

Hyperinflation: Stuck in the Swampland

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Cosmology from Home 2020

August 14, 2020

Overview

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- ▶ Single-field Slow-roll Inflation and Beyond

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- ▶ Hyperinflation-Perturbations

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- ▶ Observational Constraints

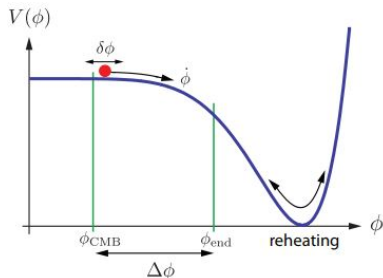
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- ▶ Motivation for Hyperinflation
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- ▶ Observational Constraints
- ▶ Motivation for Further Work

Single-field Slow-roll Inflation and Beyond

- ▶ Inflation⁰: Motivated by zero-order problems of C.S.M (Horizon & Flatness)
- ▶ Quantum fluctuations stretched out of the horizon seed LSS
- ▶ Single-field slow-roll: Scalar rolls down its potential¹
- ▶ Agreement with observations²



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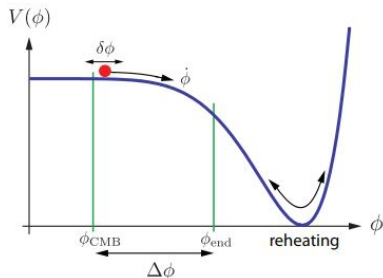
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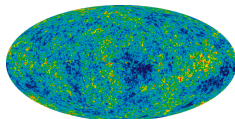
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⇒ Why bother with multiple fields, then?

- Natural from particle theory viewpoint
- Richer phenomenology³
- "Swampland" Conjectures
- Non-Gaussianity Signatures⁴



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Motivation for Hyperinflation

- ▶ Required # of e-folds: 1-D, Potential Tuning
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 2-D, Centrifugal force

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- Spinflation⁵: \mathbb{R}^2 Field-space

$$S_{R^2} = \int dt d^3x \left[a^3(t) \left(-\frac{1}{2}(\nabla_\mu \rho)^2 - \frac{1}{2}(\nabla_\mu \phi)^2 - V(\rho) \right) \right] \quad (1)$$

Velocities redshift: angular motion becomes irrelevant.

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field-space $J = M_H^2 \sinh^2 \left(\frac{\phi}{M_H} \right) \dot{\psi}$ exponentially large



Angular motion relevant **throughout** inflation.

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Angular motion relevant **throughout** inflation.

- ▶ F-S J delaying Inflation \rightarrow Adiabatic Perturbations (scale invariant)
- ▶ $\epsilon \approx 1$: Swampland Evaded⁷ (?)

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Setting the Stage

The action: Rescaling the fields w.r.t. the F-S curvature scale, $\Phi^a \rightarrow \frac{\phi^a}{M_H}$

$$S = \frac{1}{2} \int d^D x \sqrt{-g} \left[M_p^2 R - M_H^2 \mathcal{G}^{ab}(\phi) \partial_\mu \phi^a \partial_\nu \phi^b g^{\mu\nu} - 2V \right]. \quad (3)$$

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For $\rho \gg 1$, $f(\rho) = M_H \sinh(\rho) \approx M_H e^\rho$

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$$V(\rho) \approx V_0 e^{\frac{M_H^2}{M_p^2} p \rho}, \quad p \equiv \frac{M_p^2}{M_H^2} \frac{\partial [\ln V]}{\partial \rho} \quad (4)$$

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EoM \rightarrow autonomous system:

$$\begin{aligned} x' &= -(3 - \epsilon)(x + p_\rho) + y^2 \\ y' &= -(3 - \epsilon + x)y \end{aligned} \quad (5)$$

and the Hubble flow parameter, in the small field limit $M_H \leq 10^{-3} M_p$

$$\epsilon = \frac{M_H^2}{2 M_p^2} [x^2 + y^2] \rightarrow 0. \quad (6)$$

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Background Dynamics - Gradient

Scaling solutions: $\epsilon' = 0$

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► $[x, y] = [-\rho, 0] \Rightarrow$ Local Lyapunov exponents $(\lambda_1, \lambda_2) = (-3, \rho - 3)$.

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Stable solution: $\rho < 3$

- $\rho' \rightarrow -\rho$
- $\phi' \rightarrow 0$

Unstable solution: $\rho > 3$

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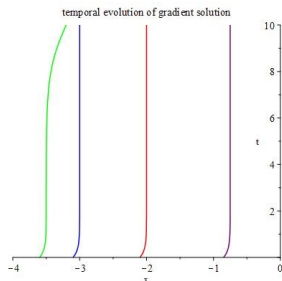
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► $[x, y] = [-p\rho, 0] \Rightarrow$ Local Lyapunov exponents $(\lambda_1, \lambda_2) = (-3, p - 3)$.

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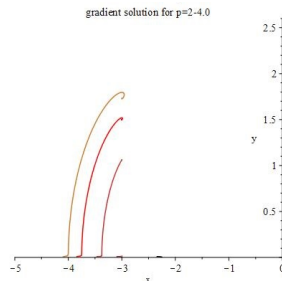
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Background Dynamics - Hyperbolic

► $[x, y]_{Hyper} = [-3, \pm\sqrt{3p-9}]$. Iff $p \geq 3$, positivity of square root argument.

⇒ The local Lyapunov exponents

$$(\lambda_1, \lambda_2) = \left(-\frac{3}{2} + \frac{3}{2}\sqrt{9 - \frac{8}{3}p}, -\frac{3}{2} - \frac{3}{2}\sqrt{9 - \frac{8}{3}p}\right)$$

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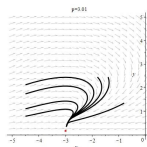
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- $3 < p < 3.375$ real eigenvalues

$$\lambda_1 \neq \lambda_2 < 0$$

critical point is a stable node.



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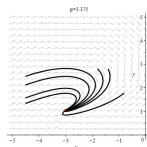
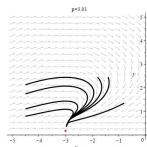
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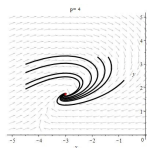
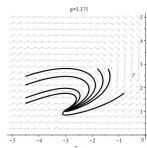
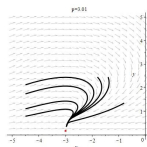
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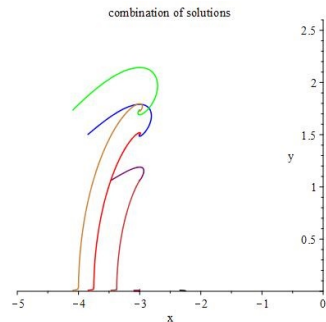
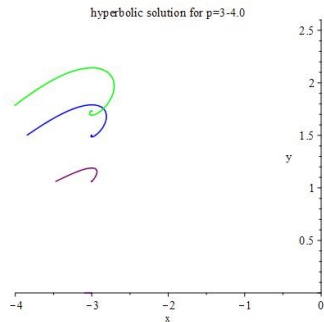
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• $p > 3.375$ compl. conjugate eigenvalues
 $Re(\lambda) < 0$
system spirals towards stable focus

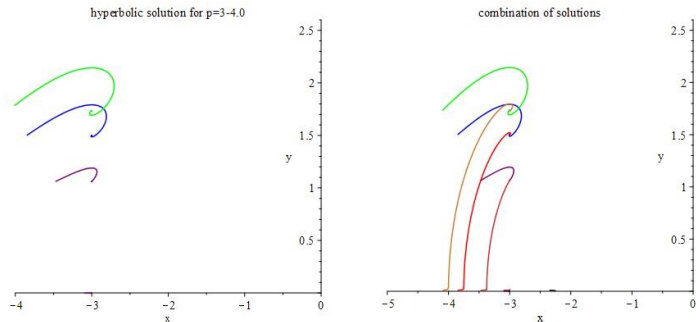


Background Dynamics - Combined



For $p > 3$ the gradient solution is unstable and for any $y_{in} \neq 0$, the system evolves exponentially fast towards the hyperbolic solution.

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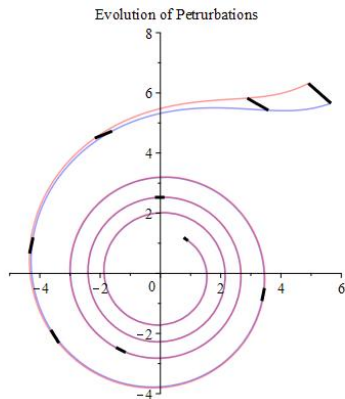


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Hyperinflation is an attractor in the regime that gradient solution is a repeller.

Perturbation Dynamics - Heuristic

Evolution of two neighboring trajectories in the configuration F-S:



Expected behaviour:

- ▶ Adiabatic (along the trajectory) perturbations asymptote to a constant
- ▶ Isocurvature (normal to the trajectory) vanish

Perturbation Dynamics - Numerical

⇒ Switch to Local Orthogonal Basis.

Define the adiabatic & entropic⁹ directions:

$$\hat{\sigma}^a = \frac{\dot{\phi}^a}{\dot{\sigma}}, \quad \hat{s}^a = \frac{1}{\sqrt{\mathcal{G}}} \epsilon^{ab} \hat{\sigma}^b \mathcal{G}_{bc}. \quad (8)$$

where $\dot{\sigma} = \sqrt{G_{ab} \dot{\phi}^a \dot{\phi}^b}$ the background field velocity.

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⇒ Zweibein transformation matrix is simple rotation for scaling solutions!

$$\begin{bmatrix} \hat{e}_\sigma \\ \hat{e}_s \end{bmatrix} = \begin{bmatrix} \frac{x}{\dot{\sigma}} & \frac{y}{f \dot{\sigma}} \\ \frac{y}{\dot{\sigma}} & -\frac{x}{f \dot{\sigma}} \end{bmatrix} \cdot \begin{bmatrix} \hat{e}_\rho \\ \hat{e}_\phi \end{bmatrix}, \quad (9)$$

Define $z = \ln\left(\frac{k_*}{k}\right)$ and $k_* = a(t) H$ the wavelength of mode that exits the horizon at time t .

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The perturbation e.o.m:

$$\begin{aligned} \partial_z^2 U_l^\sigma + 3 \partial_z U_l^\sigma + e^{-2z} U_l^\sigma + 6 y U_l^s + 2 y \partial_z U_l^s &= 0 \\ \partial_z^2 U_l^s + 3 \partial_z U_l^s + e^{-2z} U_l^s - 2 y^2 U_l^s - 2 y \partial_z U_l^\sigma &= 0. \end{aligned} \quad (10)$$

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The perturbation e.o.m:

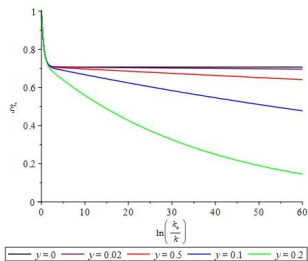
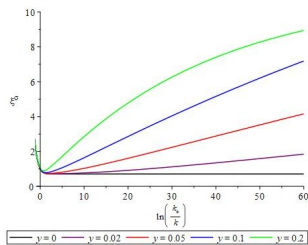
$$\begin{aligned} \partial_z^2 U_I^\sigma + 3 \partial_z U_I^\sigma + e^{-2z} U_I^\sigma + 6 y U_I^s + 2 y \partial_z U_I^s &= 0 \\ \partial_z^2 U_I^s + 3 \partial_z U_I^s + e^{-2z} U_I^s - 2 y^2 U_I^s - 2 y \partial_z U_I^\sigma &= 0. \end{aligned} \quad (10)$$

Initialisation ansatz: $z \rightarrow -\infty$, modes are decoupled, unaffected by the field-space curvature induced mixing.

$$U_{initial}^{(\sigma,s)} = \frac{H}{\sqrt{k^3}} \frac{\sqrt{\pi}}{2} e^{-(\frac{3}{2}z)} H_{\frac{3}{2}}(z) \quad (11)$$

⁹Phys. Rev. D **63** (2000), 023506 [arXiv:astro-ph/0009131 [astro-ph]]

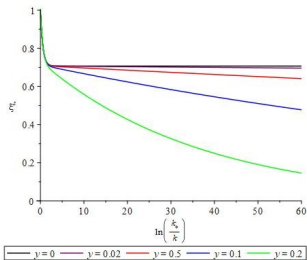
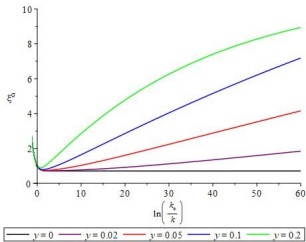
Perturbation Dynamics - Plots



⇒ As y increases, the modes reach their respective asymptote faster

$$n_s^{ad} = 1 + \frac{d \ln[\mathcal{P}_\sigma]}{d(\ln k)} \rightarrow \text{unobservable} : \text{Constraint } y < 1.$$

Perturbation Dynamics - Plots

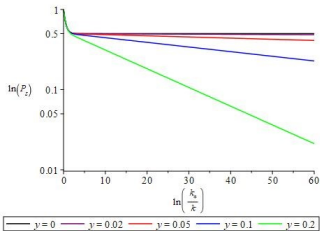
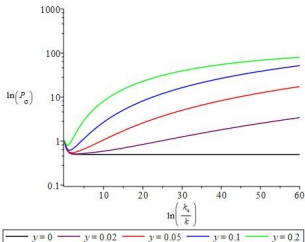


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⇒ Power Spectrum: $\mathcal{P}_\sigma = \frac{k^3}{2\pi^2} |U|^2,$

$\mathcal{P}_s = \frac{k^3}{2\pi^2} |V|^2.$

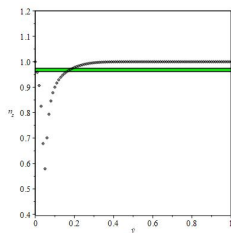


Observational Constraints

⇒ Measured¹⁰ value of adiabatic perturbations spectral index:

$$n_s = 0.968 \pm 0.006$$

$$\downarrow$$
$$0.8 \times 10^{-2} \leq y \leq 10^{-2} \text{ \& } \\ 0.16 \leq y \leq 0.19.$$



¹⁰Astron. Astrophys. **594** (2016), A13[arXiv:1502.01589 [astro-ph.CO]].

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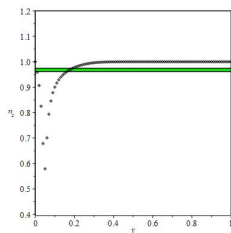
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⇒ The tensor-to-scalar ratio:

$$r_* = \frac{\Delta_t^2(k)}{\Delta_s^2(k)} = \frac{4 M_H^2}{M_p^2} \frac{9 + y^2}{\langle |U_k|^2 \rangle} \quad (12)$$



Observational bound¹⁰, $r < 0.10$.
Numerically, $|U_k|^2 < 50$

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$$\frac{M_H^2}{M_p^2} \rho < \frac{5}{12}. \quad (13)$$

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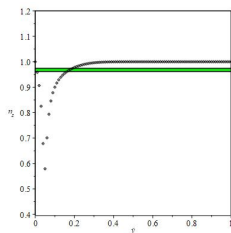
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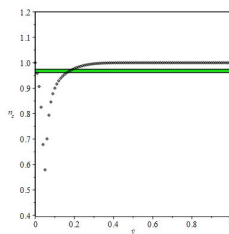
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⇒ Direct contradiction with the "Swampland" deSitter conjecture: $\epsilon \approx 1$.
Hyperinflation still stuck in the Swampland

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Motivation for Further Work

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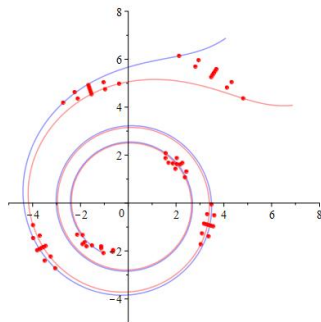
I-R limit: Stochastic Hyperinflation!

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I-R limit: Stochastic Hyperinflation!
Non-Gaussian Signature ...



Thank you for your attention!