The Distribution of Vacua in Random Landscape Potentials

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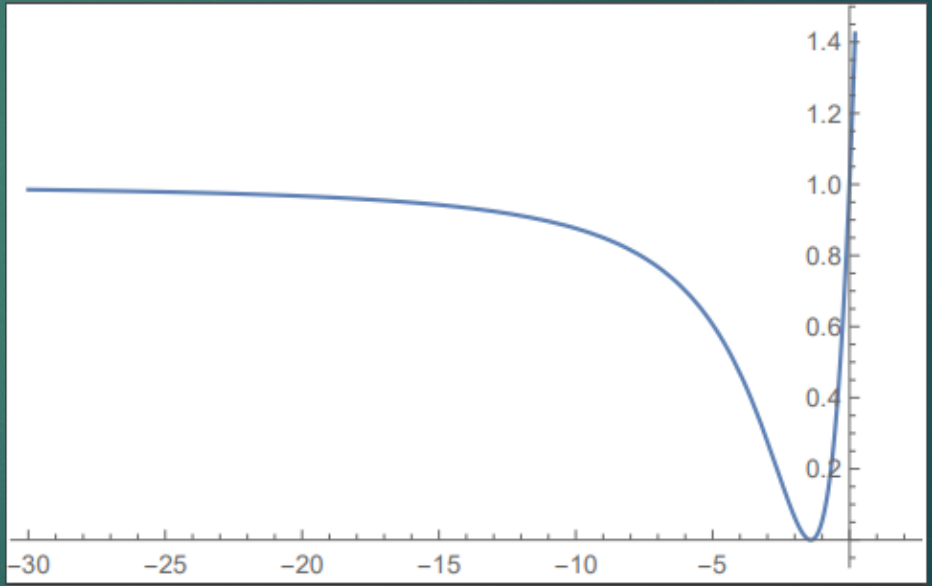
Motivation

- In the beginning there was the Big Bang, shortly after the Big Bang there was inflation.
- String theory compactification can produce a landscape, from which the *inflaton field* arises as a single degree of freedom in a 100+ dimensional potential
- In this picture, the universe inflates when the gradient of the field is small (slow-roll), and evolution ends at a local minimum slightly above zero (corresponding to dark energy).



Research questions

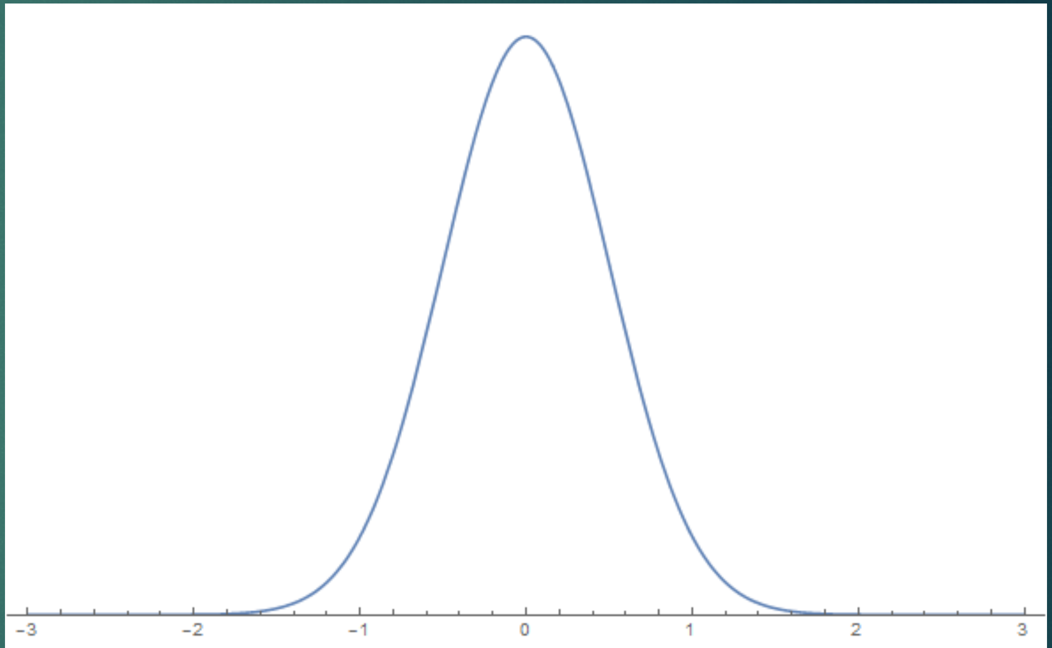
- Given a minimum, how likely is it to have potential value $\Lambda > 0$?
  - This has got to be < 0.5, because there should be more minima with $\Lambda < 0$.
- How steep are the slopes leading into this minimum?
- Before inflation, the universe was presumably at a saddle with only one downhill direction. How steep are the slopes leading out of this saddle?

Random Gaussian Fields

$$P[F(\mathbf{r}_1), F(\mathbf{r}_2), \ldots, F(\mathbf{r}_m)]dF(\mathbf{r}_1)dF(\mathbf{r}_2)\cdots dF(\mathbf{r}_m)\,, \qquad (2.1)$$

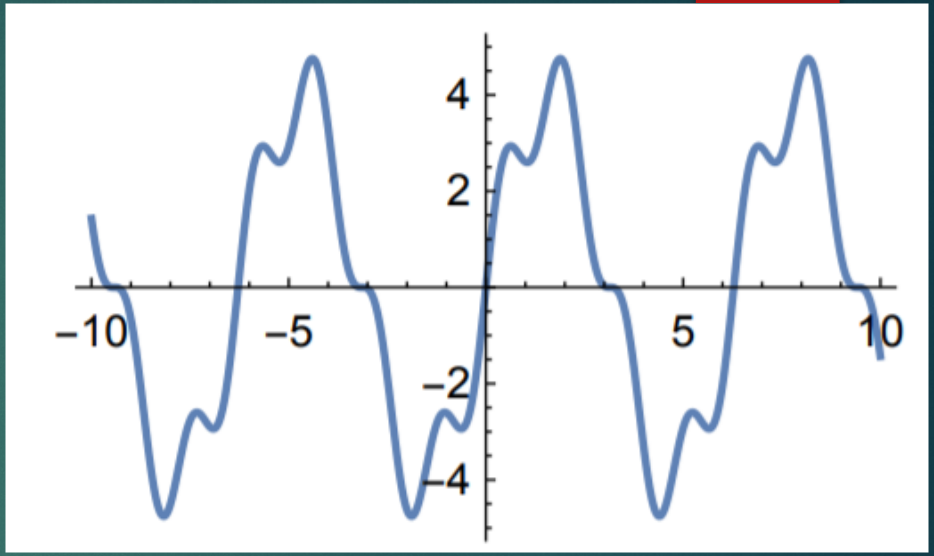
A *Gaussian random field* is one for which the various m-point probability distributions (eq. [2.1]) are multivariate Gaussians.

- Problem: the string theory landscape is formidably complex.
- Idea: model the string landscape as a *Random Gaussian Field.*
- *Random Gaussian Field*: the definition is technical, but the idea is analogous to that of *Gaussian distributions*. By the central limit theorem, independent random variables tend towards this distribution.
- By assuming that the ~100 dimensional potential of the string theory landscape is Gaussian Random, we are assuming that it arises from the superposition of a large number of independent, uncorrelated interaction terms.
- Simplest way to model a very complicated function



Math …

- Gaussians have two parameters. Random Gaussian Fields have three: σ_0, σ_1 and σ_2,corresponding to the square root of the average of the square of the potential, first derivative and second derivative respectively.
- A key result is that only the combination $\gamma = \sigma_1^2 / \sigma_0 \sigma_2$ is relevant for our purposes.
  - This is because we can rescale the potential or the field, and not affect the statistics.
- γ can be thought of as a measure of how turbulent the potential is. When it is small, the potential is more turbulent.
- It can be shown that $0 < \gamma < 1$



$$\langle FF \rangle = \sigma_0^2$$

$$\langle \eta_i \eta_j \rangle = \frac{\sigma_1^2}{3} \delta_{ij}$$

$$\langle \zeta_{ij} \zeta_{kl} \rangle = \frac{\sigma_2^2}{15} (\delta_{ij} \delta_{kl} + \delta_{ik} \delta_{jl} + \delta_{il} \delta_{jk})$$

Math …

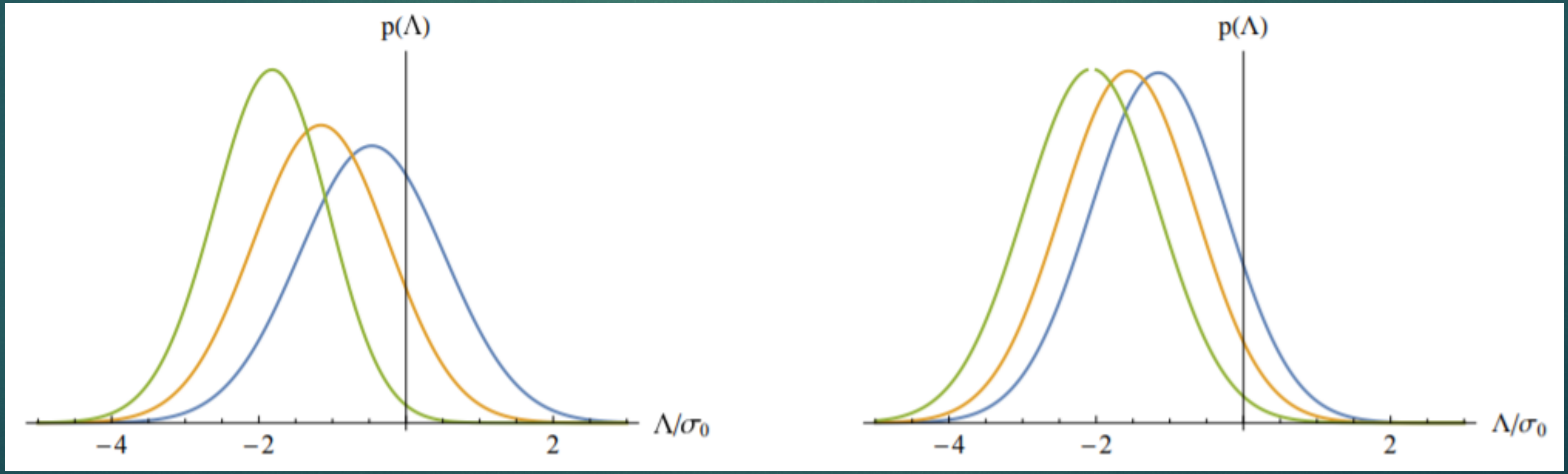
- The technical details are in our paper on the arXiv.
- Ultimately, the integral we want to compute is (for N = 4):

$$\int_0^\infty d\lambda_1 \int_0^{\lambda_1} d\lambda_2 \int_0^{\lambda_2} d\lambda_3 \int_0^{\lambda_3} d\lambda_4 (\lambda_1-\lambda_2)(\lambda_1-\lambda_3)(\lambda_1-\lambda_4)(\lambda_2-\lambda_3)(\lambda_2-\lambda_4)(\lambda_3-\lambda_4)\lambda_1\lambda_2\lambda_3\lambda_4 \exp\left[-\frac{\alpha K \alpha}{2}\right]$$

- Where the λ's are the eigenvalues of the Hessian at the point, and the αKα are polynomial functions of the λ's (with some contribution from γ and V).
- If anyone knows how to compute this integral in 100 dimensions efficiently – let me know.

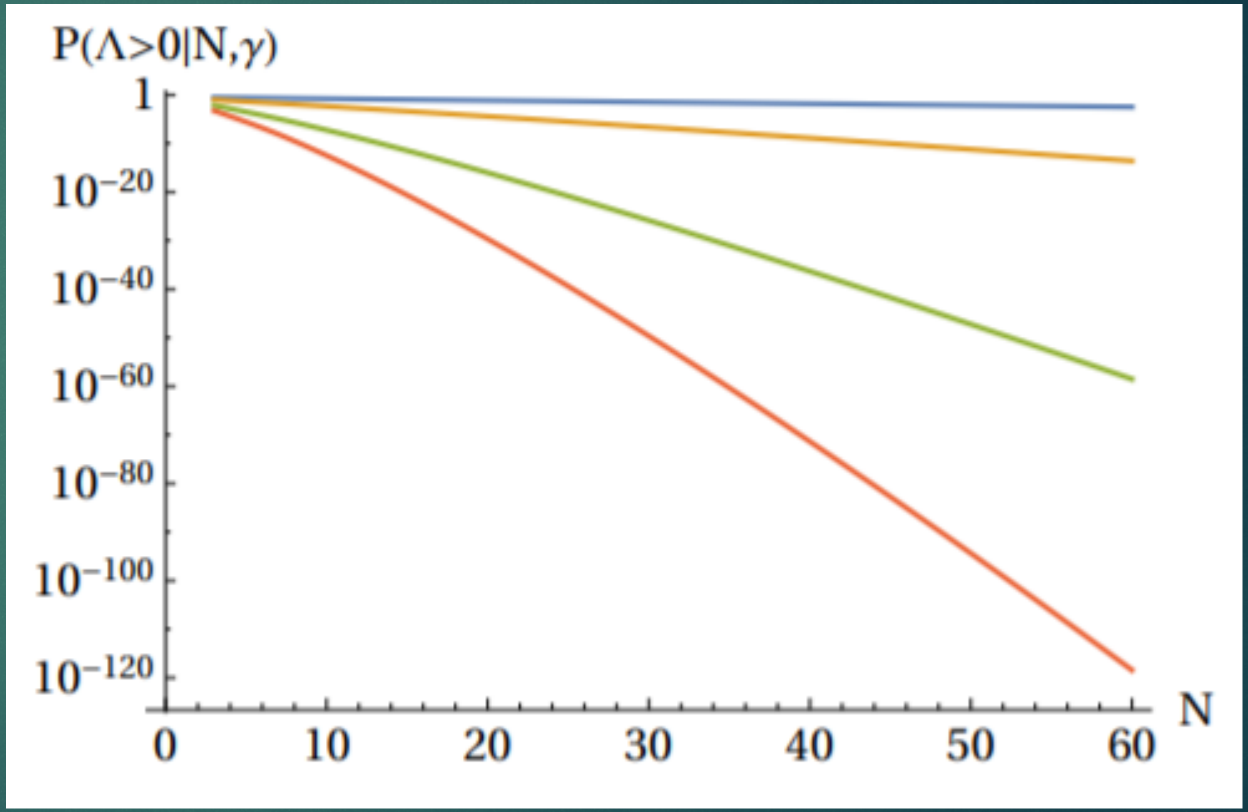
How does probability vary with γ and N?

- Note that as N increases or γ increases, P(min) decreases.
  - The more fields there are, the less likely it is that all fields reach a minimum at a point. Similarly, the less turbulent a potential is, the fewer minima we get.
- But it is not so obvious how $P(\Lambda > 0 | \text{min})$ behaves.
- Left figure: $P(\Lambda > 0 | \text{min})$ for $\gamma = 0.2$, 0.5 and 0.8; right figure: $P(\Lambda > 0 | \text{min})$ for $N = 3$, 5 and 8.



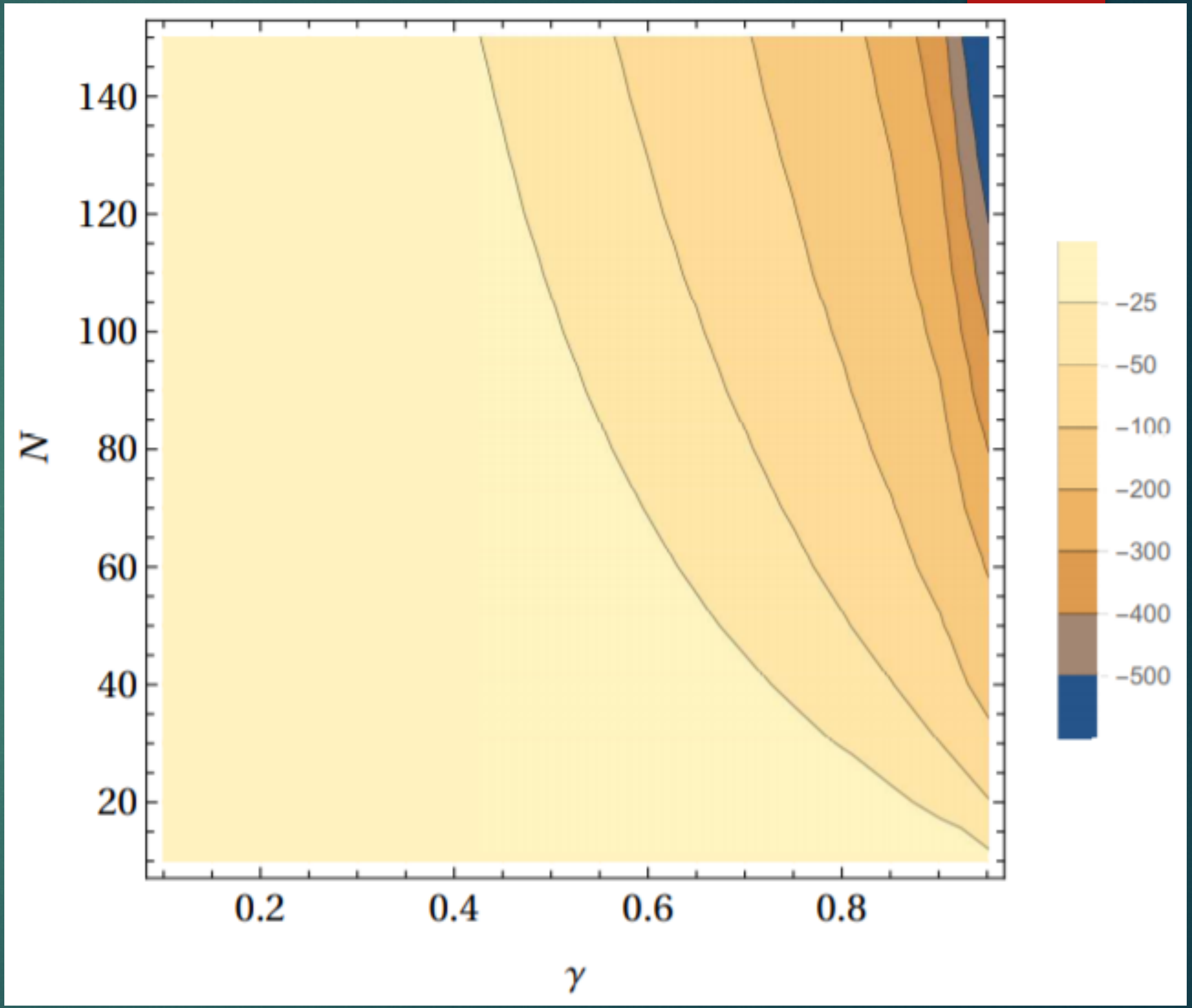
Results

- From top to bottom: γ = 0.2, 0.5, 0.8 and 0.9.



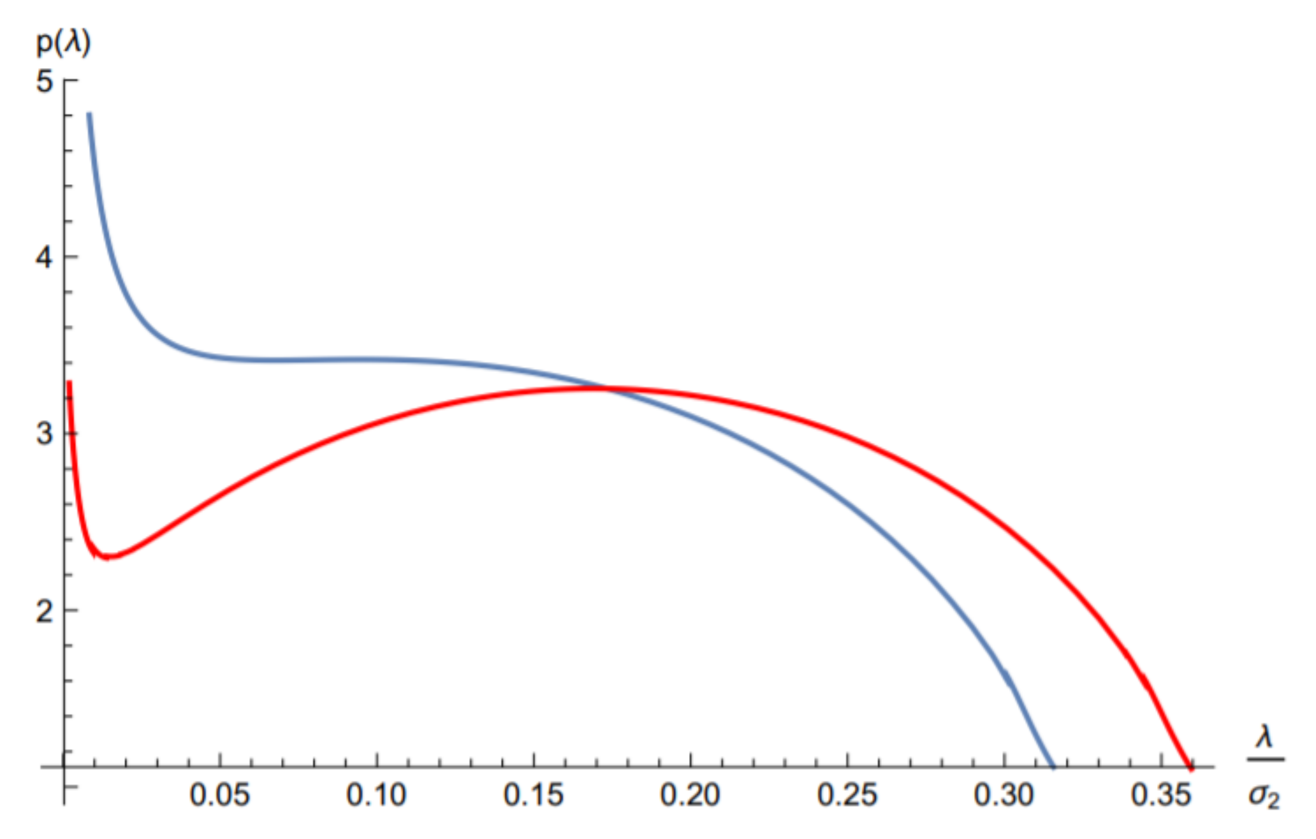
Results

- Another view of the same calculation.
- String theory is said to have ~ 10^{500} minima, but there's a region of parameter space where even 10^{500} minima might not suffice to have one that has $\Lambda > 0$!
- In the specific case of a Gaussian power spectrum, $\gamma = \sqrt{\frac{N}{N+2}}$. At N = 100, $P(\Lambda > 0 \mid \text{min}) \sim 10^{-780}$



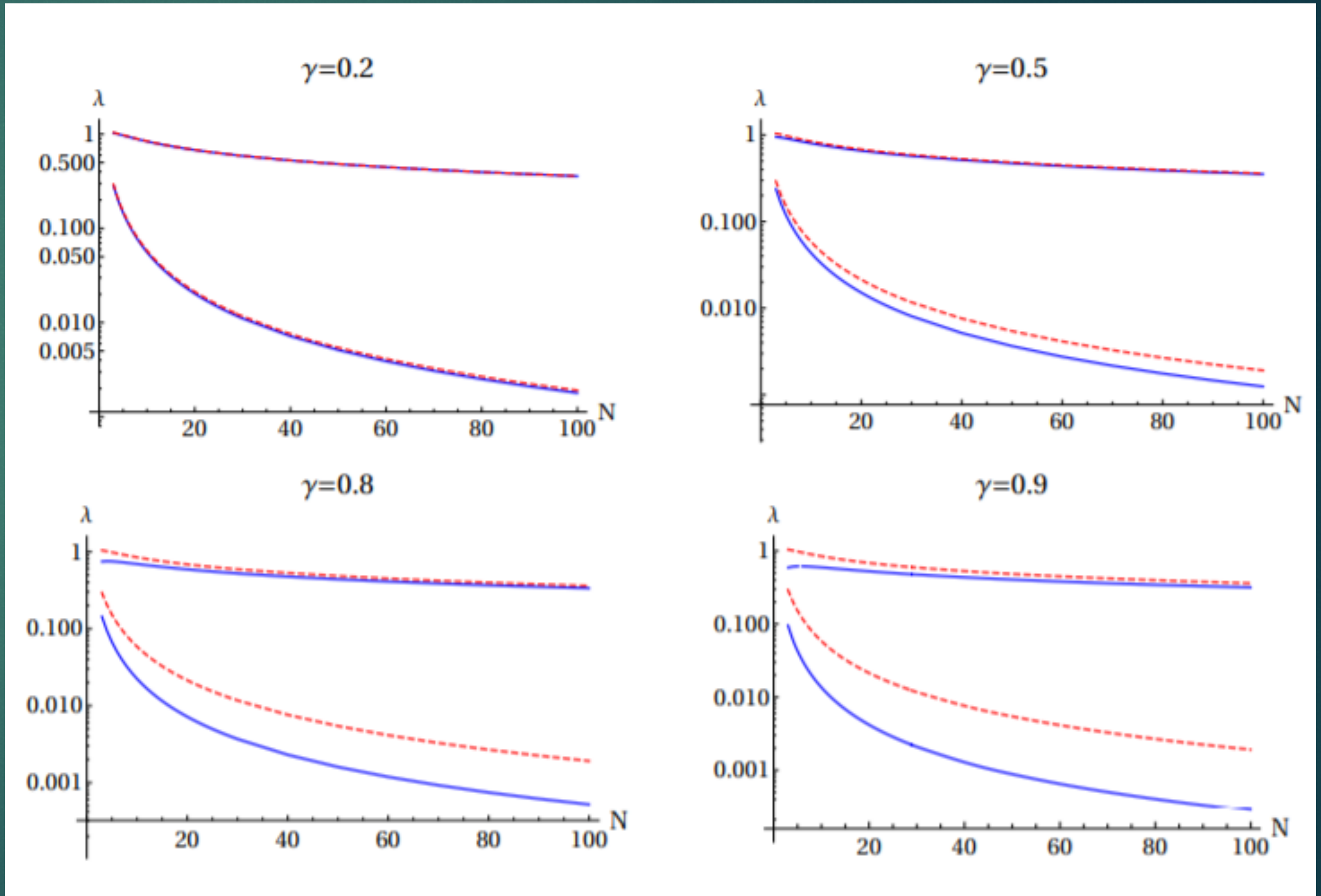
Eigenvalue distribution

- The eigenvalues of the Hessian at a point roughly correspond to the slopes at the point.
- For these calculations, we examine the eigenvalues at the single most-likely point.
- Figure for N = 100, γ = 0.9. Red line: most likely point; blue line: most likely point with constraint Λ = 0.
- It can be shown that the red line is independent of γ.
- Looks like a Wigner semicircle, with deviations at very small eigenvalues.



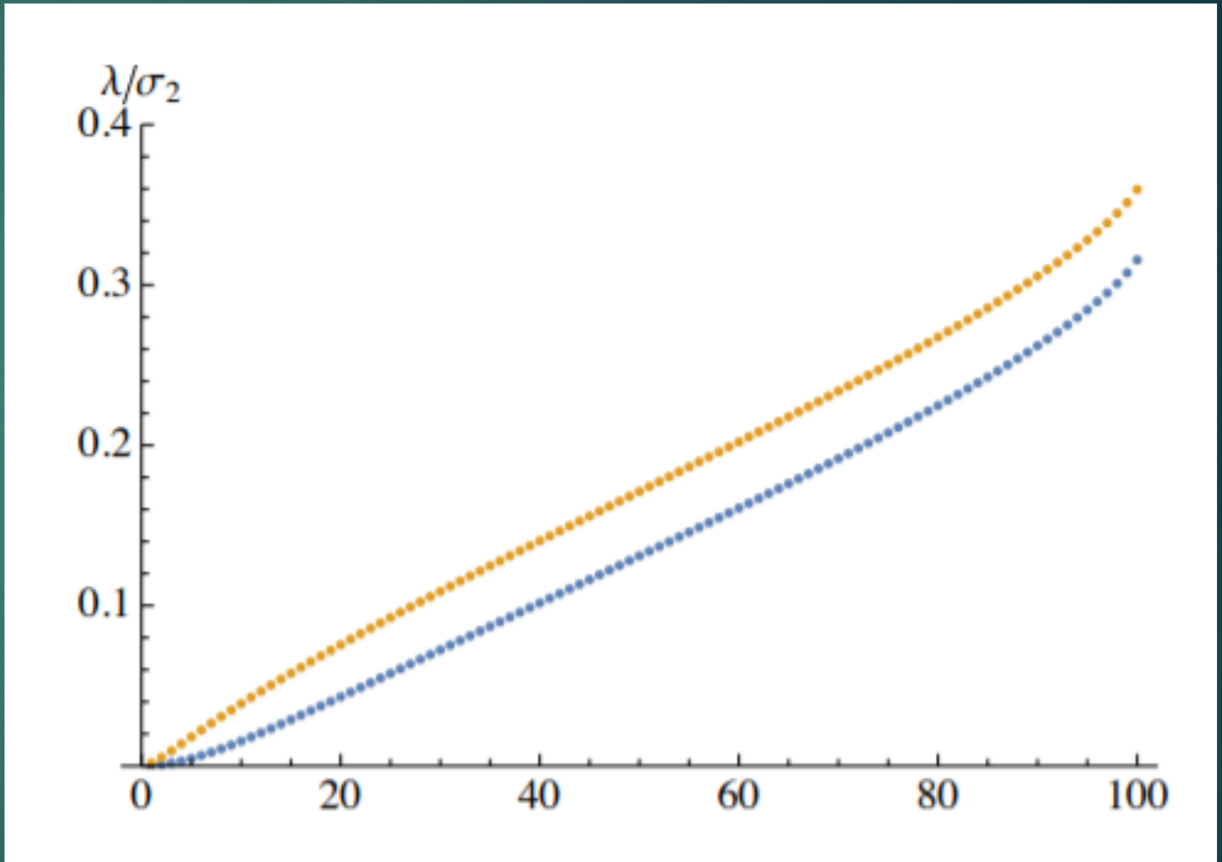
Eigenvalue distribution

- These are the biggest and smallest expected eigenvalues.
- Dashed lines: overall peak; solid lines: peak with $\Lambda = 0$.



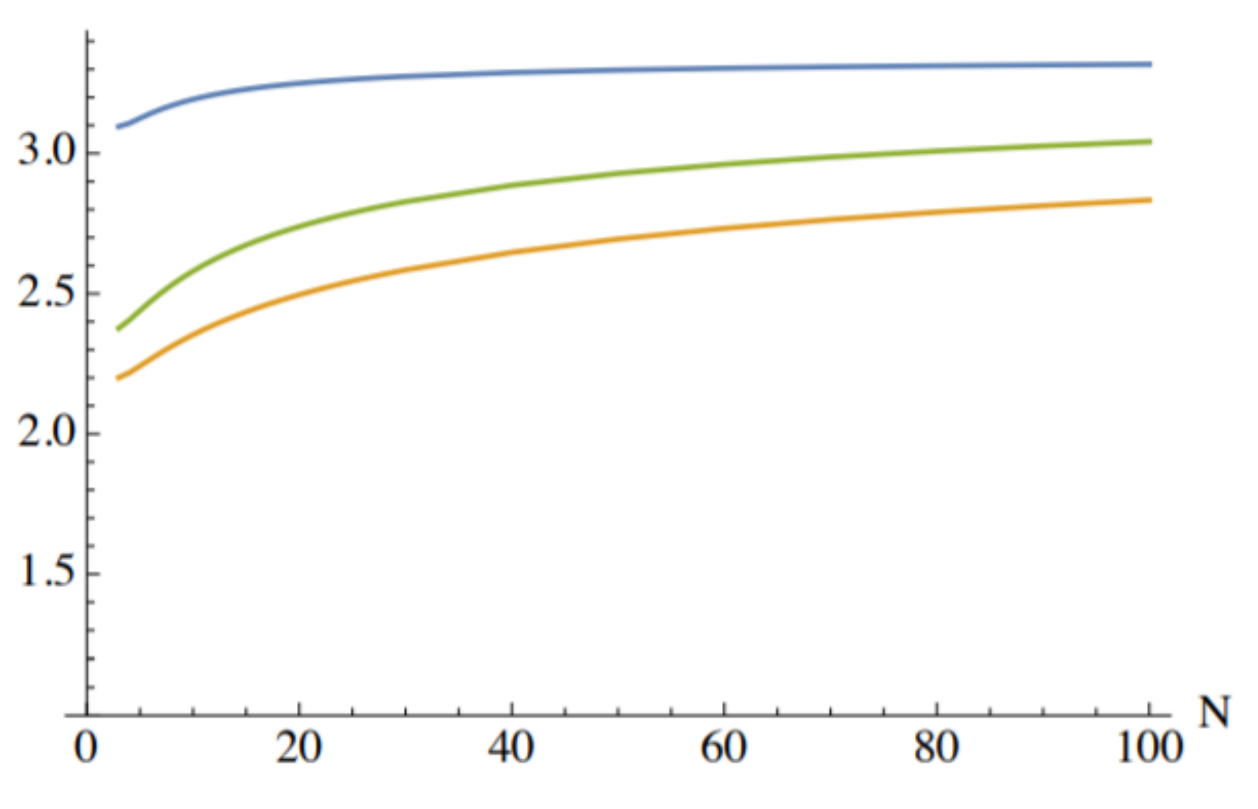
Eigenvalue distribution

- We can plot all the eigenvalues …
- N = 100, γ = 0.9. Orange line: overall peak, blue line: peak with Λ = 0.



Eigenvalue distribution

- Or the ratio of the two smallest eigenvalues …
- Orange line: ratio at overall peak (does not depend on γ); green line: γ = 0.5, Λ = 0; blue line: γ = 0.9, Λ = 0.



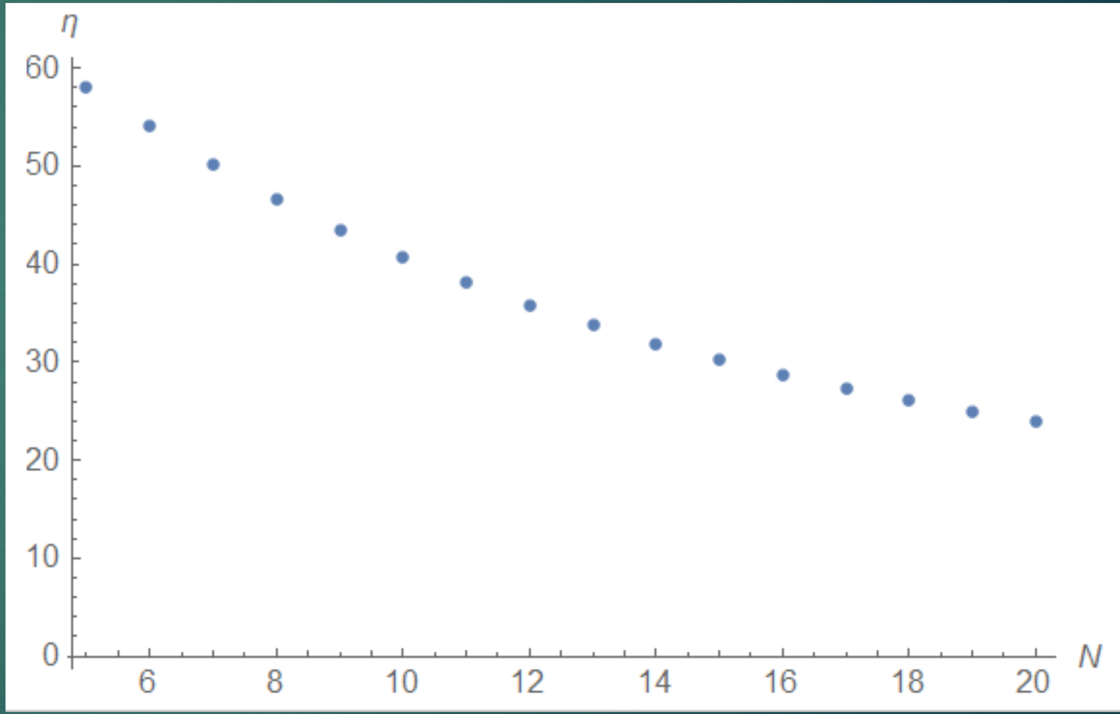
Saddles

$$\int_0^{\infty} d\lambda_1 \int_0^{\lambda_1} d\lambda_2 \int_0^{\lambda_2} d\lambda_3 \int_0^{\lambda_3} d\lambda_4 (\lambda_1-\lambda_2)(\lambda_1-\lambda_3)(\lambda_1-\lambda_4)(\lambda_2-\lambda_3)$$
$$(\lambda_2-\lambda_4)(\lambda_3-\lambda_4)\lambda_1\lambda_2\lambda_3\lambda_4 \exp\left[-\frac{\alpha K \alpha}{2}\right]$$

- We can also investigate saddles. The only difference is that we require one eigenvalue to be below zero.
- In this case, our primary focus is the slow-roll inflation parameter η = 1/8πG (V''/V). V'' is related to the eigenvalues, and V (i.e. Λ) is in the aKa factor. We can directly calculate it!
- Inflation requires η to be << 1
- We also need the *correlation length* to not be large (or small η would be trivial)
  - Correlation length measures how much knowledge of the inflaton at one point reveals about the value at a more distant point. Physically, at some point, correlation must drop to zero.
  - A large correlation length is also unphysical in string theory.

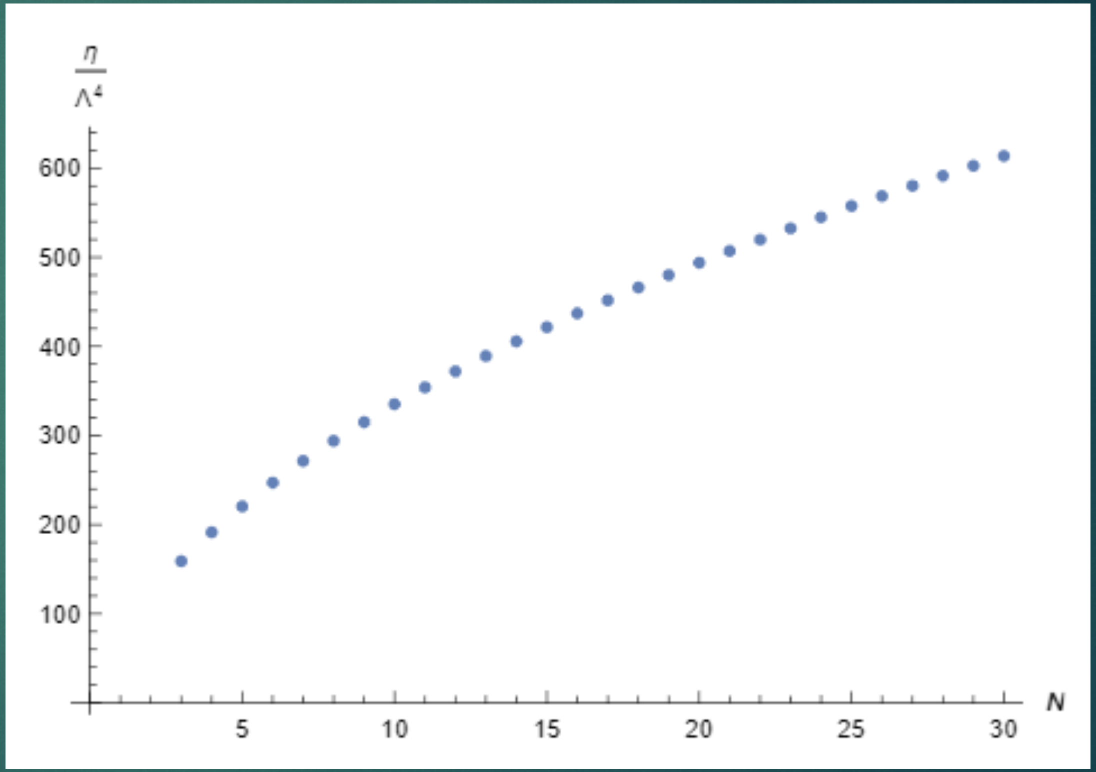
Results

- The results depend on what γ is. In turn, γ depends on what the power spectrum is.
- A nice intermediate result is that if γ is constant, then we observe η decreasing with dimensions!
  - Figure for γ = 0.95, V = 0.01 m_{pl}. V < ~0.01 m_{pl} is observationally required because of the non-detection of b-mode polarization.
- This implies that if η is not << 1 at N = 100, we can just increase N and still "get there".



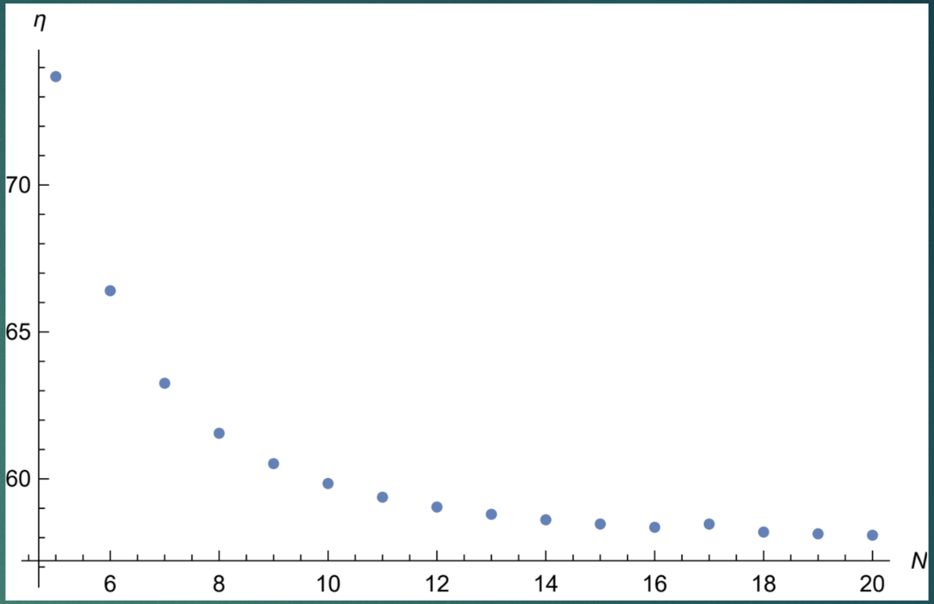
Results

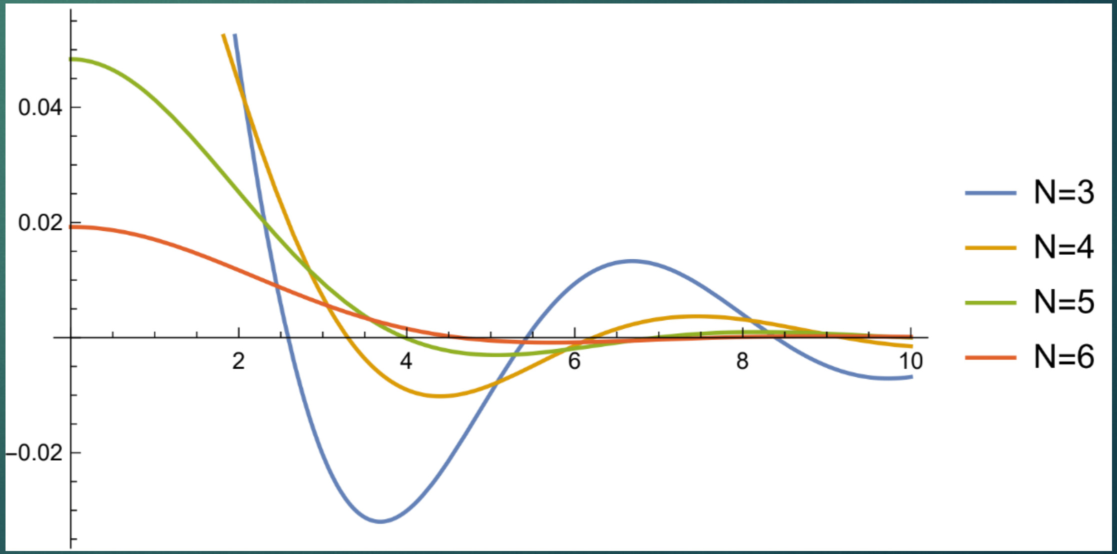
- Gaussian power spectra
  - $\Upsilon = \sqrt{\dfrac{N}{N+2}}$
  - Plot at V = 0.01 m_{pl}
  - On the y-axis, Λ is the correlation length



Results

- Power law power spectrum, P(k)=Ak^{-n}, with infrared cutoff
  - $Y = \sqrt{\dfrac{(n-4-N)(n-N)}{n-N-2}}$
  - Top plot: n = 2N+1/2, V = 0.01 m_{pl}
- Looks good, but …
  - Bottom plot: correlation length increases with N





Results

- Other power spectra?
- Try: convolution of two Gaussians
- Can we vary the four free parameters such that γ is constant?
  - Unfortunately, the answer is probably "no". Details are complicated, and there is no conclusive proof, but there is reason to believe that no such combination exists to have constant γ but also non-increasing Λ

$$U_0^2 e^{-\phi^2/2L_0^2} + U_1^2 e^{-\phi^2/2L_1^2} = \frac{1}{(2\pi)^N}\int d^N k P(k) e^{ik\phi\cos\theta}$$

Conclusion

- The Random Gaussian Field approximation lets us ask physical questions of the landscape and get calculable answers.
- Under the Random Gaussian Field approximation, inflation is not favoured for common power spectra.
  - This doesn't mean inflation is impossible (all calculations above are at the most likely saddle), but it does mean inflation is not likely.
  - It's possible a more complicated power spectrum works, but that would invoke complex physics and therefore defeat the purpose of the approximation.
- The end?