Three-point Gaussian streaming model for redshift-space distortions

(Based on Kuruvilla & Porciani, JCAP, 2020; arXiv:2005.05331)

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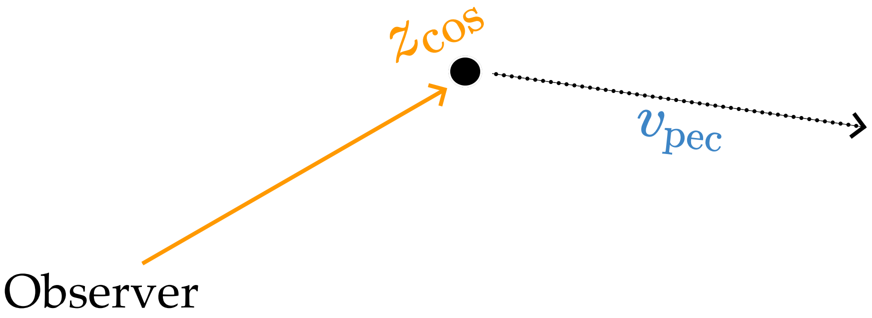
Redshift-Space Distortions (RSD)



- Observed redshift, is affected by both cosmological expansion and the peculiar velocity along the line-of-sight.

$$1 + z_{\rm obs} \approx (1 + z_{\rm cos})\left(1 + \frac{v_{\parallel}}{c}\right)$$

- Our 3D maps of the Universe are “distorted”.

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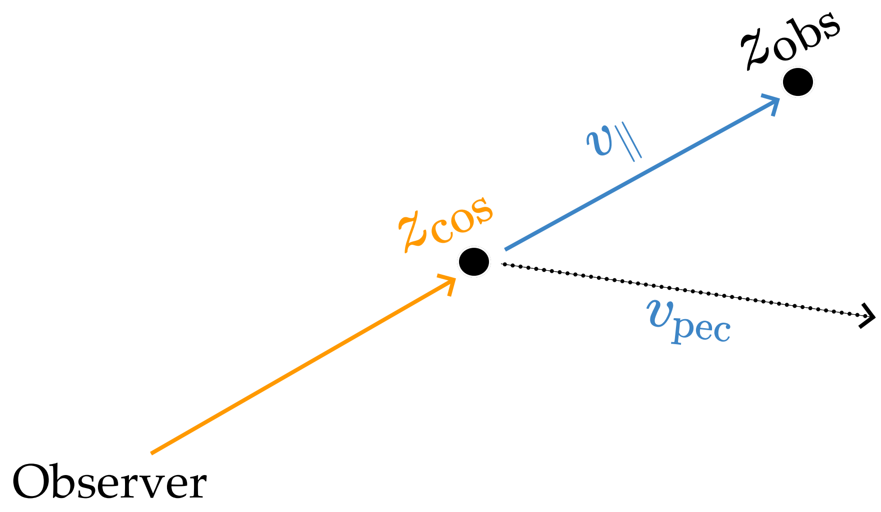

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$$1 + z_{\rm obs} \approx (1 + z_{\rm cos})\left(1 + \frac{v_{\parallel}}{c}\right)$$

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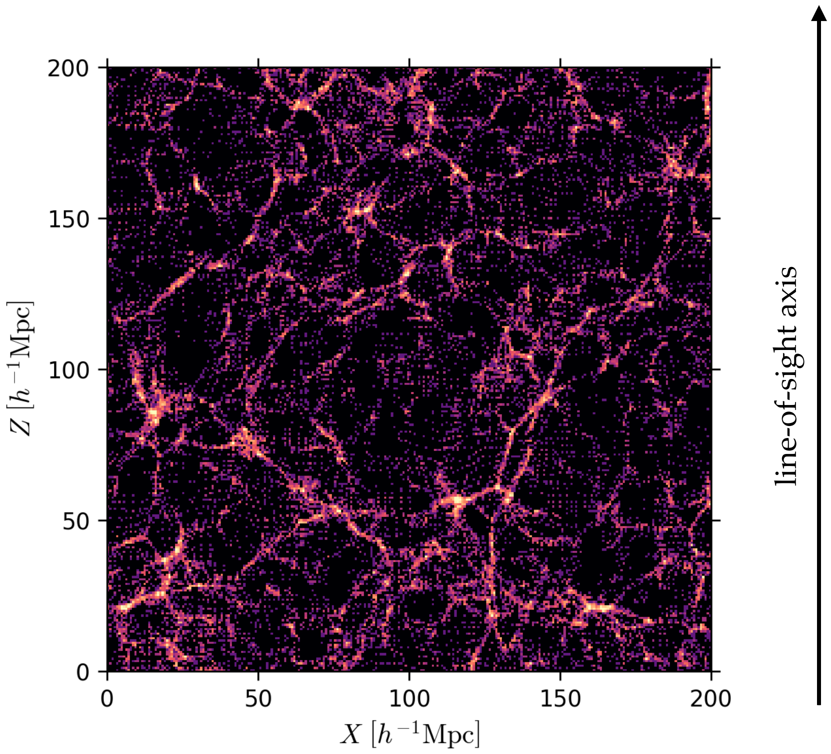
Real space

(Unobserved)

Redshift space

(Observed)



$$\vec{s} = \vec{x} + (\vec{v} \cdot \hat{z})\hat{z}$$

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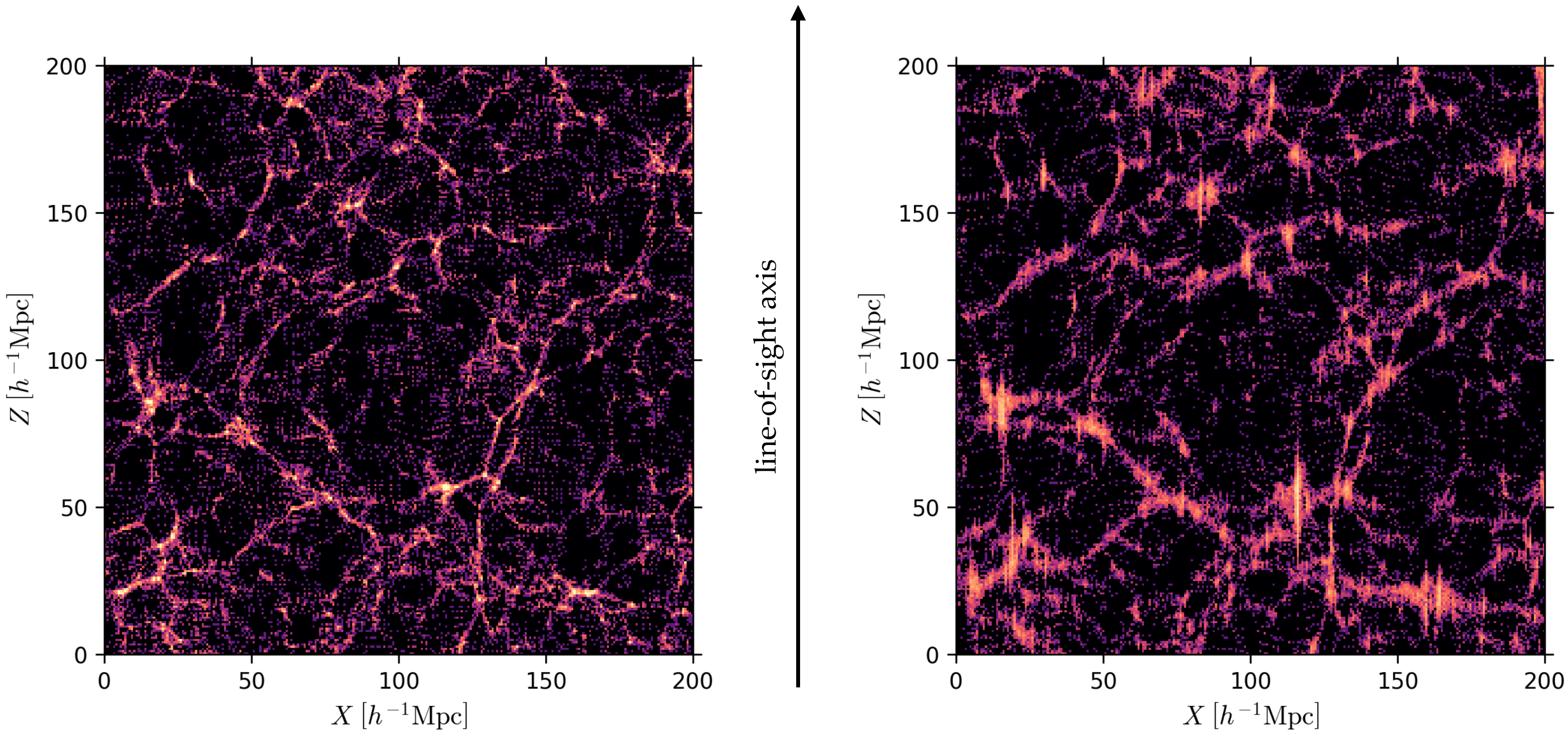
Real space
(Unobserved)

Redshift space
(Observed)



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Real space
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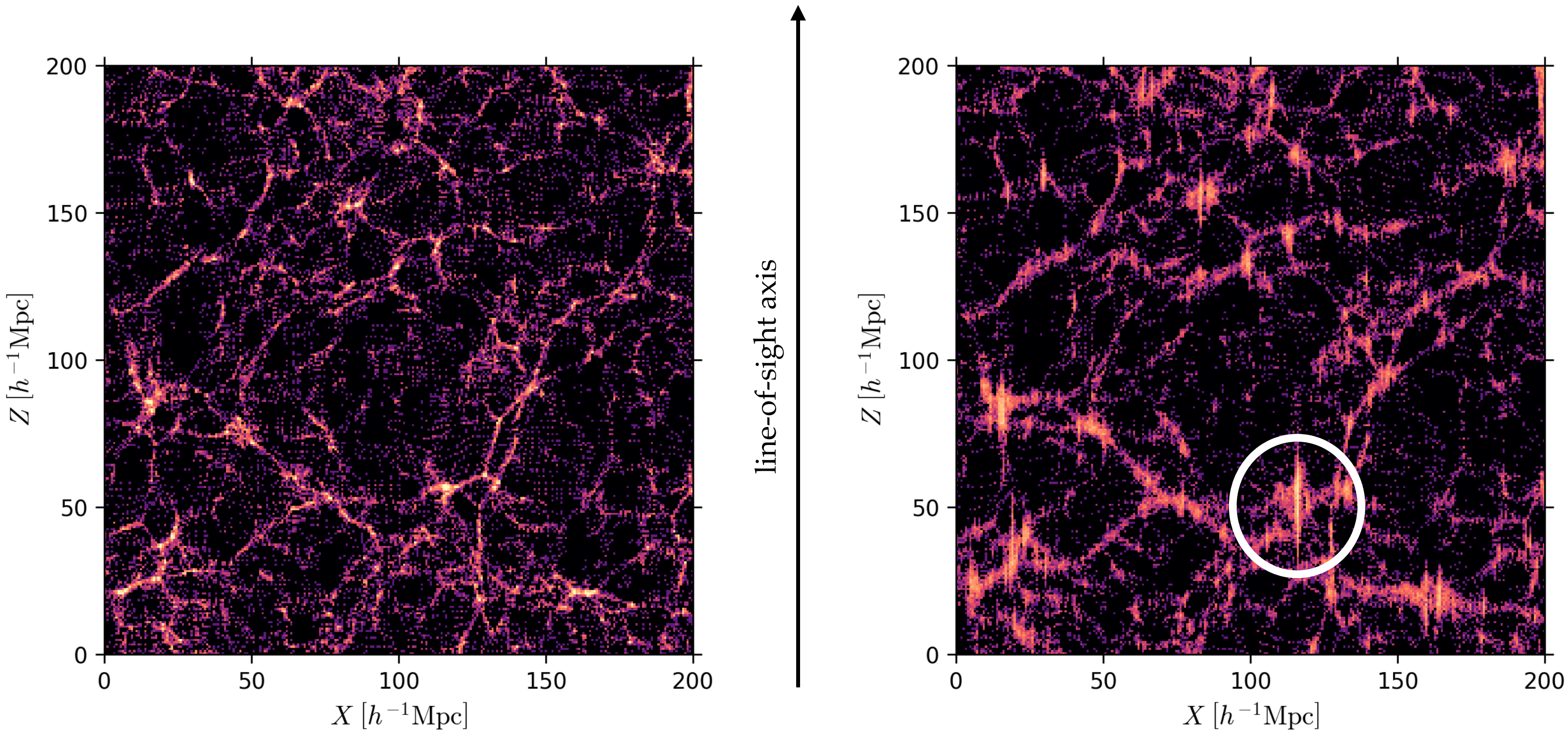
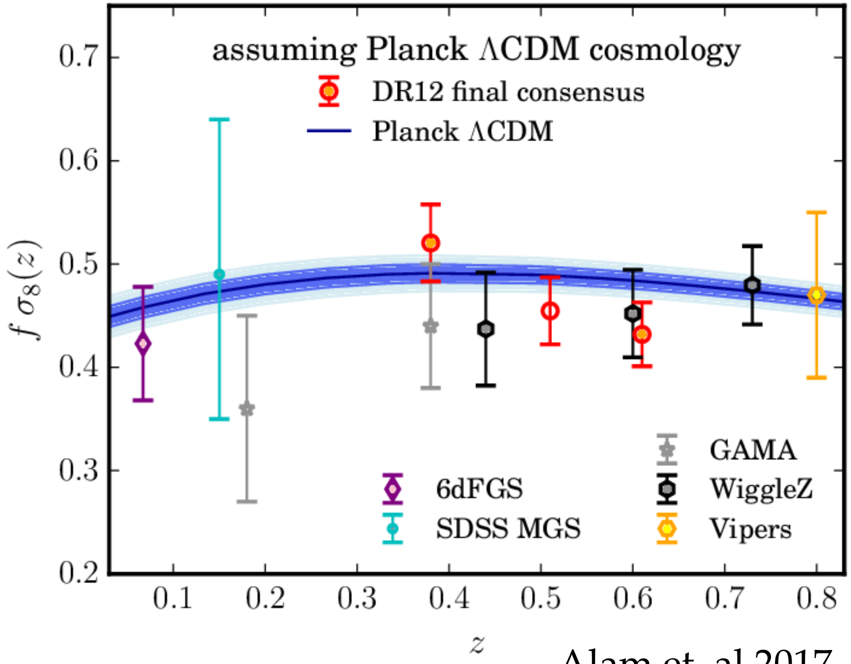
Redshift space
(Observed)



$$\vec{s} = \vec{x} + (\vec{v} \cdot \hat{z})\hat{z}$$

Motivation

- Tool to test theories of gravity.
- Growing interest in extending RSD studies to smaller scales, and also to extend to higher order clustering statistics.



Alam et. al 2017

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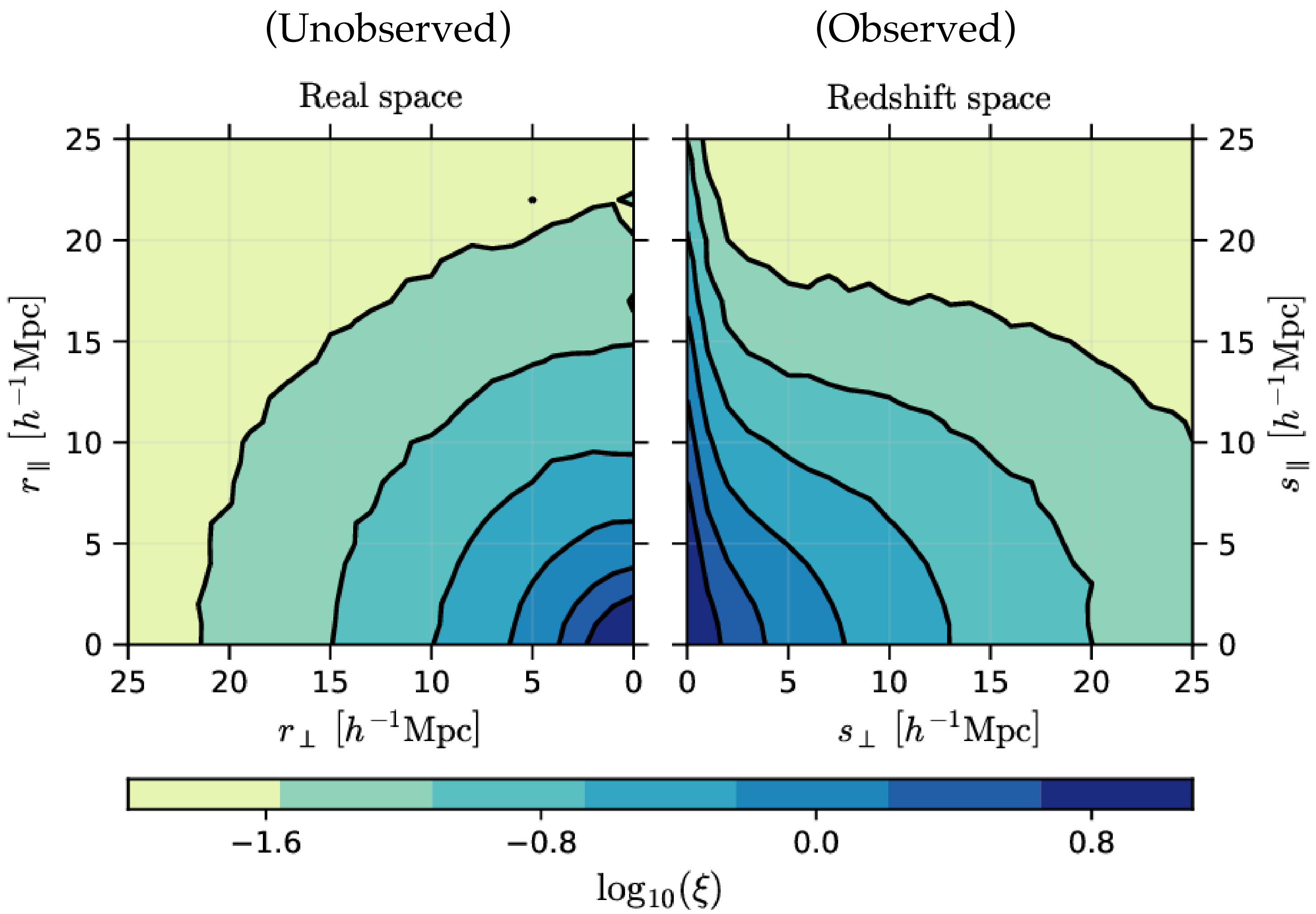
Two-point correlation function



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Two-point streaming model

$$1 + \xi_{\mathrm{s}}(s_\perp, s_\parallel) = \int_{-\infty}^{+\infty} [1 + \xi(r)]\; \mathcal{P}^{(2)}_{w_\parallel}(w_\parallel \mid \vec{r}\,)\, \mathrm{d}r_\parallel$$

(Peebles 80, Fisher 95, Scoccimarro 04, Kuruvilla & Porciani 18, Vlah & White 19)

$\mathcal{P}^{(2)}_{w_\parallel}(w_\parallel \mid \vec{r}\,)$: Relative pairwise line-of-sight velocity distribution

- Current workhorse is the Gaussian streaming model (GSM).
(Fisher 95, Reid & White 11)

- GSM used in the BOSS analyses (e.g. Reid et al. 12, Samushia et al. 14; Alam et al. 17, Chuang et al. 17, Satpathy et al. 17) and the recent eBOSS analyses. (e.g. Zarrouk et al. 18, Bautista et al. 20, Tamone et al. 20, Wang et al. 20)

Can we generalise it to three-point and higher order statistics?

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Yes we can!

n-point streaming model

- Generalised the streaming model framework to n-point statistics.

$$\mathcal{G}_n = \int \mathcal{F}_n \, \mathcal{P}^{(n)}_{\boldsymbol{w}_\parallel} \, \mathrm{d}w_{12\parallel} \ldots \mathrm{d}w_{mn\parallel}$$

where,

$\mathcal{G}_n$: the (anisotropic) n-point full CF in redshift space

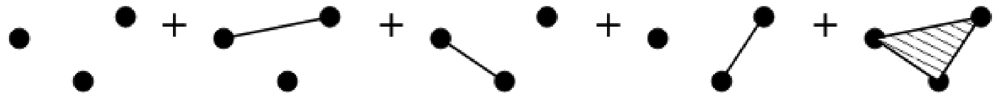
$\mathcal{F}_n$: the (isotropic) n-point full CF in real space

$\mathcal{P}^{(n)}_{\boldsymbol{w}_\parallel}$: the joint probability density of n-1 relative line-of-sight peculiar velocity

The equation is exact under distant observer approximation

Full correlation function in three-point

$\langle \delta_1 \, \delta_2 \, \delta_3 \rangle =$

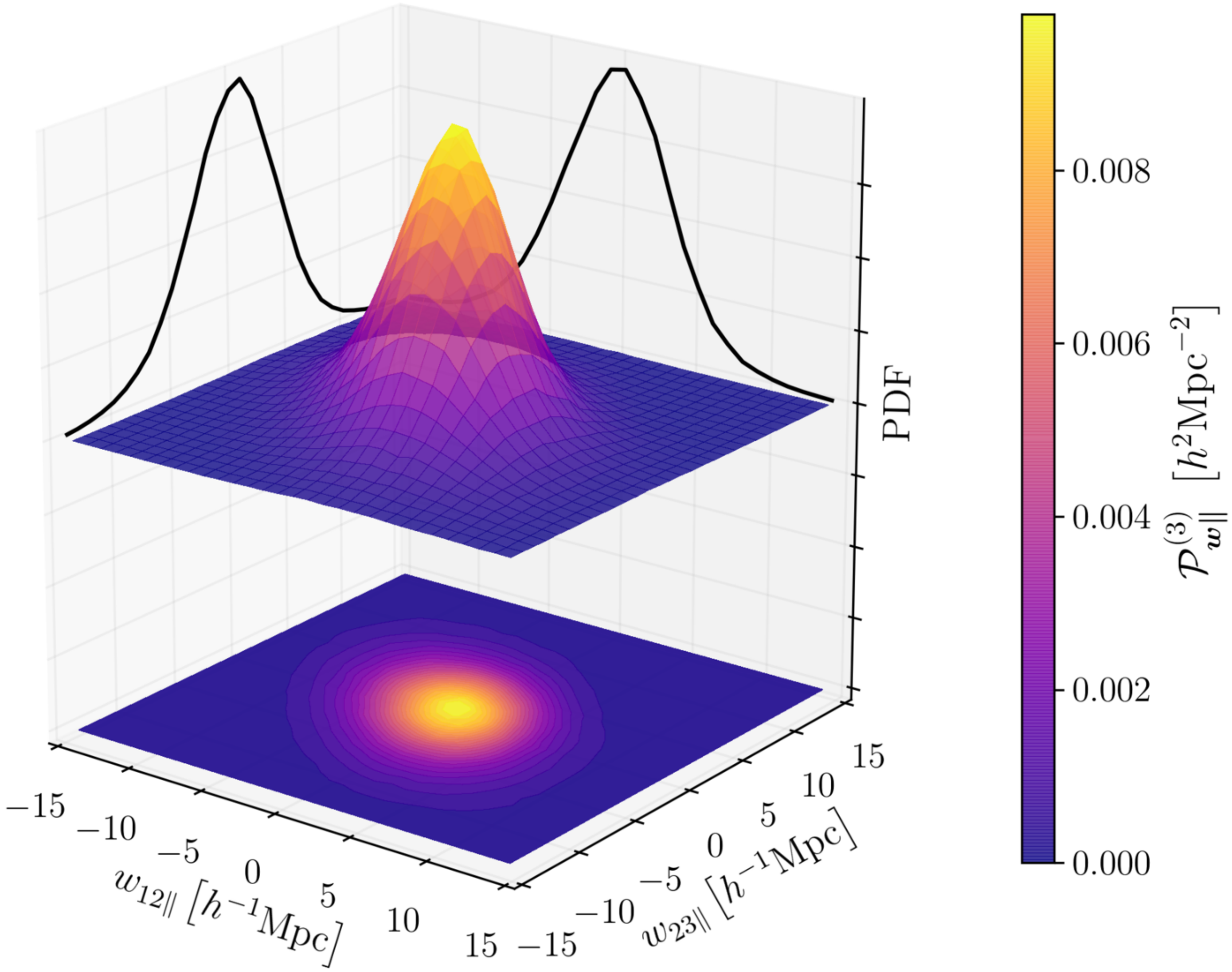


Credits: Bernardeau et al. 01

Thus the three-point streaming model is

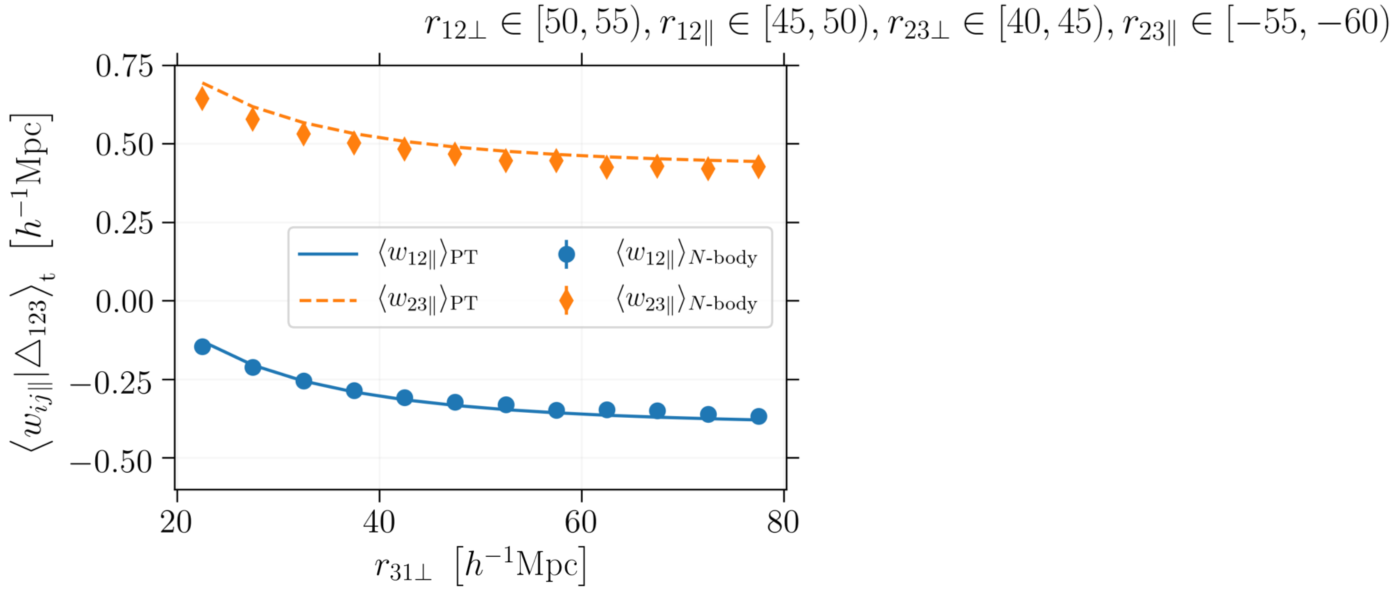
$$
\begin{aligned}
&1 + \xi_{\mathrm{s}}\left(s_{12\parallel}, s_{12\perp}\right) + \xi_{\mathrm{s}}\left(s_{23\parallel}, s_{23\perp}\right) + \xi_{\mathrm{s}}\left(\check{s}_{31\parallel}, s_{31\perp}\right) + \zeta_{\mathrm{s}}\left(\boldsymbol{s}_{12}, \boldsymbol{s}_{23}\right) \\
&= \int \left[1 + \xi(\check{r}_{12}) + \xi(\check{r}_{23}) + \xi(\check{r}_{31}) + \zeta(\check{r}_{12}, \check{r}_{23}, \check{r}_{31})\right] \\
&\qquad \mathcal{P}^{(3)}_{\boldsymbol{w}_\parallel}\left(s_{12\parallel} - r_{12\parallel}, s_{23\parallel} - r_{23\parallel} \,\middle|\, \check{\boldsymbol{r}}_{12}, \check{\boldsymbol{r}}_{23}\right) \mathrm{d}r_{12\parallel} \, \mathrm{d}r_{23\parallel}
\end{aligned}
$$

What does $\mathcal{P}^{(3)}_{w_\parallel}$ look like?


PDF
$\mathcal{P}^{(3)}_{w\|}$ $[h^2\mathrm{Mpc}^{-2}]$
0.000
0.002
0.004
0.006
0.008
$w_{12\|}$ $[h^{-1}\mathrm{Mpc}]$
$w_{23\|}$ $[h^{-1}\mathrm{Mpc}]$
−15 −10 −5 0 5 10 15

Can we predict the velocity moments?

Mean

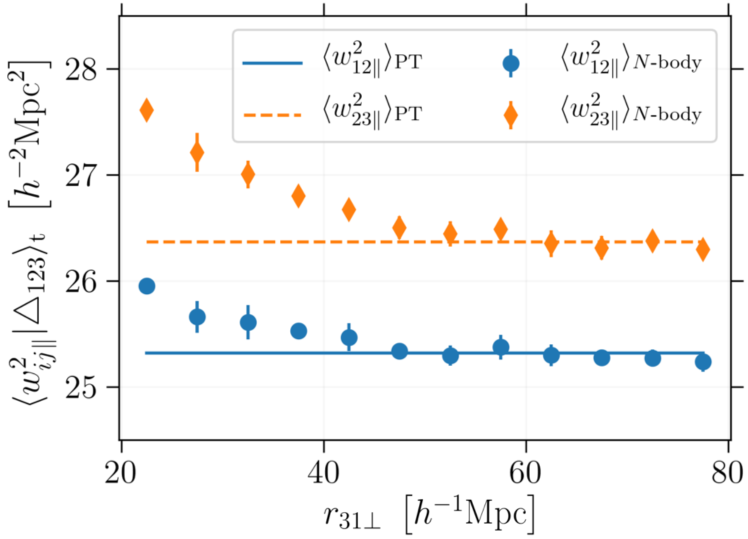


$$\langle w_{12\parallel} | \triangle_{123} \rangle_{\rm t} \simeq \bar{w}(r_{12})\, \mu_{12} - \frac{1}{2} \left[\bar{w}(r_{23})\, \mu_{23} + \bar{w}(r_{31})\, \mu_{31} \right]$$

$$\langle w_{23\parallel} | \triangle_{123} \rangle_{\rm t} \simeq \bar{w}(r_{23})\, \mu_{23} - \frac{1}{2} \left[\bar{w}(r_{12})\, \mu_{12} + \bar{w}(r_{31})\, \mu_{31} \right]$$

Second moment

$r_{12\perp} \in [50, 55), r_{12\parallel} \in [45, 50), r_{23\perp} \in [40, 45), r_{23\parallel} \in [-55, -60)$



$$\langle w^2_{ij\parallel} | \triangle_{123} \rangle_{\rm t} \simeq 2 \left[\sigma_v^2 - \psi_\parallel(r_{ij}) \right]$$

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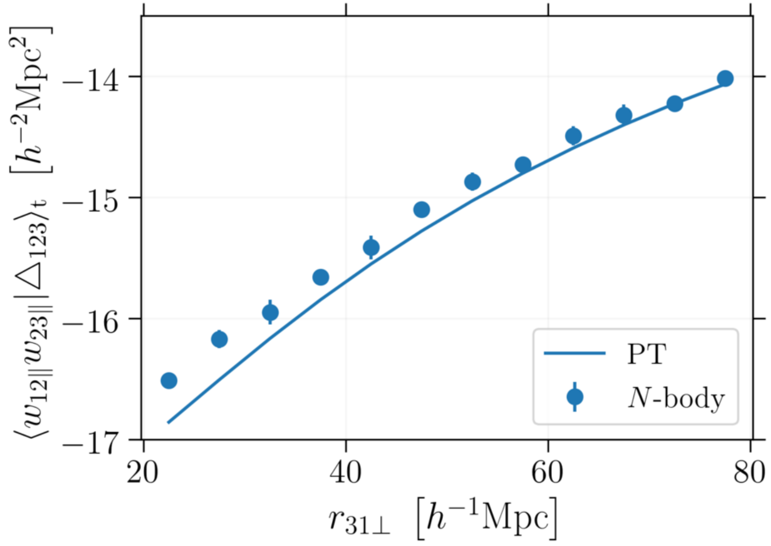
Second cross moment

$r_{12\perp} \in [50, 55), r_{12\|} \in [45, 50), r_{23\perp} \in [40, 45), r_{23\|} \in [-55, -60)$

$$\langle w_{12\|}\, w_{23\|} | \triangle_{123} \rangle_{\rm t} \simeq \psi_{\|}(r_{12}) + \psi_{\|}(r_{23}) - \psi_{\|}(r_{31}) - \sigma_v^2 \;.$$



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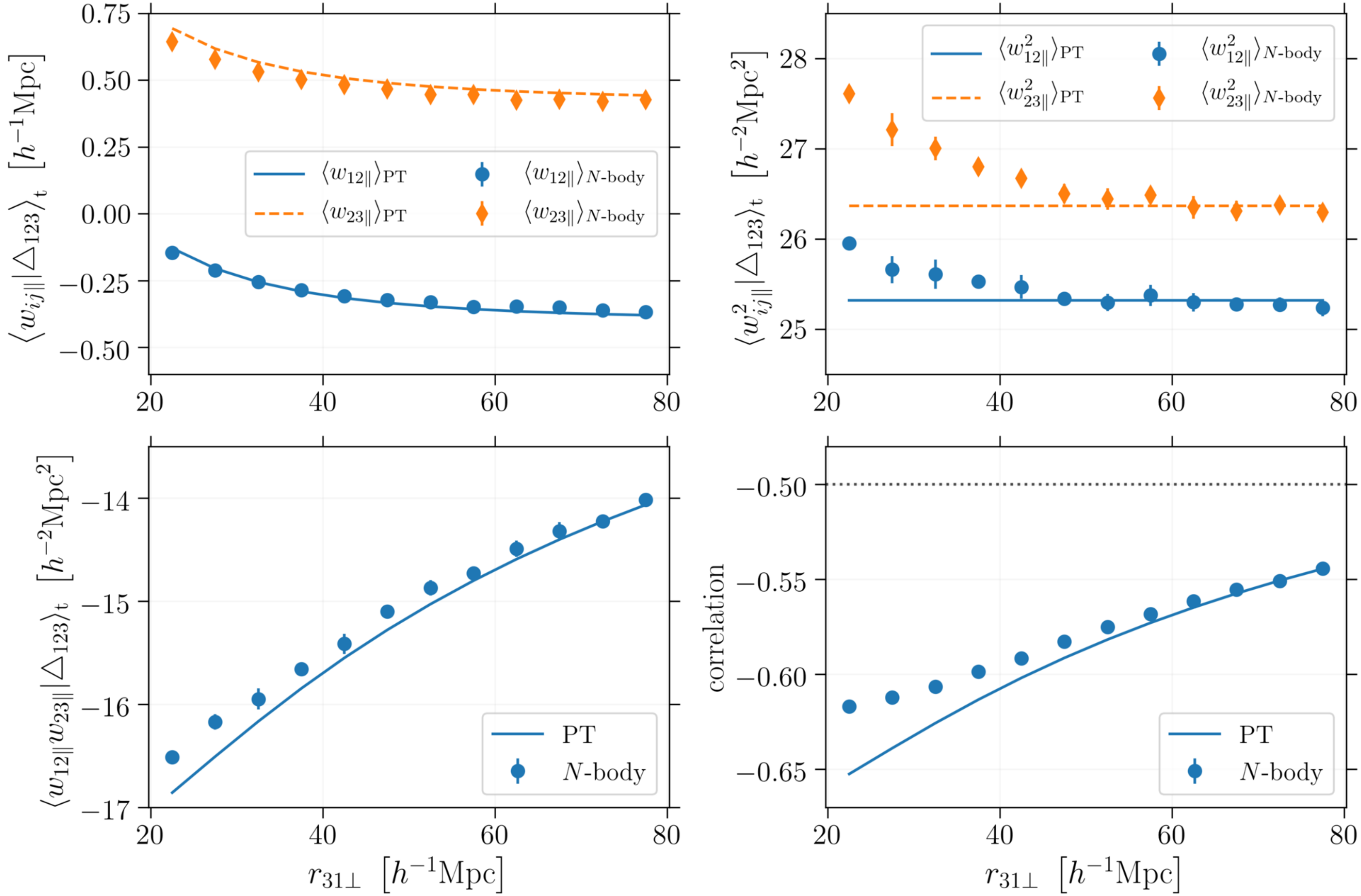

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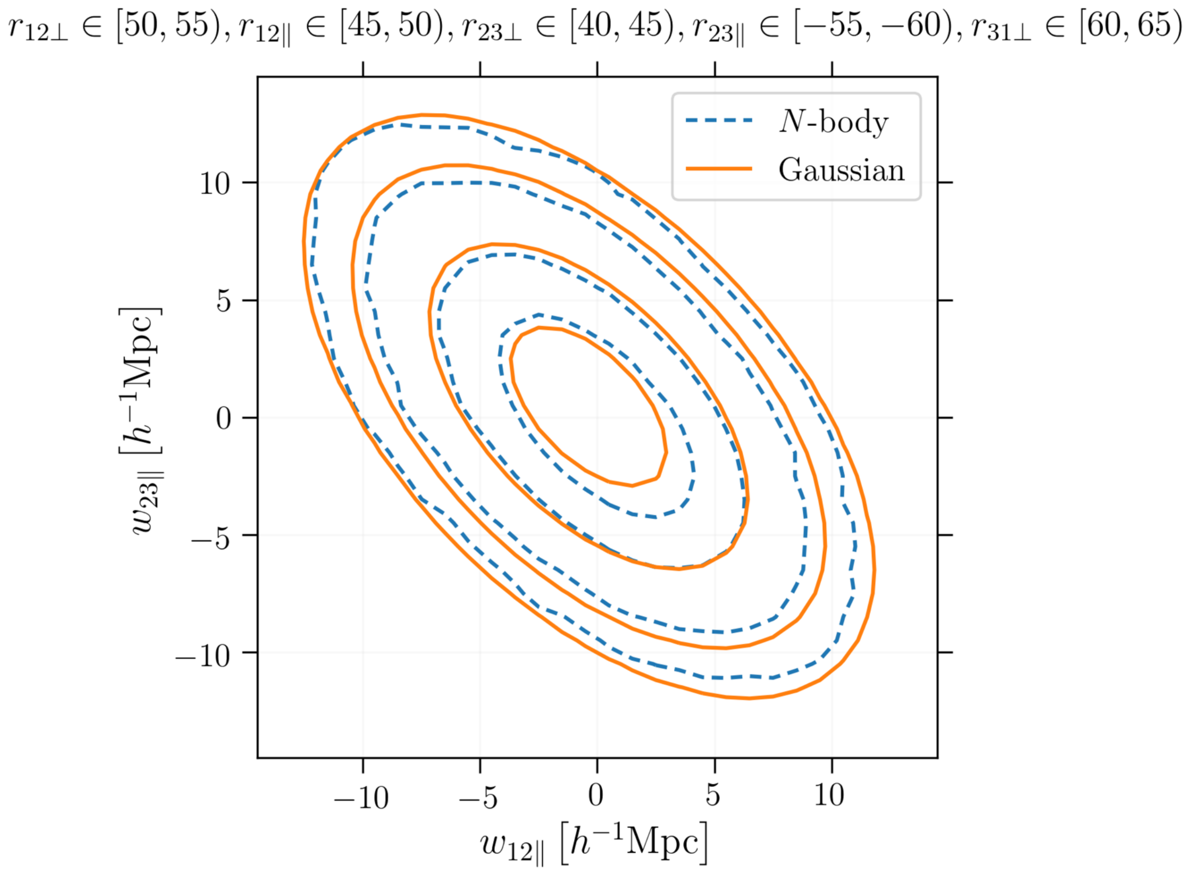
Bivariate Gaussian + velocity moments from PT

“Truth is much too complicated to allow anything but approximations.” -- John von Neumann



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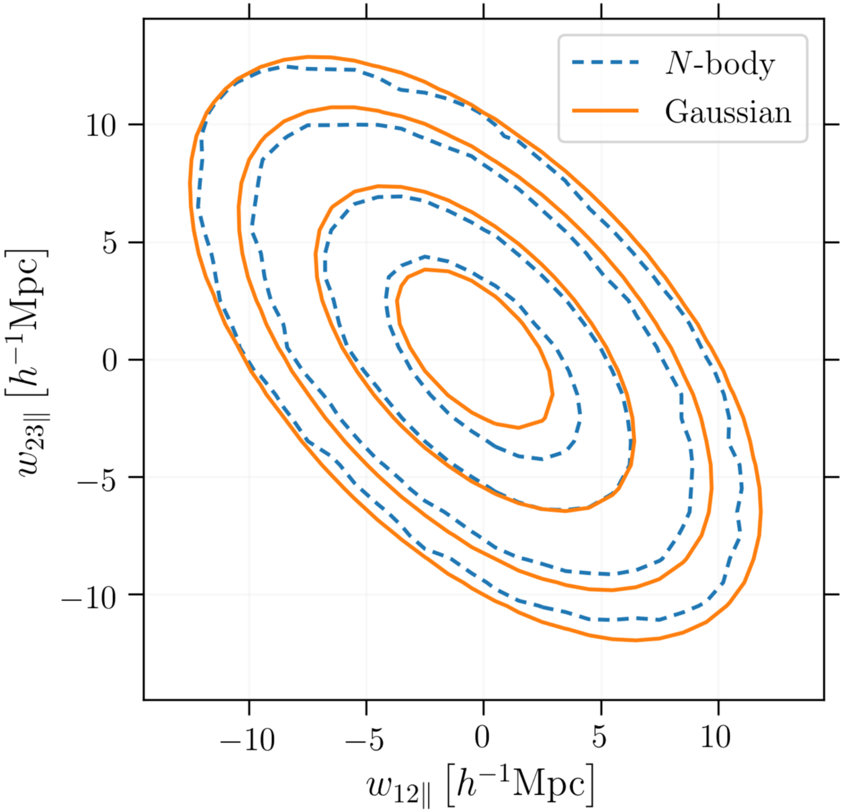


- For these triangular configurations, Gaussian approximation is nearly lossless.
- Jensen-Shannon divergence = 0.005 nats.
- JS divergence is bound between 0 and 0.693.

Three-point Gaussian Streaming Model (3ptGSM)

Simplest implementation:

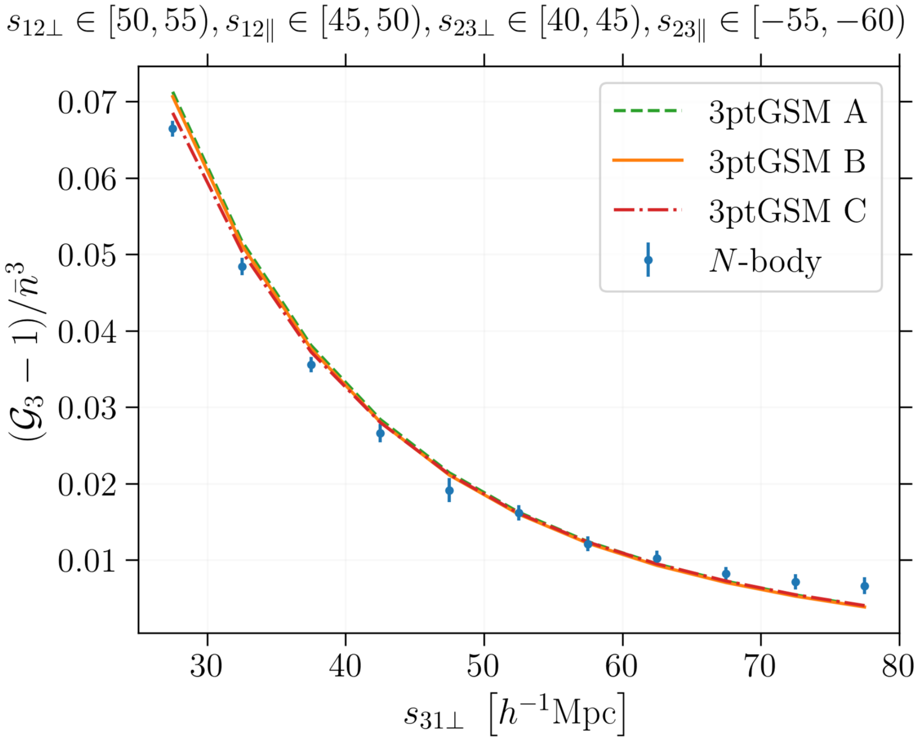
- Velocity moments from the standard perturbation theory at leading order

- Bivariate Gaussian as the approximating PDF for $\mathcal{P}^{(3)}_{\boldsymbol{w}_\parallel}$

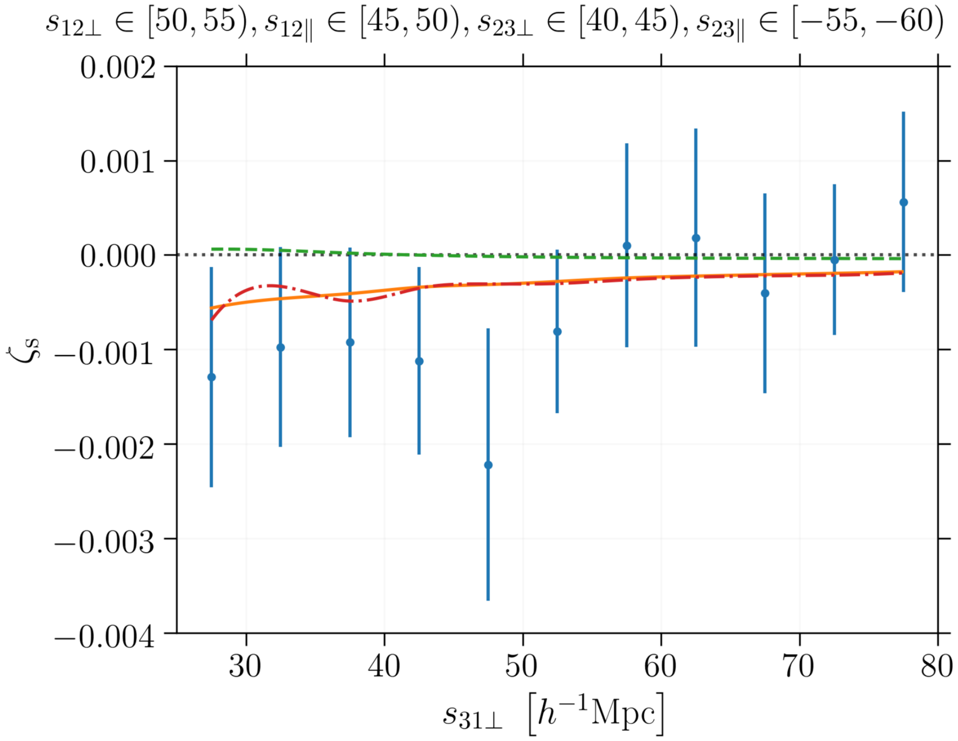
- Evaluating real space 2PCF and 3PCF at leading order (LO) in perturbation theory (PT).

Anisotropic 3PCF

(Full 3PCF)



(Connected 3PCF)



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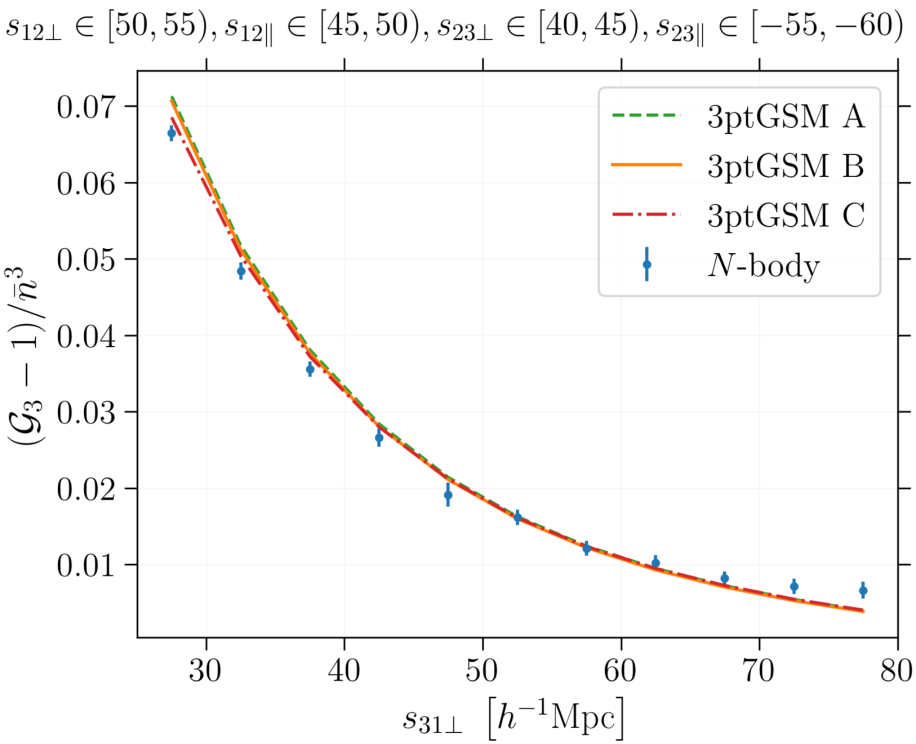
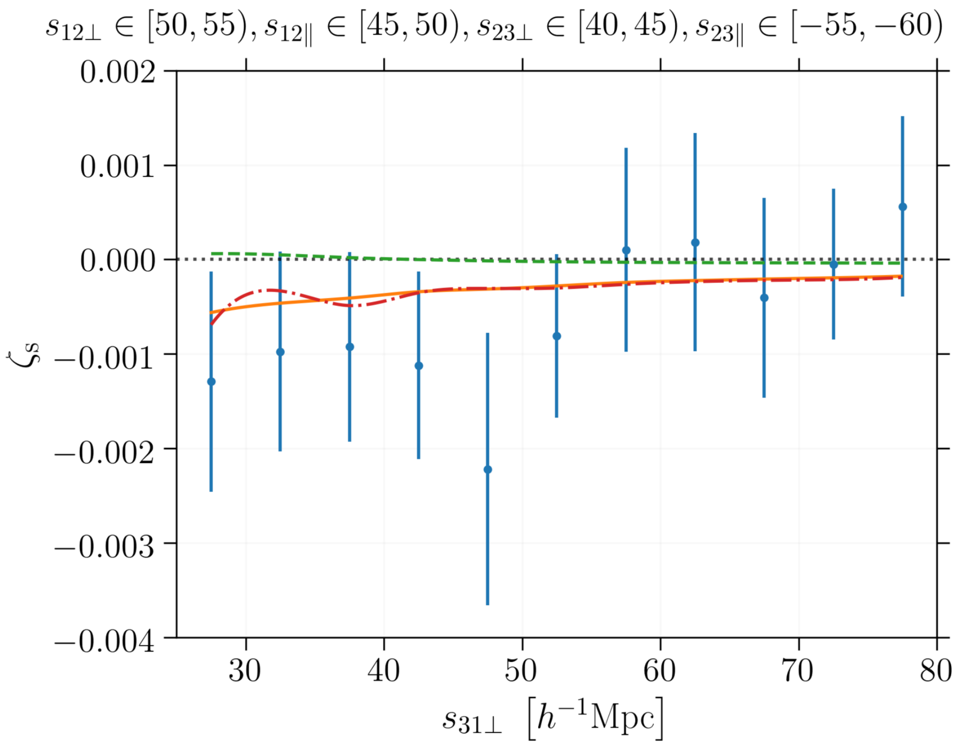


Natural estimator

$$\frac{DDD}{RRR}$$

(Connected 3PCF)



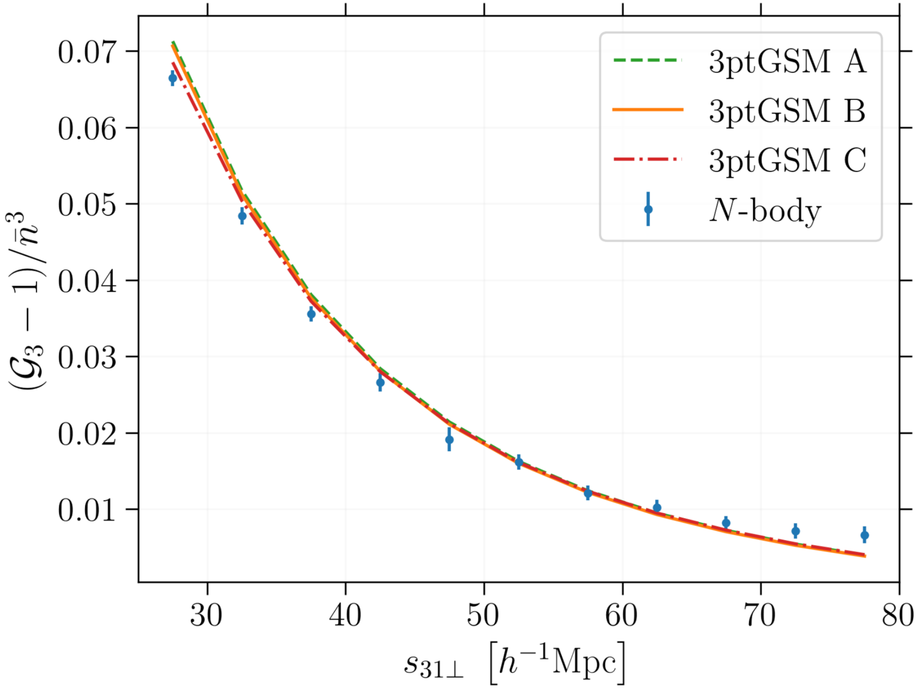
Szapudi-Szalay estimator

$$\frac{(D-R)(D-R)(D-R)}{RRR}$$

Anisotropic 3PCF

(Full 3PCF)

$s_{12\perp} \in [50, 55), s_{12\parallel} \in [45, 50), s_{23\perp} \in [40, 45), s_{23\parallel} \in [-55, -60)$



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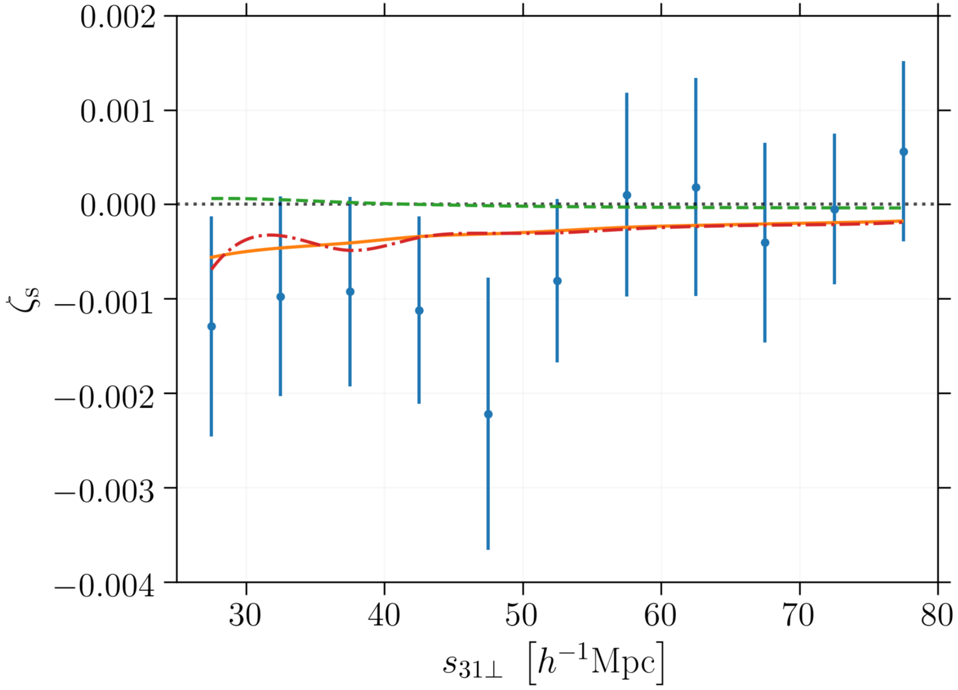

Case A: ξ (LO-PT) and $\zeta = 0$

Case B: ξ (LO-PT) and ζ (LO-PT)

Case C: ξ (halo model) and ζ (LO-PT)

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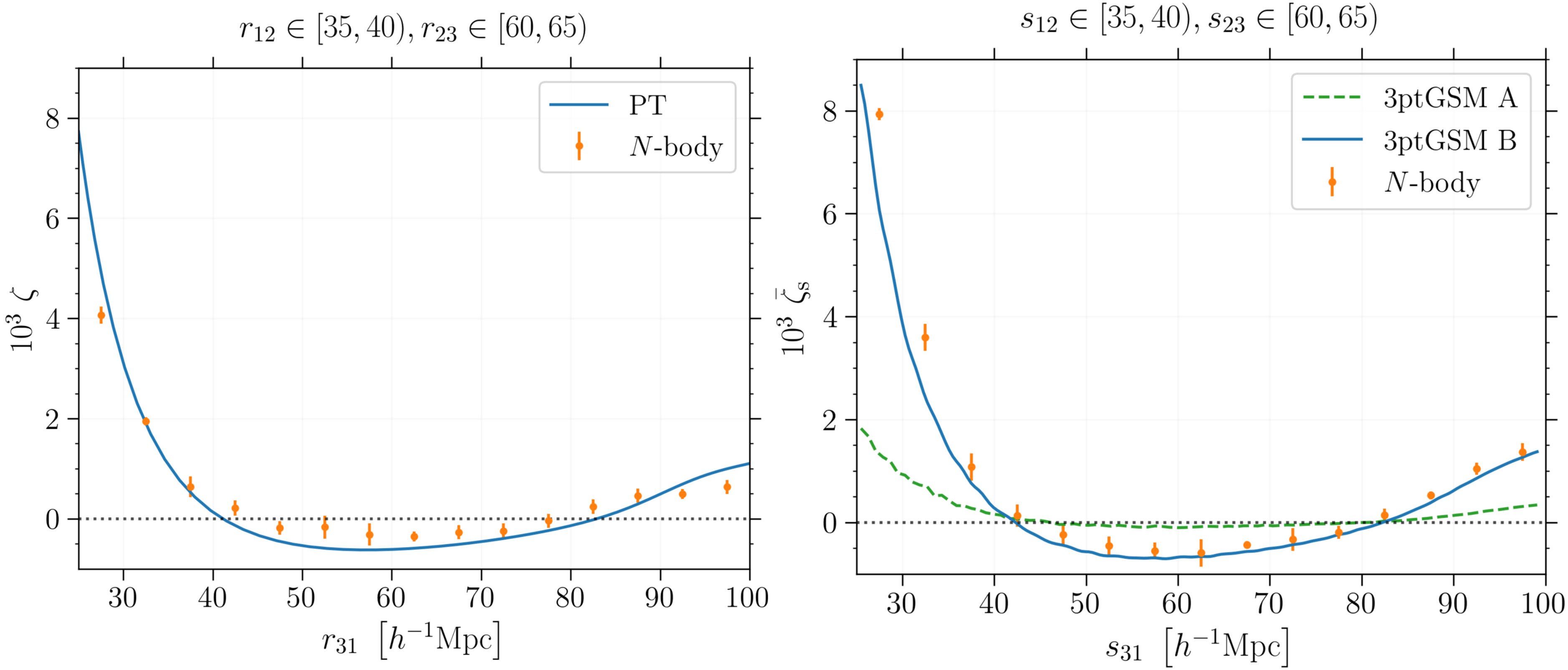
Connected 3PCF: spherically averaged

(real space) (redshift space)



Case A: ξ (LO-PT) and $\zeta = 0$

Case B: ξ (LO-PT) and ζ (LO-PT)

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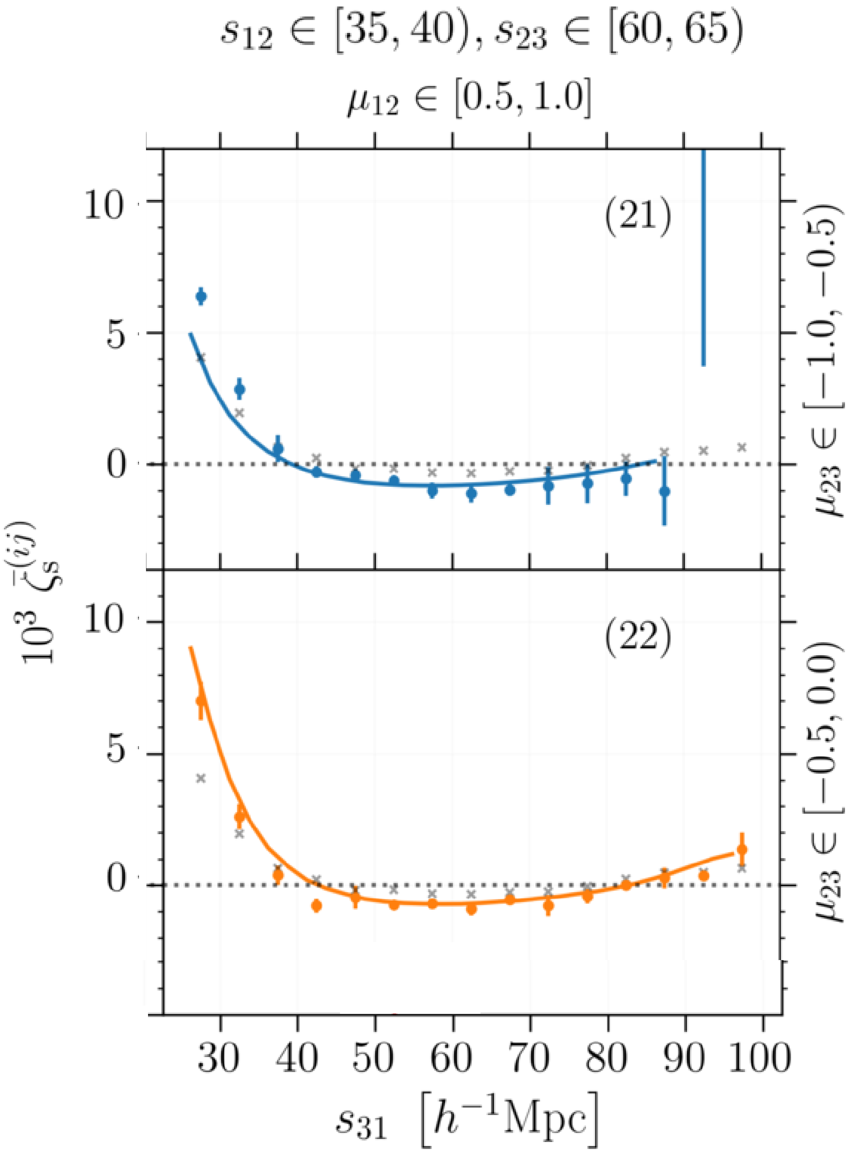
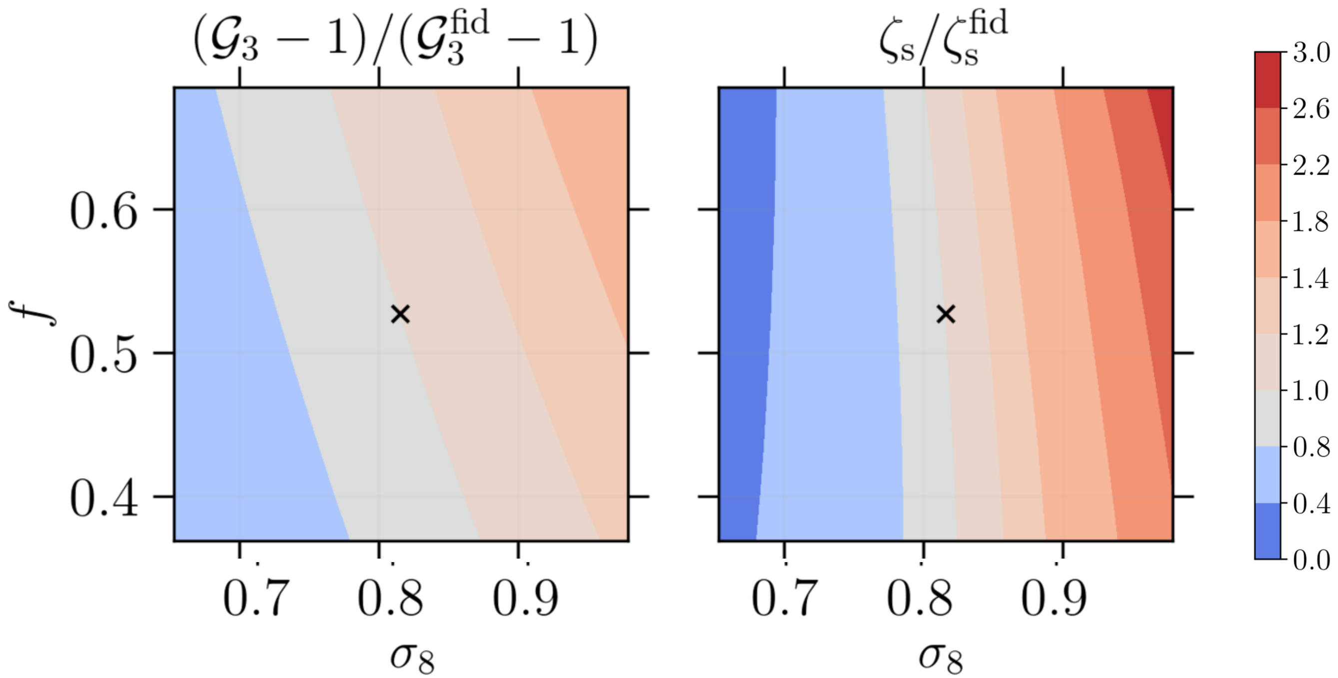
Connected 3PCF: wedges



Dependence on the growth rate of structure

Breaking f-σ_8 degeneracy

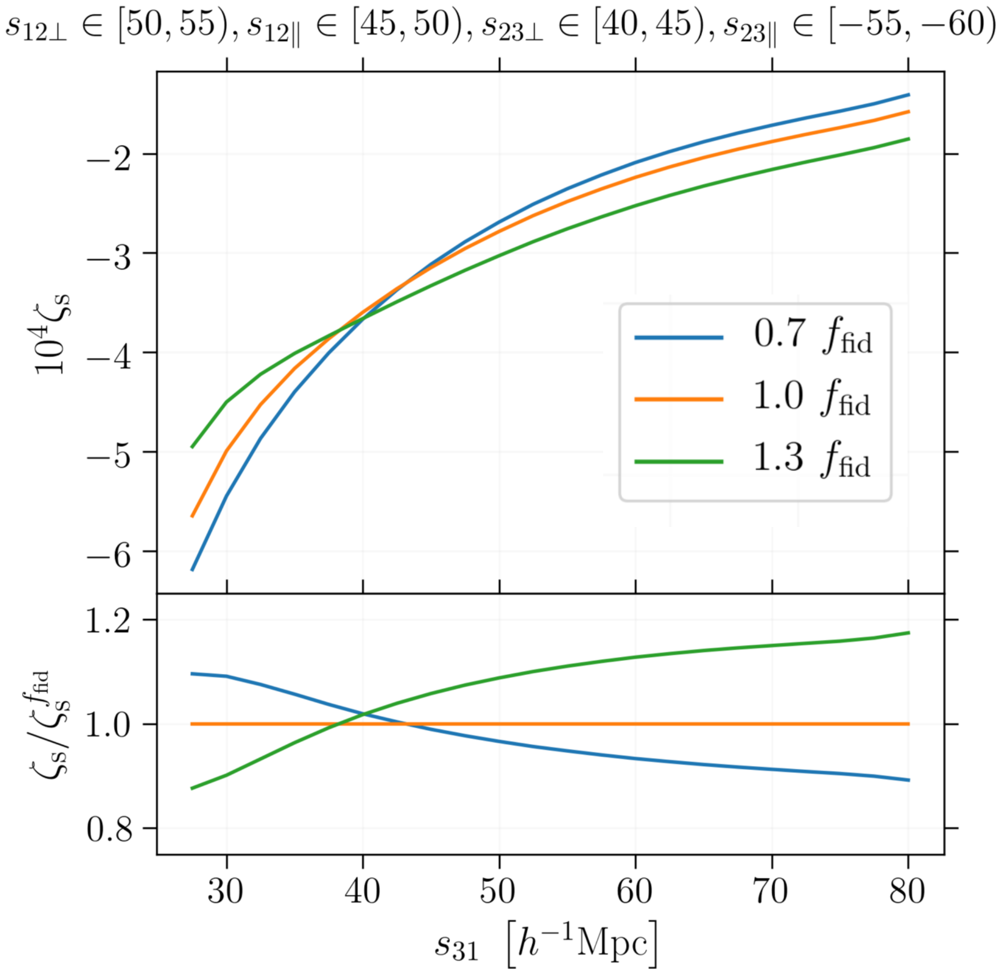
$$s_{12\perp} = 52.5, s_{12\|} = 47.5, s_{23\perp} = 42.5, s_{23\|} = -57.5, s_{31\perp} = 50.0$$



- Suggests that measuring redshift space 3PCF with sufficient accuracy can help in breaking f-σ_8 degeneracy.
- Similar conclusion was reached based on power spectrum and bispectrum.

(Gil-Marin et al. 14)

Varying growth rate



Conclusions

"All models are wrong, but some are useful" -- George Box

- For three-point: velocity moments were predicted using perturbation theory at leading order and compared to simulations.
- Introduced a phenomenological model for 3PCF: the three-point Gaussian streaming model.
- Our results suggests that 3PCF in redshift space can help in breaking f-σ_8 degeneracy

Outlook:

- Apply 3ptGSM to biased tracers.
- Use more sophisticated PT flavours like CLPT or CLEFT

Thank you for listening!