Kinetic Field Theory: A comparison between KFT and SPT

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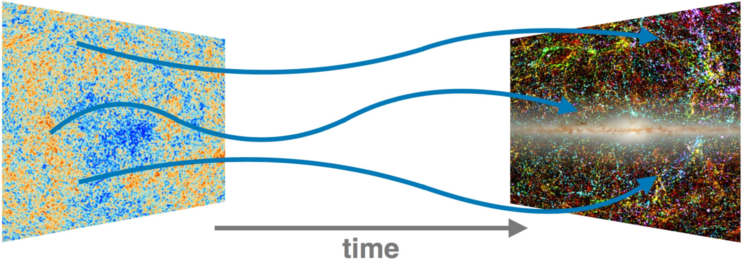
at Cosmology From Home

Cosmic structure formation

How can we understand cosmic structure formation from first principle?



Kinetic Field Theory in a nutshell

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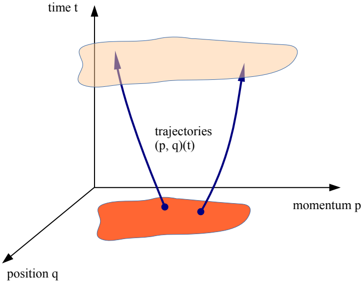
- Non-equilibrium statistics of N correlated classical particles.

$$Z[\mathbf{J}] = \int \mathrm{d}\mathbf{q}\,\mathrm{d}\mathbf{p}\; \underbrace{P(\mathbf{q},\mathbf{p})}_{\text{initial conditions}}\; e^{\overbrace{i\int \mathrm{d}t'\langle \mathbf{J}, \bar{\mathbf{x}}\rangle}^{\text{dynamics}}}$$



- density-fluctuation power spectrum:

$$P_\delta(k,a) \propto \hat{\rho}(1)\,\hat{\rho}(2)\,Z[J]\Big|_{J=0}$$

The KFT power spectrum

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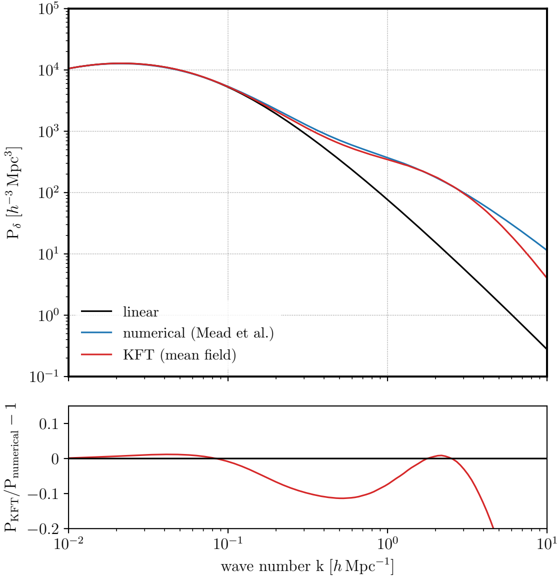

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Theory of cosmic structure formation

In general:

$$Z = \int \mathcal{D}\varphi \, P[\varphi]$$

Kinetic Field Theory:

- Z_{KFT} in terms of particles and their trajectories
- Solution to Hamiltonian e.o.m.

$$\vec{q}(t) = \vec{q}^{(i)} + g_{qp}(t,0)\,\vec{p}^{(i)} + \int_0^t dt'\, g_{qp}(t,t')\,\vec{f}(t')$$

- Derive statistical properties by functional derivatives

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Eulerian SPT:

- Z_{SPT} in terms of averaged fields (velocity and density) and their e.o.m
- Fluid equations:

$$\partial_t\,\rho + \rho\,\theta + v^i\,\partial_i\,\rho = 0$$
$$\partial_t\,\theta + \partial_i\left(v^j\,\partial_j v^i\right) + 4\pi G\rho = 0$$

$(\theta = \partial_i v^i)$

- Derive statistical properties by functional derivatives

Loss of information in SPT I

- Fluid equations obtained from a finite truncation of Vlasov-Boltzmann hierarchy:

$$\partial_t \rho + \partial_i \left(\rho \, v^i\right) = 0 \tag{1}$$

$$\partial_t v^i + v^j \, \partial_j v^i + \partial^i \tilde{\phi} + \frac{\partial_j \left(\rho \, \sigma^{ij}\right)}{\rho} = 0 \tag{2}$$

$$\nabla^2 \tilde{\phi} = 4\pi G \, \rho \tag{3}$$

- The truncation means that we neglect all information contained in the higher moments of the phase-space distribution!

Loss of information in SPT II

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- To solve the fluid equations, we set the velocity dispersion $\sigma_{\mathrm{ij}} = 0$ (single-stream approximation):

$$\partial_t \rho + \rho\,\theta + v^i\,\partial_i\,\rho = 0$$
$$\partial_t\,\theta + \partial_i\left(v^j\,\partial_j v^i\right) + 4\pi G\rho = 0$$

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- This assumes that the system is described at every point by a single-valued velocity field.
- This assumption breaks down once streams start to cross! $\rightarrow$ shell-crossing problem
- Further simplification: $\nabla \times \vec{v} = 0$ initially. But $\sigma_{\mathrm{ij}} \neq 0$ could source vorticity.

Information content

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has already lost information!

Comparison of the perturbation theory (PT)

Kinetic Field Theory:

- PT in terms of deviation of particle from inertial trajectories due to interactions.
- For

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Eulerian SPT:

- PT is not in terms of the interaction potential.
- Linear theory describes the independent evolution of Fourier field modes.
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→ Direct comparison of perturbative expansion for KFT and SPT difficult.

Comparison in the free-streaming regime

- Expanding the KFT density power spectrum to 2^{nd} order in $\mathrm{P}_{\delta}^{\mathrm{ini}}$ gives the one-loop result of SPT.

→ By construction, KFT represents a complete resummation of the SPT perturbation series in the advective kinematics in the free-streaming regime.

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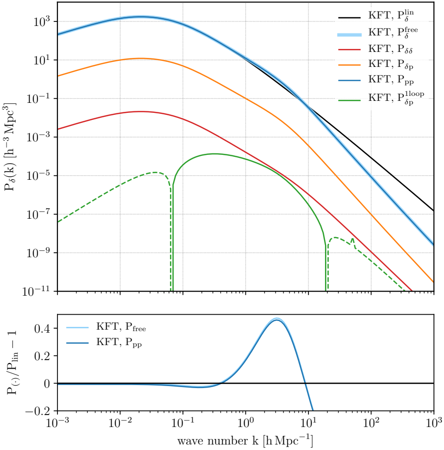
KFT: exact free evolution



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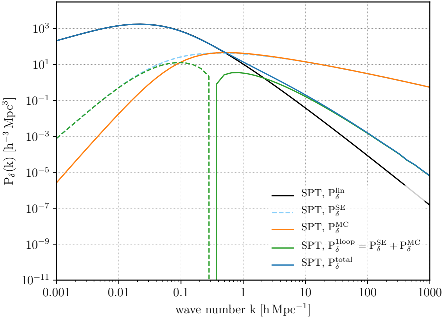
1loop SPT: recovered from KFT to 2^{nd} order in P_δ^{ini}



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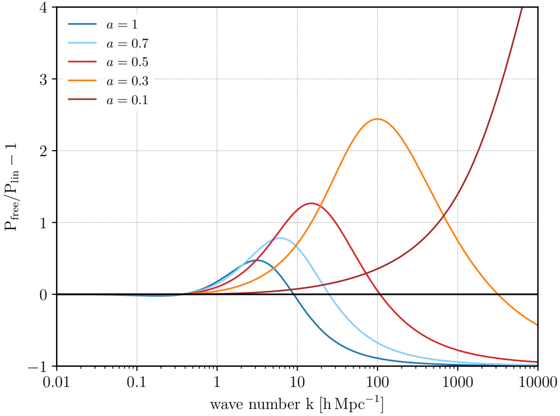
Transient bahaviour



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Indicates shell-crossing.

Summary

- Parameter free, analytic description of non-linear structure formation.
- Successful far into the non-linear regime of structure formation.
- Does not require model assumptions on halo profiles and halo mass functions.
- Gain understanding of LSS formation on a fundamental particle-level.
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