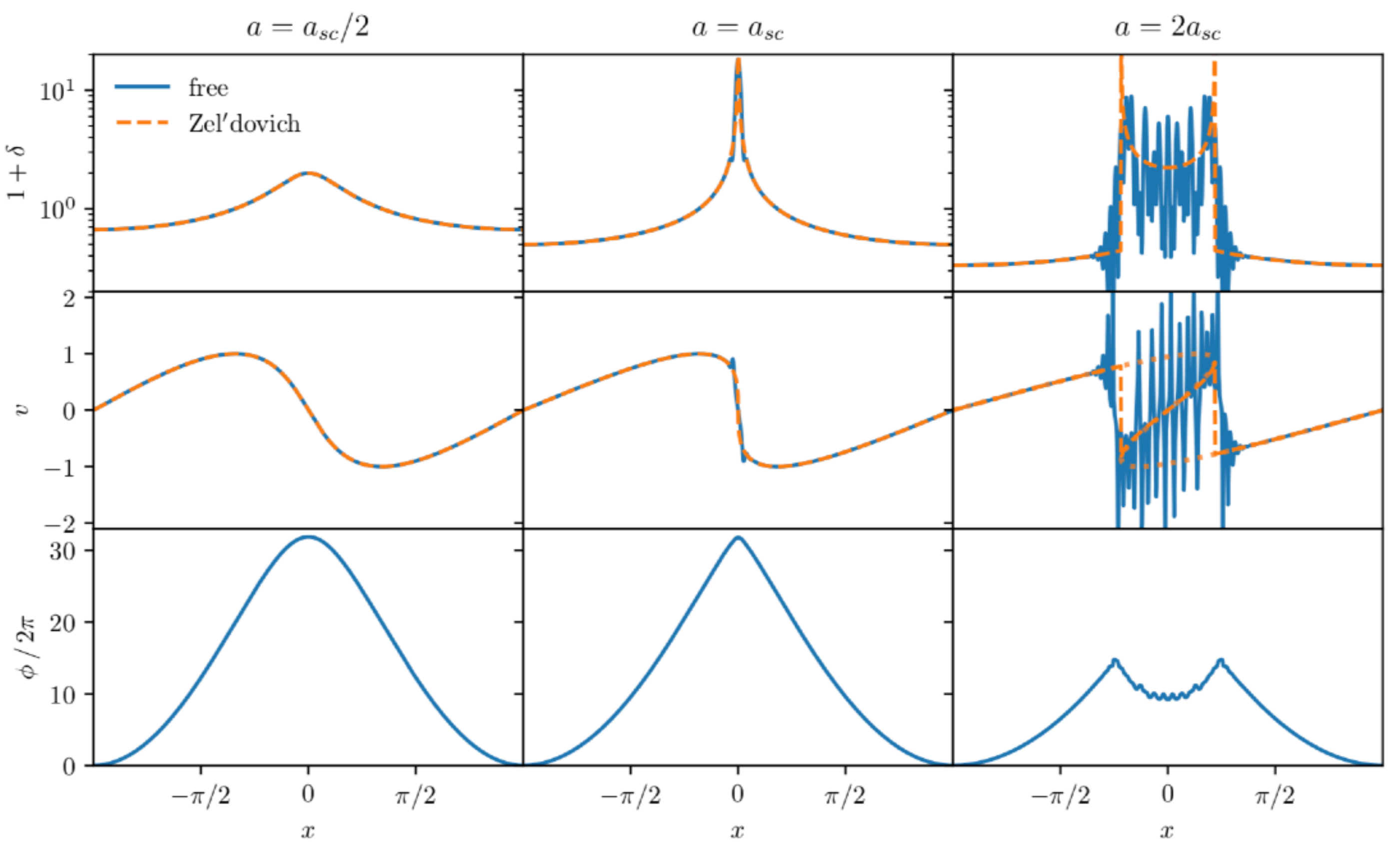
Uhlemann, Rampf, Gosenca & OH (2019)
Rampf, Uhlemann & OH (2020)

see also Short&Coles (2006)

PPT dynamics

Obtain a field version of Zeldovich trajectories:





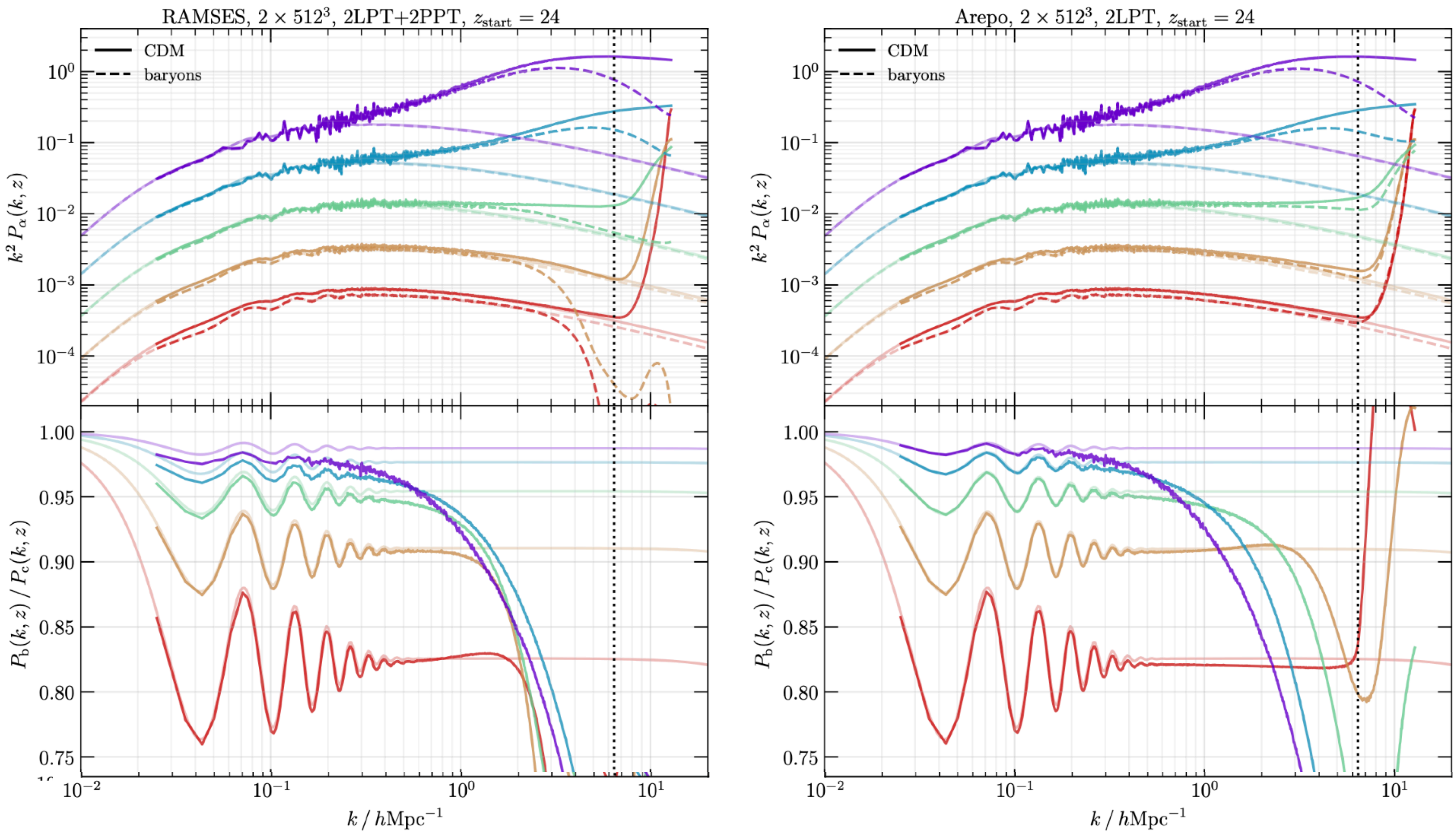
dynamics 'smoothed' by hbar scale

Interference = multi-streaming

See Cora Uhlemann's talk "A semiclassical path to the cosmic web"

Uhlemann, Rampf, Gosenca & OH (2019)

2LPT Arepo vs. 2PPT Ramses

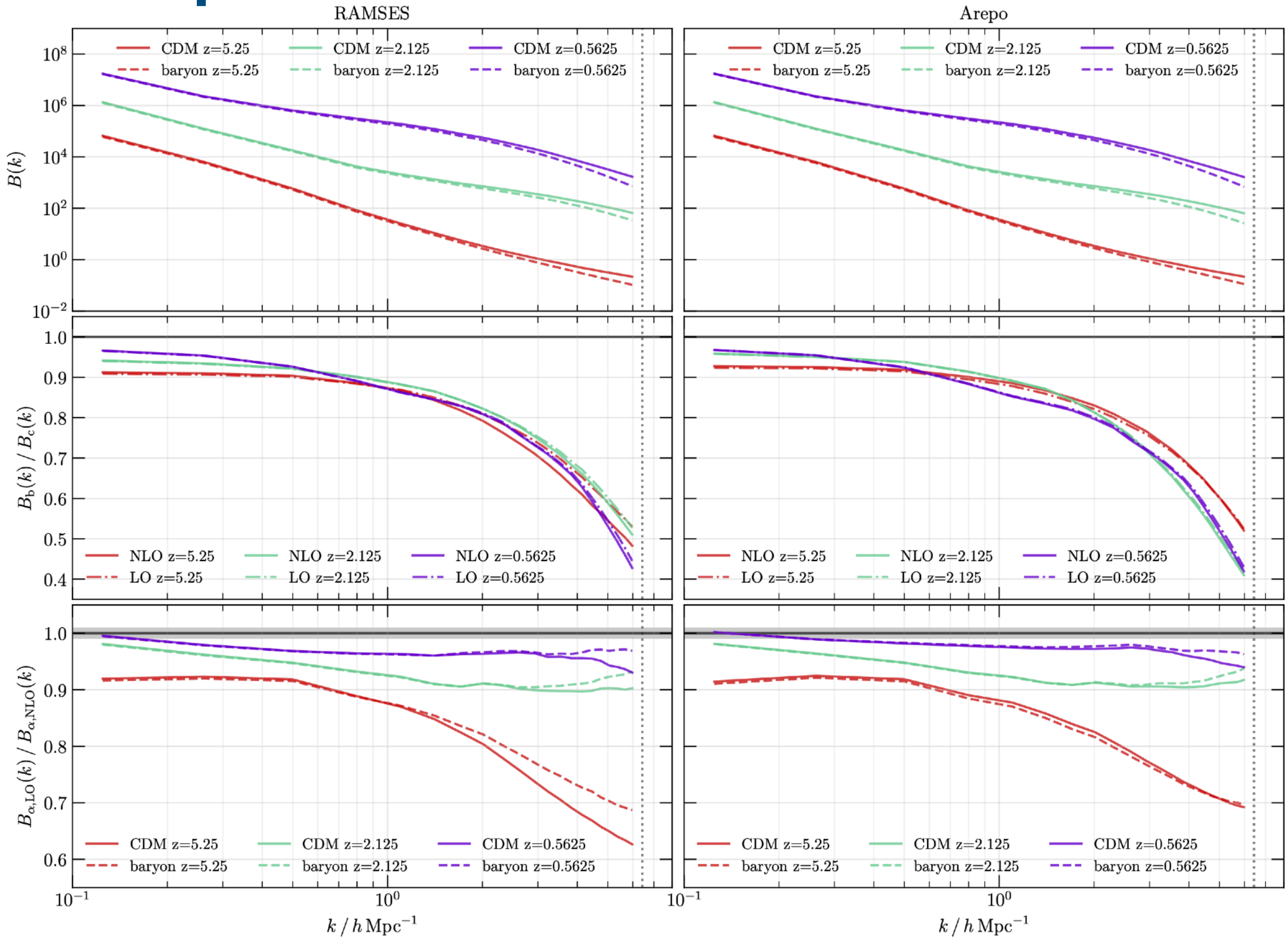


Due to PPT and advection errors, high-z Ramses more smooth

At z<2.5 results become very similar in the power spectra

Hahn, Rampf & Uhlemann 2020, submitted

2LPT Arepo vs. 2PPT Ramses



Bispectra more similar between codes than power spectra, large improvement from LO to NLO!!

Hahn, Rampf & Uhlemann 2020, submitted

MUSIC 2 — towards a whole ecosystem for ICs

The roadmap… **Do get in touch if you want to be early adopter!**

MUSIC2 monofonIC https://bitbucket.org/ohahn/monofonic

single resolution (=only full cosmological volume) version

- direct integration of CLASS
- **up to 3LPT, incl. baryons**
- PLT corrections
- **new propagator approach for Eulerian baryons**
- still modular architecture: multi code, easily extensible
- MPI+threads (no more limits)
- call directly from within your sim code (in prep.)

MUSIC2 cosmICweb beta release: fall 2020 or get in touch!

- **cosmological ICs in the cloud**
- reproducibility of zooms
- towards “one” numerical universe
- integrates with MUSIC1 update

MUSIC2 polyfonIC next year

multi resolution (=zoom) version

- will replace MUSIC1
- MPI+threads

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