Kinetic Field Theory: An Introduction

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at Cosmology From Home

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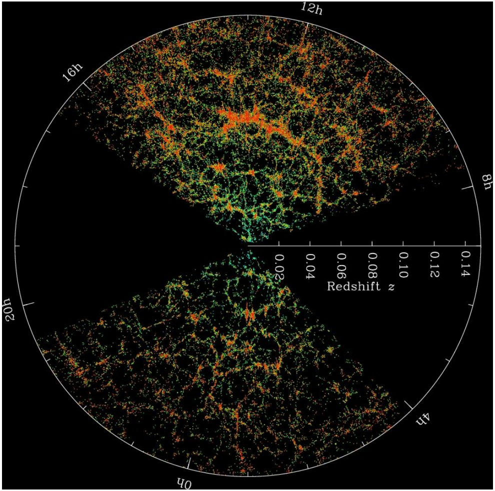
How do non-linear structures form and evolve?

What is the nature of dark matter and dark energy?

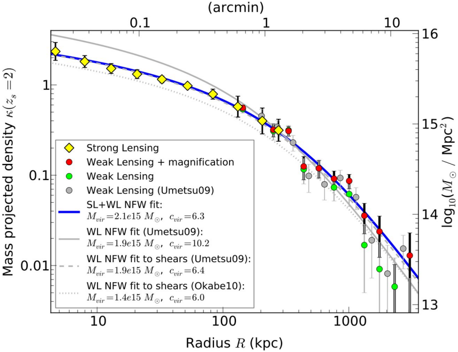


Sloan Digital Sky Survey

How do DM halos form?

How can we understand the self-similarity of cosmic structure?

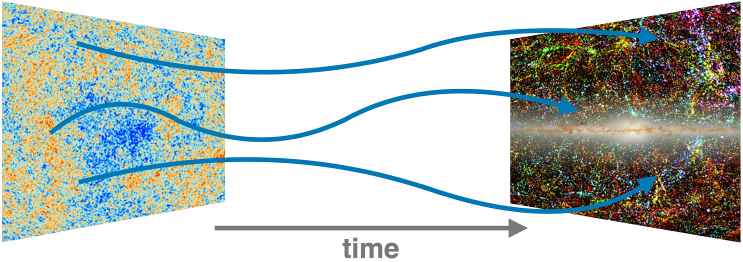
Abell 2261, CLASH Project



Coe et al. 2012

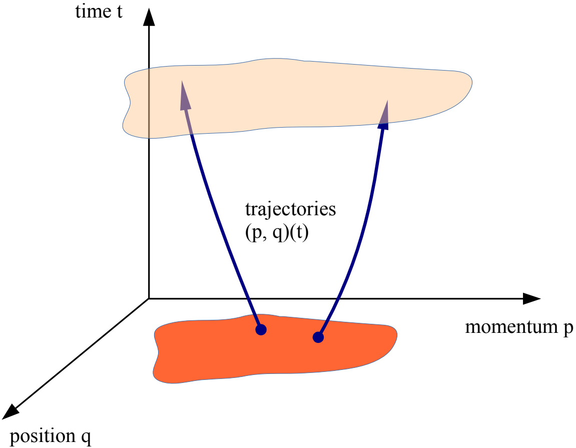
Cosmic structure formation

How can we understand cosmic structure formation from first principle?



Kinetic Field Theory –

A particle based approach

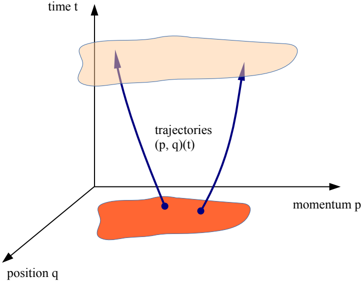


Trajectories in phase space do not cross.

Kinetic Field Theory in a nutshell

- Non-equilibrium statistics of N correlated classical particles.

$$Z[\mathbf{J}] = \int \mathrm{d}\mathbf{q}\,\mathrm{d}\mathbf{p}\ \underbrace{P(\mathbf{q},\mathbf{p})}_{\text{initial conditions}}\ e^{\overbrace{i\int \mathrm{d}t' \langle \mathbf{J}, \bar{\mathbf{x}}\rangle}^{\text{dynamics}}}$$



- Gaussian initial conditions fully specified by CMB power spectrum.
- solution to e.o.m
  $\bar{\mathbf{x}}$ = inertial motion + interactions

Particle dynamics

- Hamiltonian dynamics

$$\mathcal{H} = \frac{\vec{p}^2}{2m} + m\varphi \,, \quad \vec{\nabla}^2\varphi = A_\varphi \delta$$

(m and A_φ depend on time)

- Solution to e.o.m.

$$\vec{q}(t) = \vec{q}^{(i)} + g_{qp}(t,0)\,\vec{p}^{(i)} + \int_0^t dt'\, g_{qp}(t,t')\,\vec{f}(t')$$

Particle dynamics

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- Hamiltonian dynamics

$$\mathcal{H} = \frac{\vec{p}^2}{2m} + m\varphi \,, \quad \vec{\nabla}^2\varphi = A_\varphi \delta$$

(m and A_φ depend on time)

- Solution to e.o.m. for all N particles

$$\bar{\mathbf{x}}(t) = \underbrace{\mathcal{G}(t,0)\mathbf{x}^{(i)}}_{\text{inertial motion}} + \underbrace{\int_0^t dt' \, \mathcal{G}(t,t') \, \mathcal{F}(t')}_{\text{interactions}}$$

$(\mathbf{x} = \vec{x}_i \otimes \vec{e}_i$ and $\vec{x}_i = (\vec{q}_i, \, \vec{p}_i))$

Clever choice of propagators

$$Z[\mathbf{J}] = \int \mathrm{d}\mathbf{q}\,\mathrm{d}\mathbf{p}\,P(\mathbf{q},\mathbf{p})\,e^{i\int dt' \langle \mathbf{J}, \bar{\mathbf{x}} \rangle}$$

- Choose $g_{qp}(t,t')$ to reproduce linear growth

$$g_{qp}(t,t') = D_+(t) - D_+(t') =: t - t'$$

(Zel'dovich trajectories)

- Relative force

$$\vec{f} = -\vec{\nabla}\varphi + \frac{\dot{m}h}{m^2}\int_0^t dt'\, m\vec{\nabla}\varphi$$

Correlation functions and density operator

- number density operator:

$$\rho = \sum_{i=1}^{N} \delta_D \left(\vec{q} - \vec{q}_i\right) \quad \rightarrow \quad \sum_{i=1}^{N} \exp\left(-i\vec{k} \cdot \vec{q}_i\right)$$

$$\hat{\rho} = \sum_{i=1}^{N} \exp\left(-i\vec{k} \cdot \frac{\delta}{\delta J_{q_i}(t)}\right)$$

- density-fluctuation power spectrum:

$$P_\delta(k, a) \propto \hat{\rho}(1)\, \hat{\rho}(2)\, Z[J] \Big|_{J=0}$$

Include particle interactions

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Option 1: Perturbative

- Interaction operator

$$Z[\mathbf{J},\mathbf{K}] = e^{i\hat{S}_I} Z_0[\mathbf{J},\mathbf{K}]$$

$$P_\delta(k,t) \propto \hat{\rho}\hat{\rho}\left[1 + i\hat{S}_I + \ldots\right] Z_0[\mathbf{J},\mathbf{K}]$$

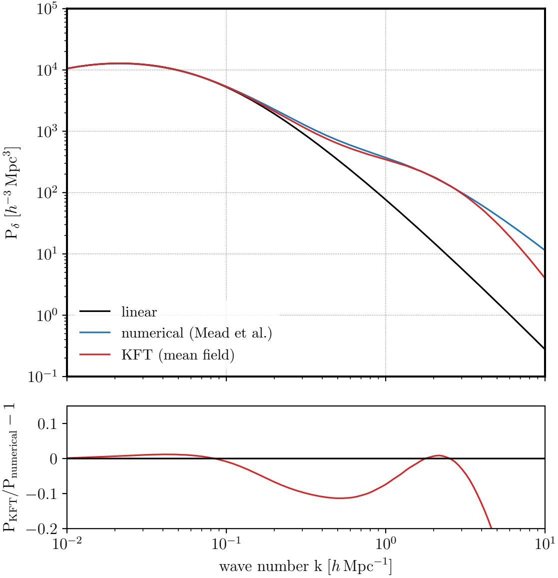
Option 2: Mean-field

- Evaluated interaction term $\langle S_I \rangle$: force between particle pairs weighted by correlation function ξ_{12}

$$P_\delta(k,t) = e^{i\langle S_I \rangle}\mathcal{P}(k)$$

(closed expression)

The power spectrum



PRELIMINARY, Bartelmann et al. in prep

Take away message

(1) KFT is an analytical, particle-based approach for cosmic large-scale structure formation.

(2) It does not suffer from shell-crossing, because it lives in phase-space.

(3) It yields results comparable to those obtained with numerical simulations (in a fraction of the time).

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