A Real-Time (Semiclassical) Picture of Vacuum Decay

Jonathan Braden
Canadian Institute for Theoretical Astrophysics
www.cita.utoronto.ca/~jbraden

w/ Matt Johnson, Hiranya Peiris, Andrew Pontzen, and Silke Weinfurtner
1712.02356, 1806.06069, 1904.07873, and in progress

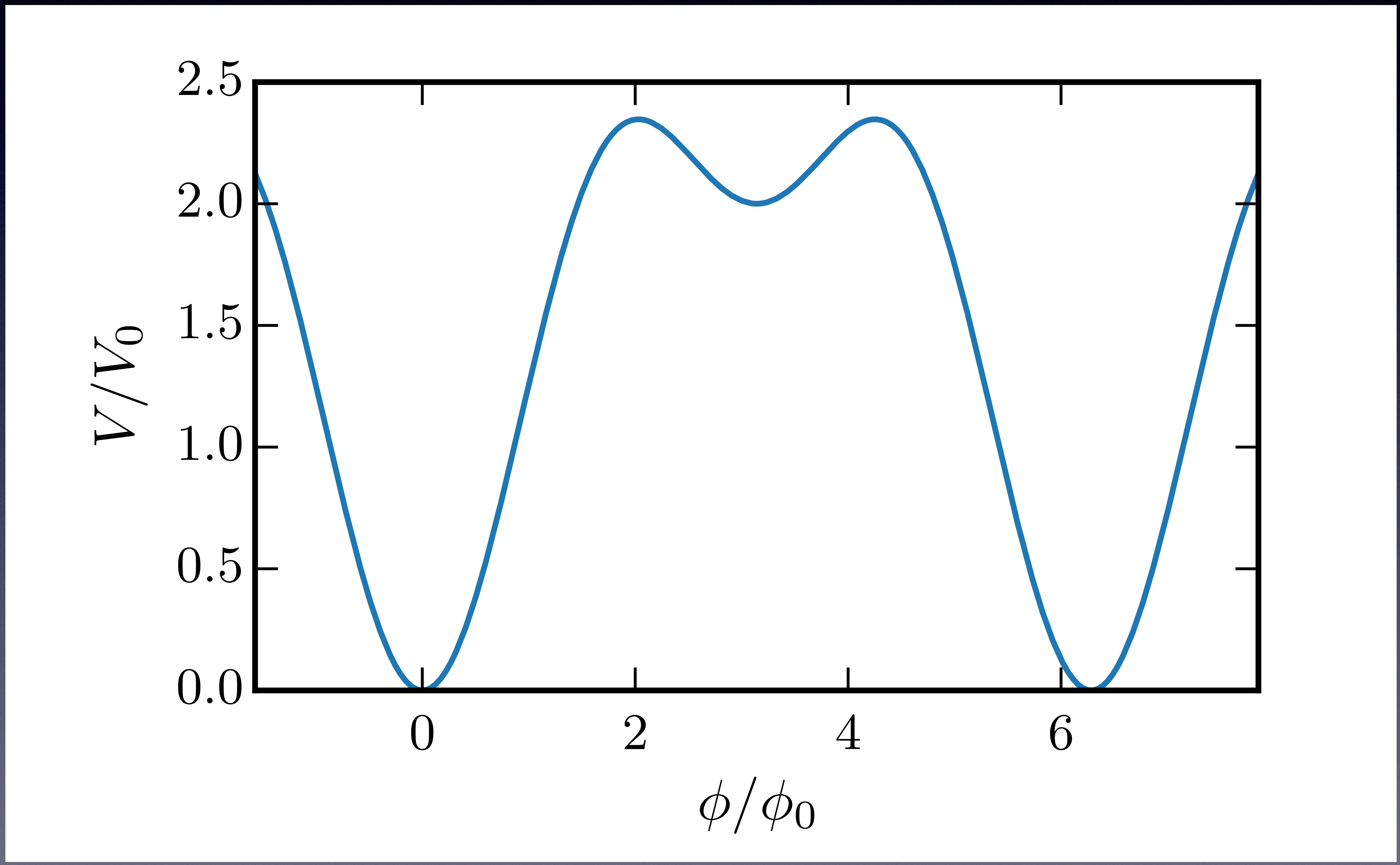
2018 Buchalter Cosmology 3rd Prize

Cosmology from Home

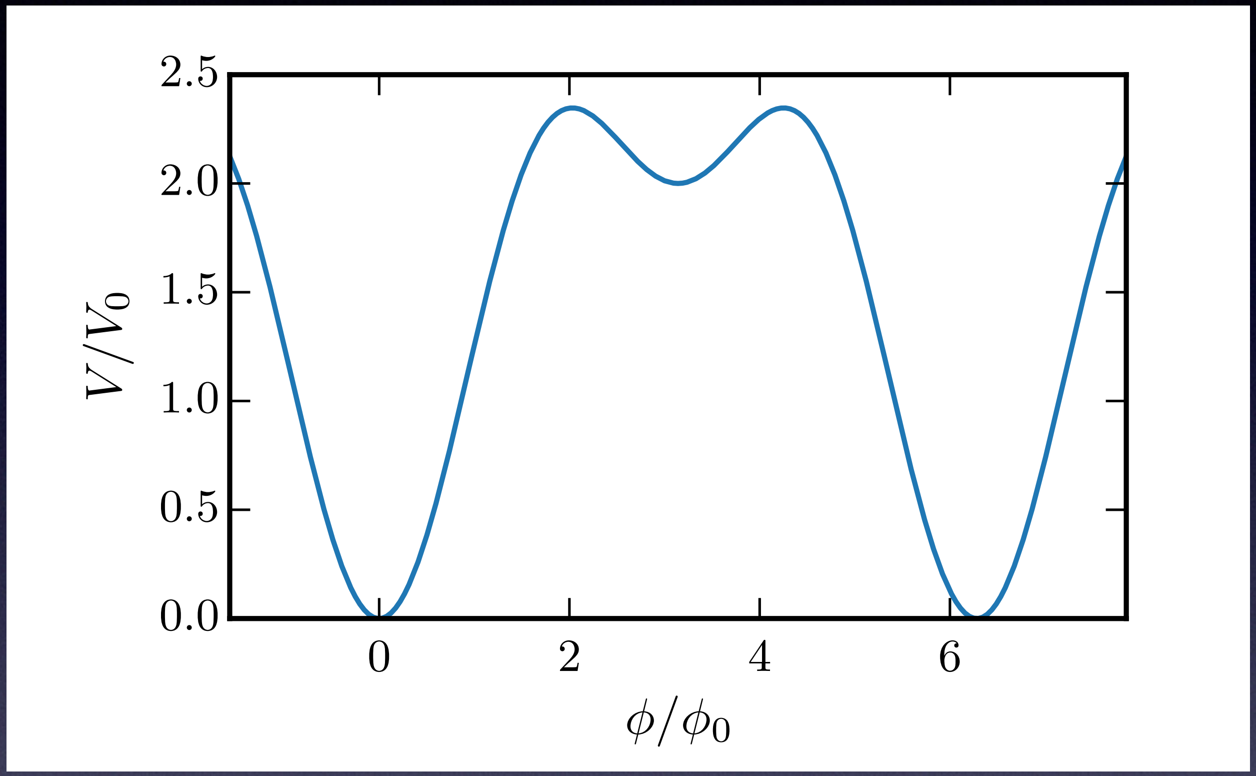
Outline

- Review of Vacuum Decay and 1st Order Phase Transitions
- Euclidean Description
- Real-Time Description of Decay
- Simulation in the Lab
- Conclusions / Novel Future Applications

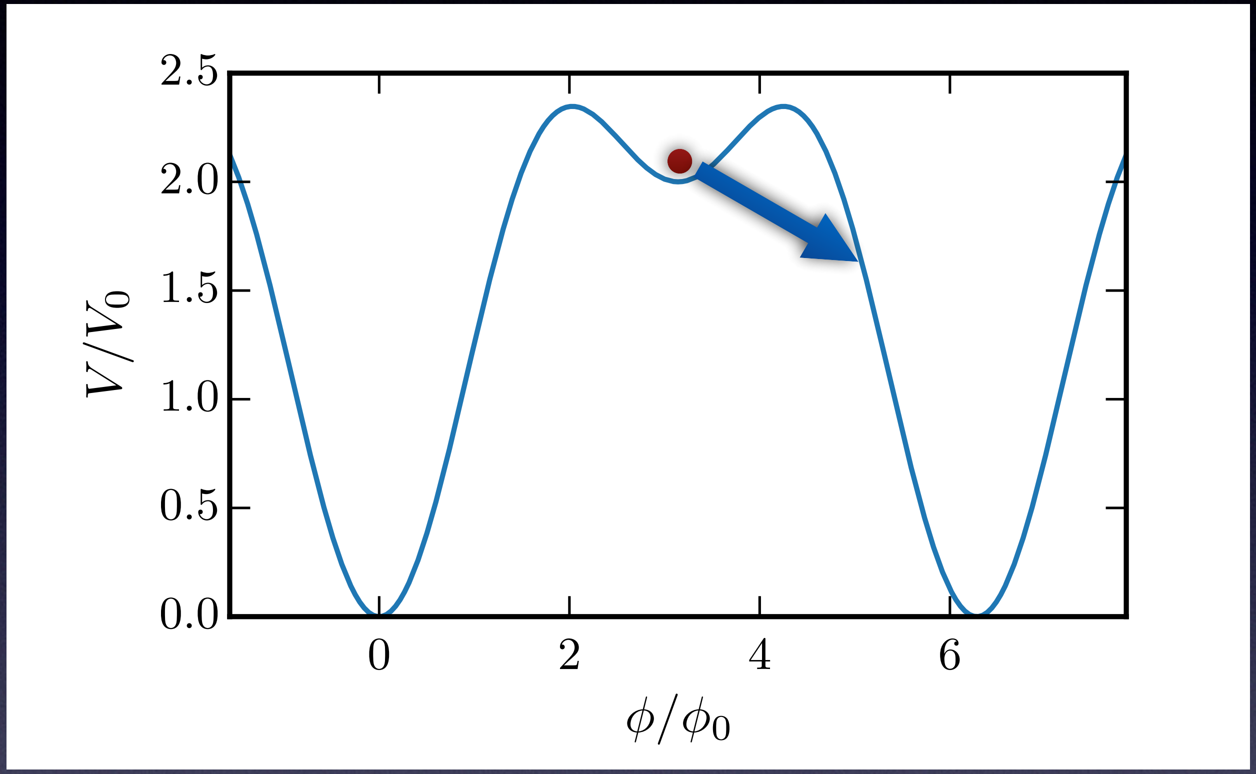
First Order Phase Transitions

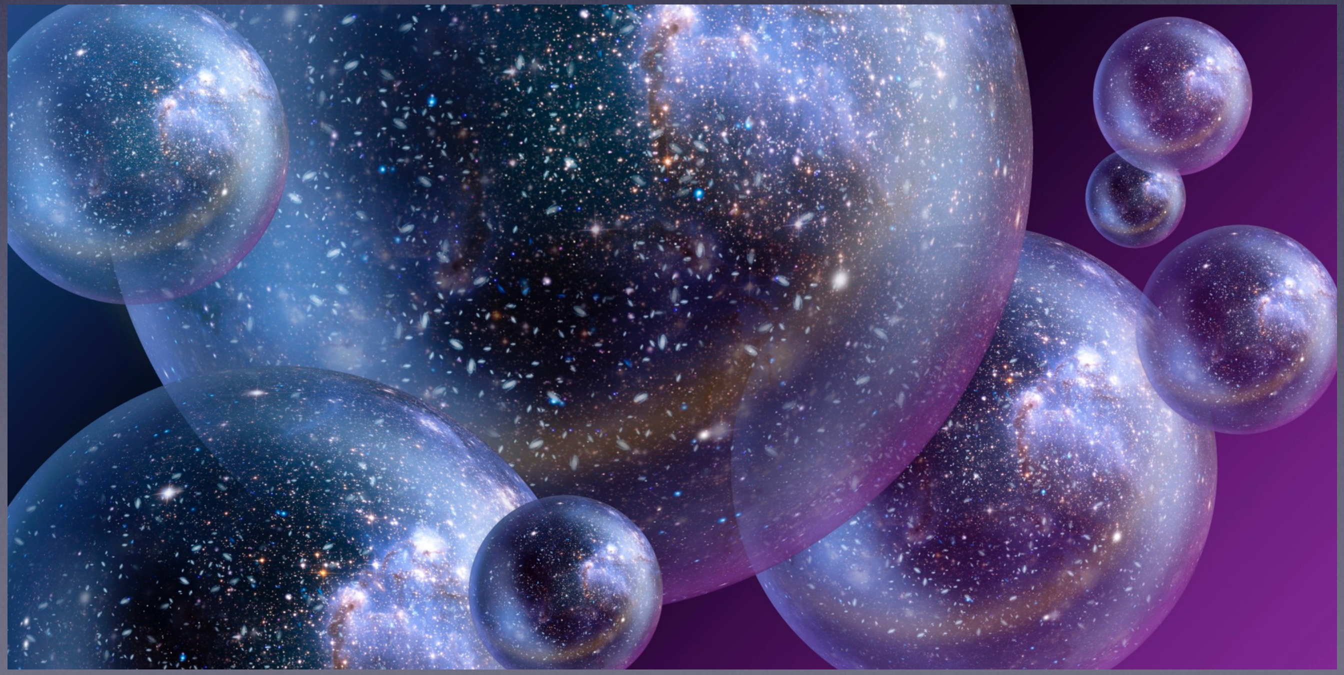


First Order Phase Transitions

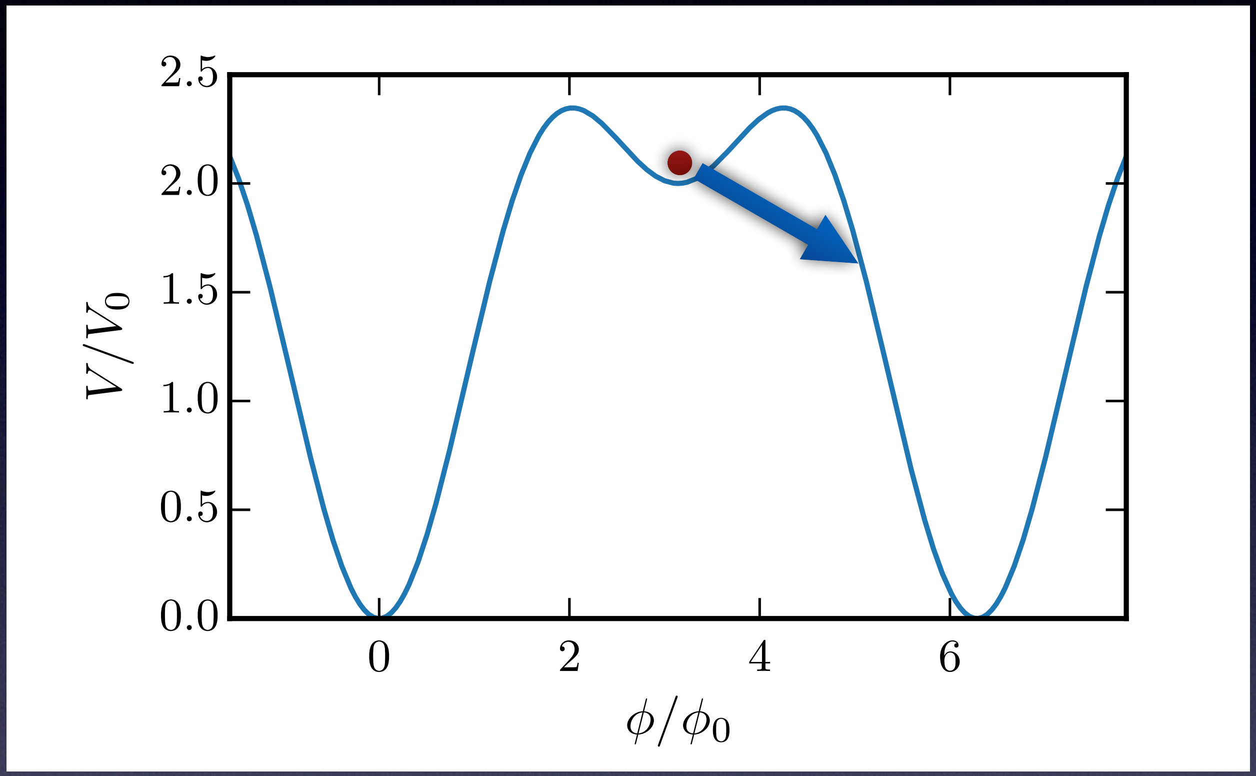


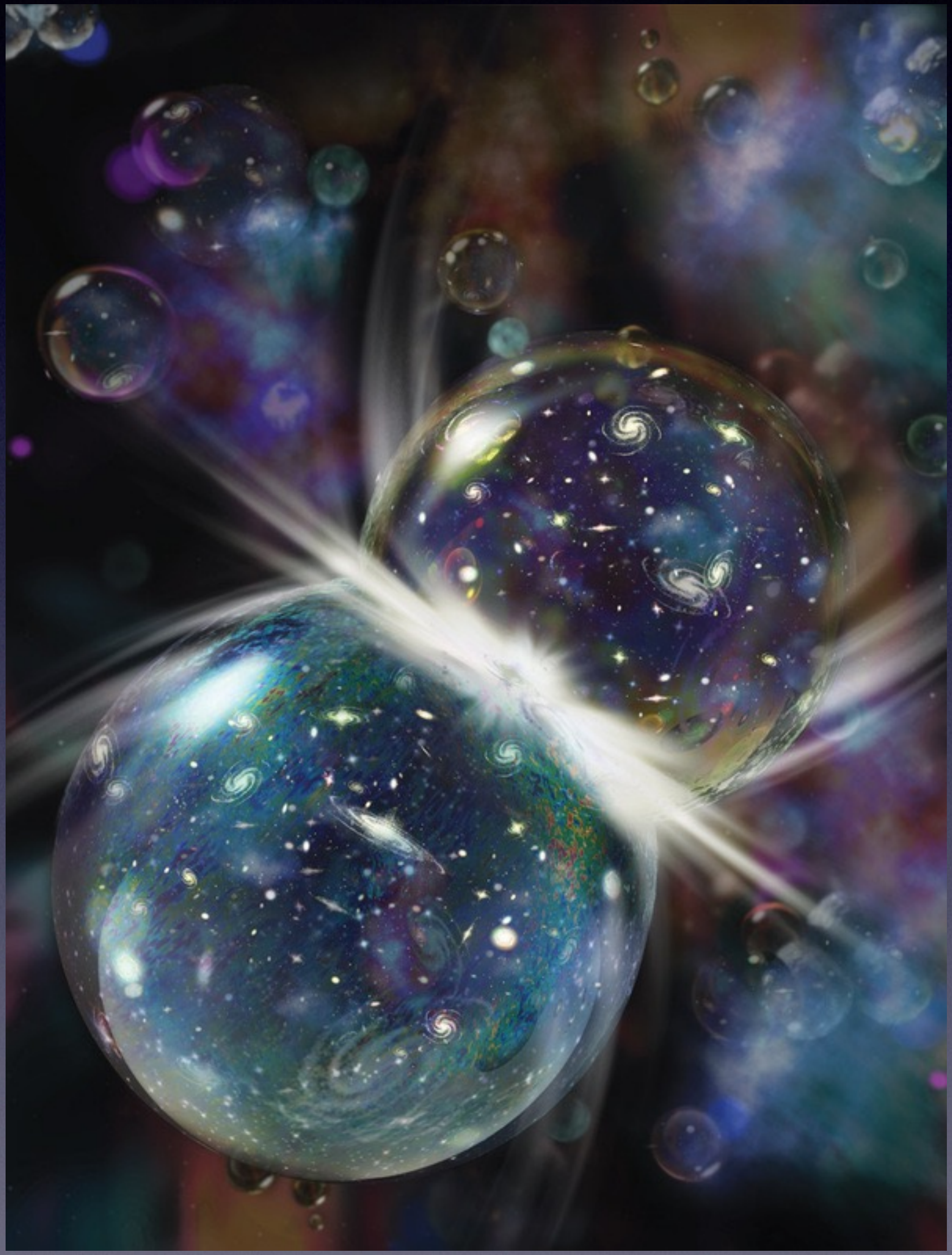
First Order Phase Transitions

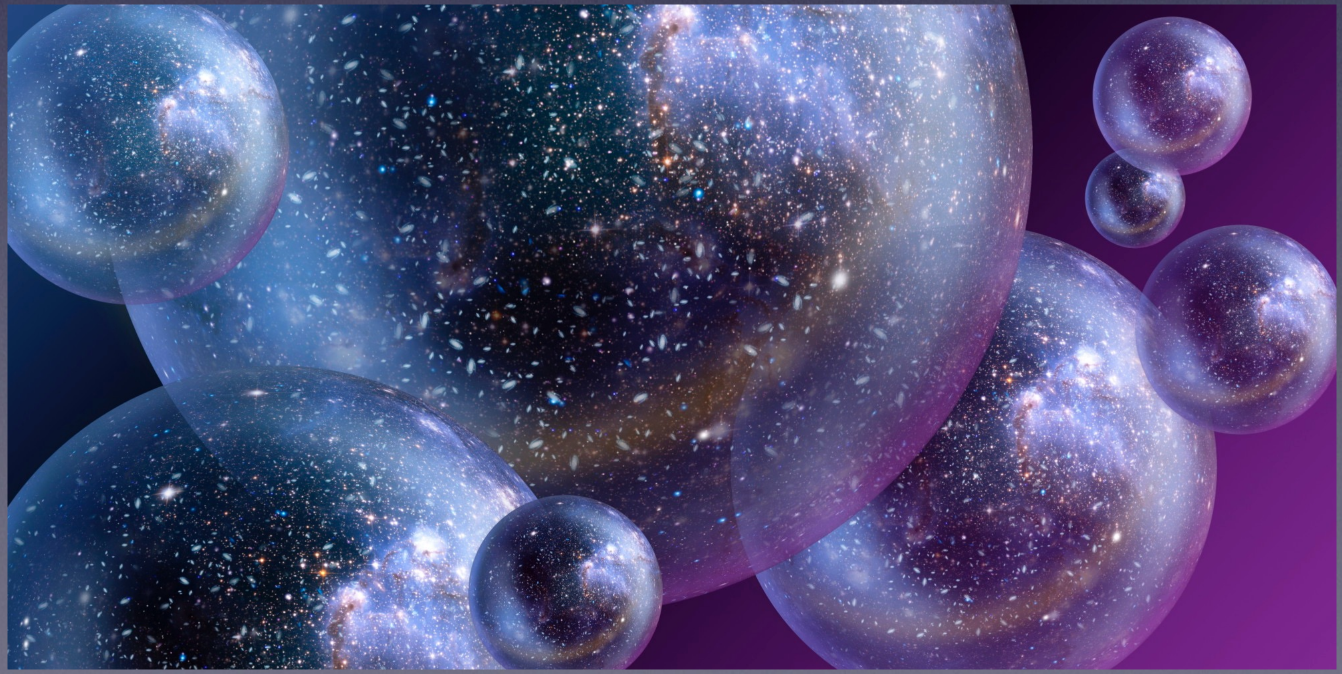




First Order Phase Transitions








INSTA


INSTA

0th Order Questions

- How fast does the vacuum decay?
- Do bubbles form?
- What do the bubbles look like?

Can be tackled with Euclidean bounce formalism

Bounce Formalism

[Coleman 1977 + many others]

$$P_{\text{undecayed}} = |\langle \Omega_{\text{FV}}(t) | \Omega_{\text{FV}}(t=0) \rangle|^2 \sim e^{-\Gamma t}$$

Work in Euclidean Time and Extremise Euclidean Action

$$S_{\text{E}} = A_{\text{d+1}} \int dr_{\text{E}} r_{\text{E}}^d \left(\frac{\phi'^2}{2} + V(\phi) \right)$$

Solve the following BVP

$$\frac{\partial^2 \phi_{\text{I}}}{\partial r_{\text{E}}^2} + \frac{d}{r_{\text{E}}} \frac{\partial \phi_{\text{I}}}{\partial r_{\text{E}}} - \frac{\partial V}{\partial \phi} = 0 \qquad \frac{\partial \phi_{\text{I}}}{\partial r_{\text{E}}}(0) = 0 \qquad \phi(\infty) = \phi_{\text{fv}}$$

$$r_{\text{E}}^2 = \tau^2 + \mathbf{r}^2 \qquad \tau = it$$

Decay Rates

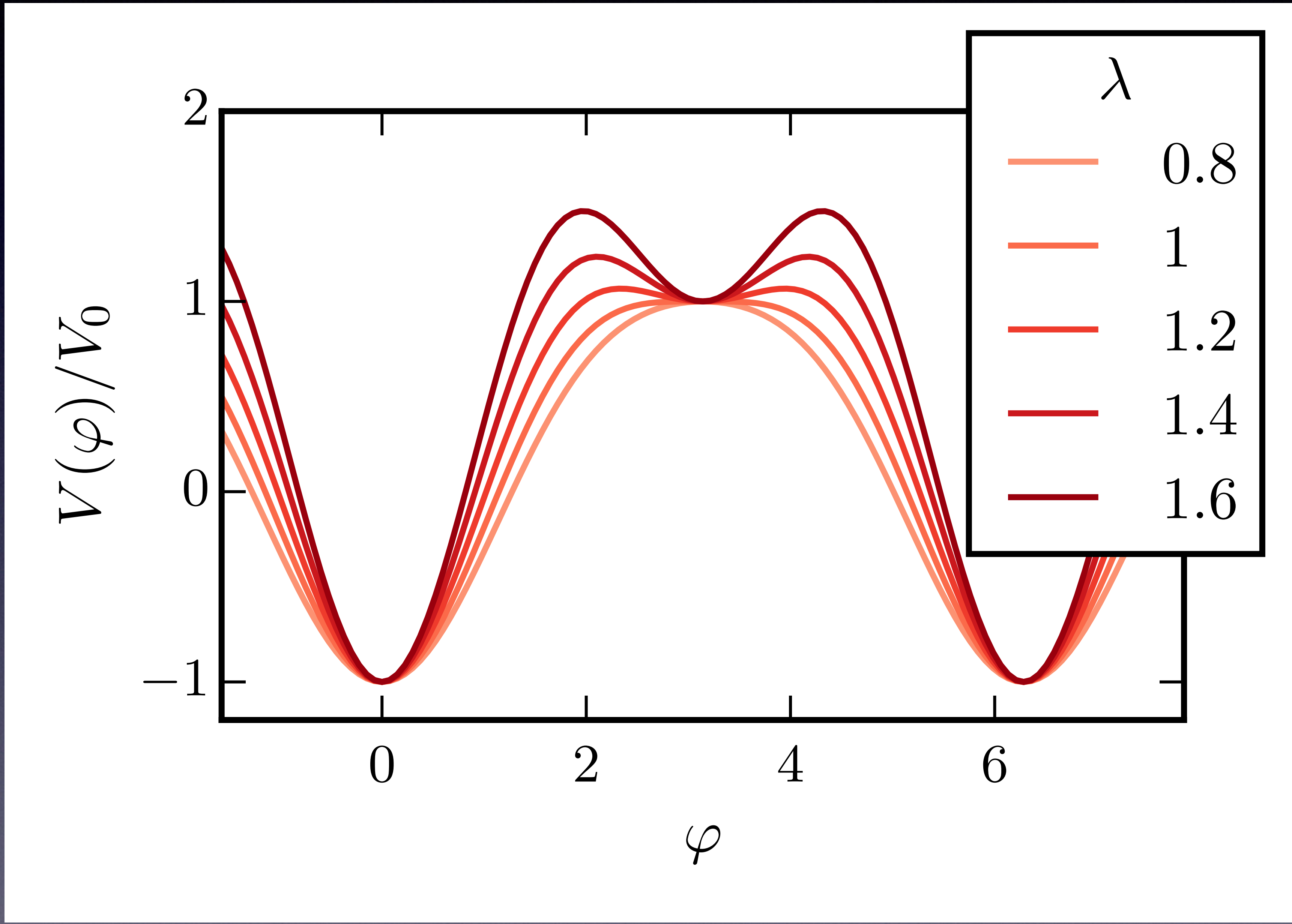
$$S_{\mathrm{E}} = A_{\mathrm{d+1}} \int dr_{\mathrm{E}} r_{\mathrm{E}}^{d} \left(\frac{\phi'^2}{2} + V(\phi) \right)$$

$$S_{\mathrm{I}} = S_{\mathrm{E}}[\phi_{\mathrm{B}}] - S_{\mathrm{E}}[\phi_{\mathrm{fv}}]$$

- Single negative eigenmode

$$\frac{\Gamma}{V} = \left(\frac{S_{\mathrm{I}}}{2\pi} \right)^{D/2} \sqrt{\frac{\det \delta^2 S_{\mathrm{E}}[\phi_{\mathrm{fv}}]}{\det' \delta^2 S_{\mathrm{E}}[\phi_{\mathrm{B}}]}} e^{-S_{\mathrm{I}}} \left(1 + \mathcal{O}(\hbar)\right)$$

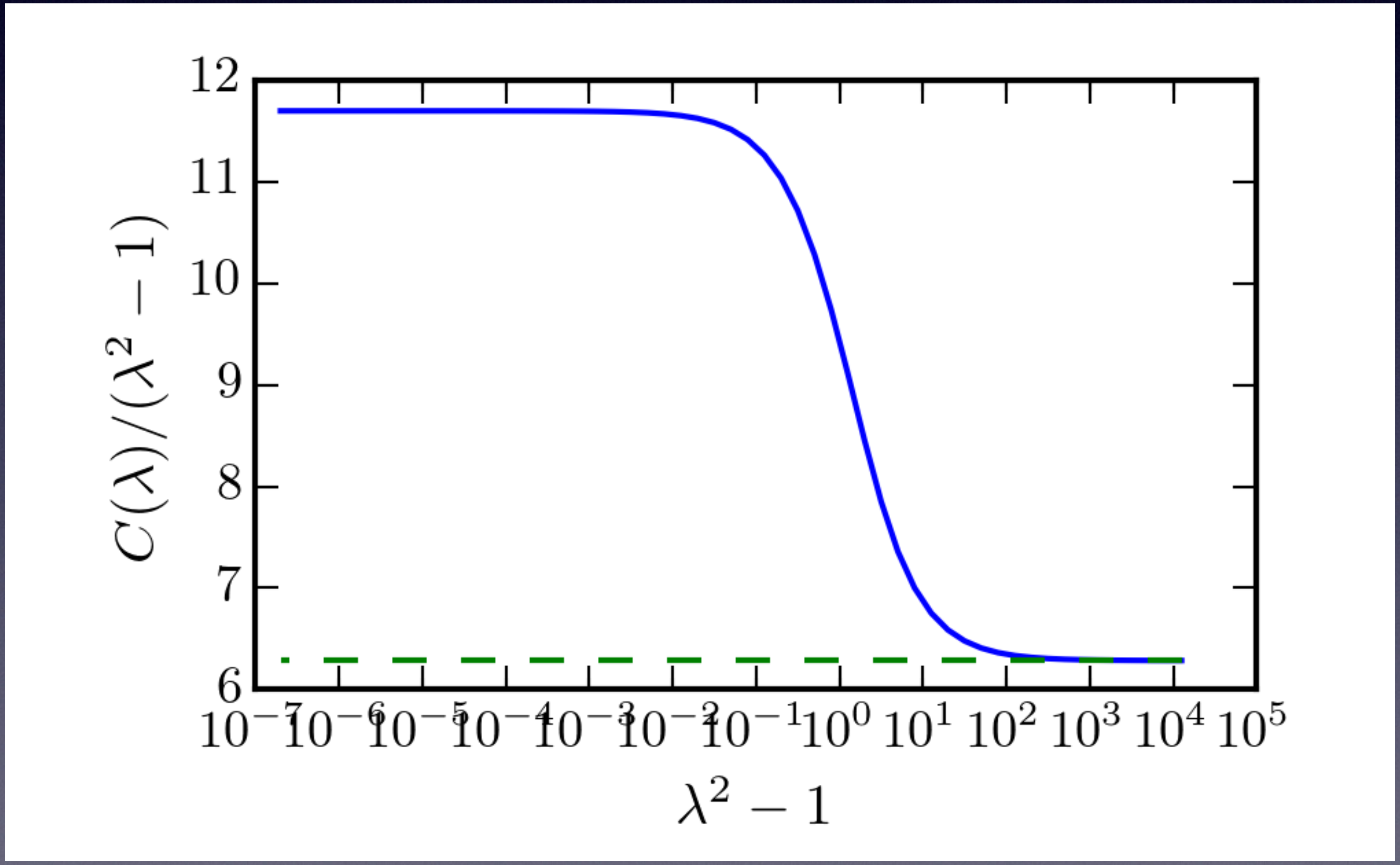
Model



$$V(\phi) = V_0 \left(-\cos\left(\frac{\phi}{\phi_0}\right) + \frac{\lambda^2}{2}\sin^2\left(\frac{\phi}{\phi_0}\right) + 1 \right)$$

Nucleation Rates

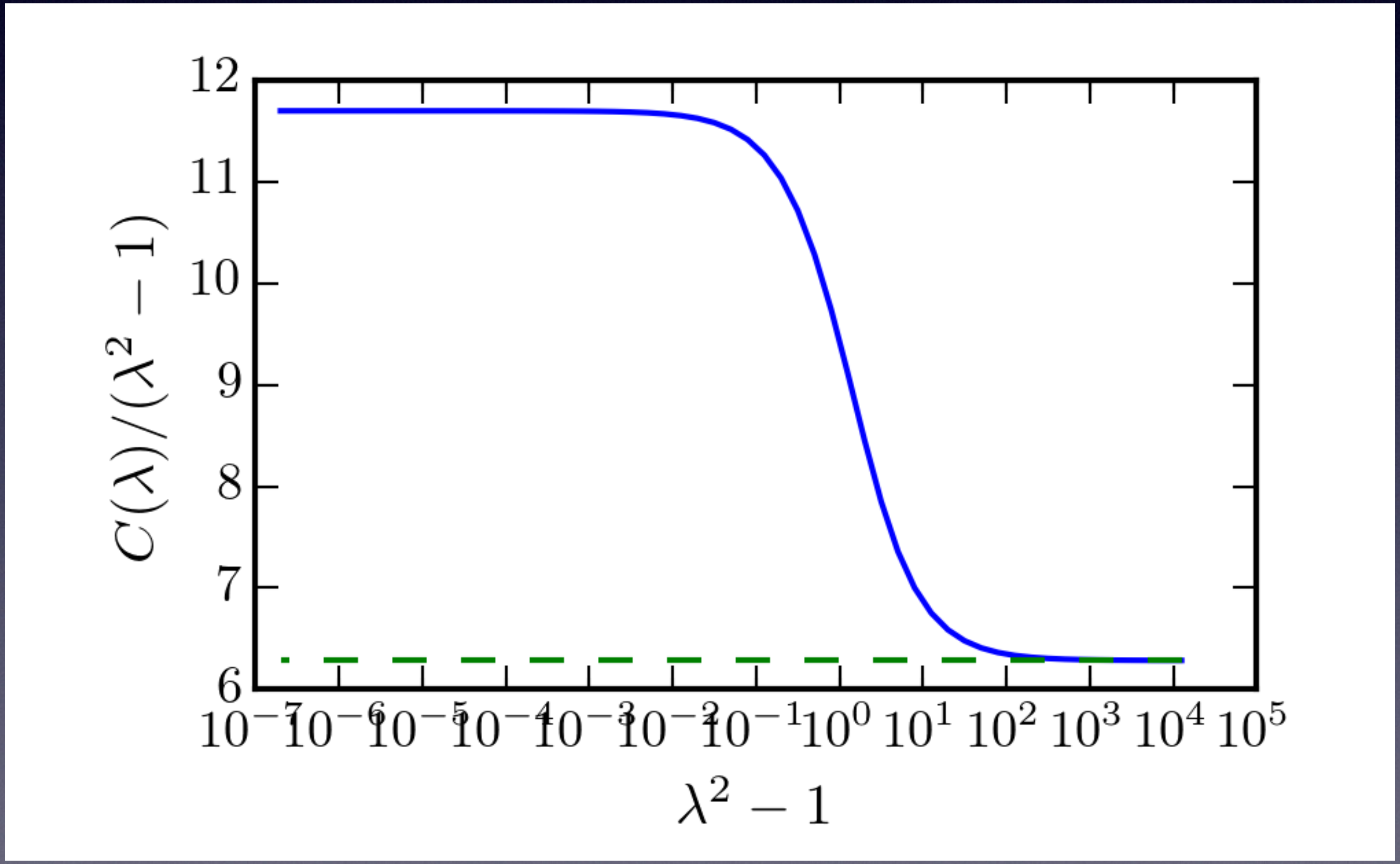
$$\frac{\Gamma}{V} \approx g(\lambda)\left[m_{\mathrm{eff}}^2\right]^{\frac{D}{2}}\left(\frac{S_{\mathrm{I}}}{2\pi}\right)^{\frac{D}{2}} e^{-S_{\mathrm{I}}}$$



$$V(\phi) = V_0\left(-\cos\left(\frac{\phi}{\phi_0}\right) + \frac{\lambda^2}{2}\sin^2\left(\frac{\phi}{\phi_0}\right) + 1\right)$$

Nucleation Rates

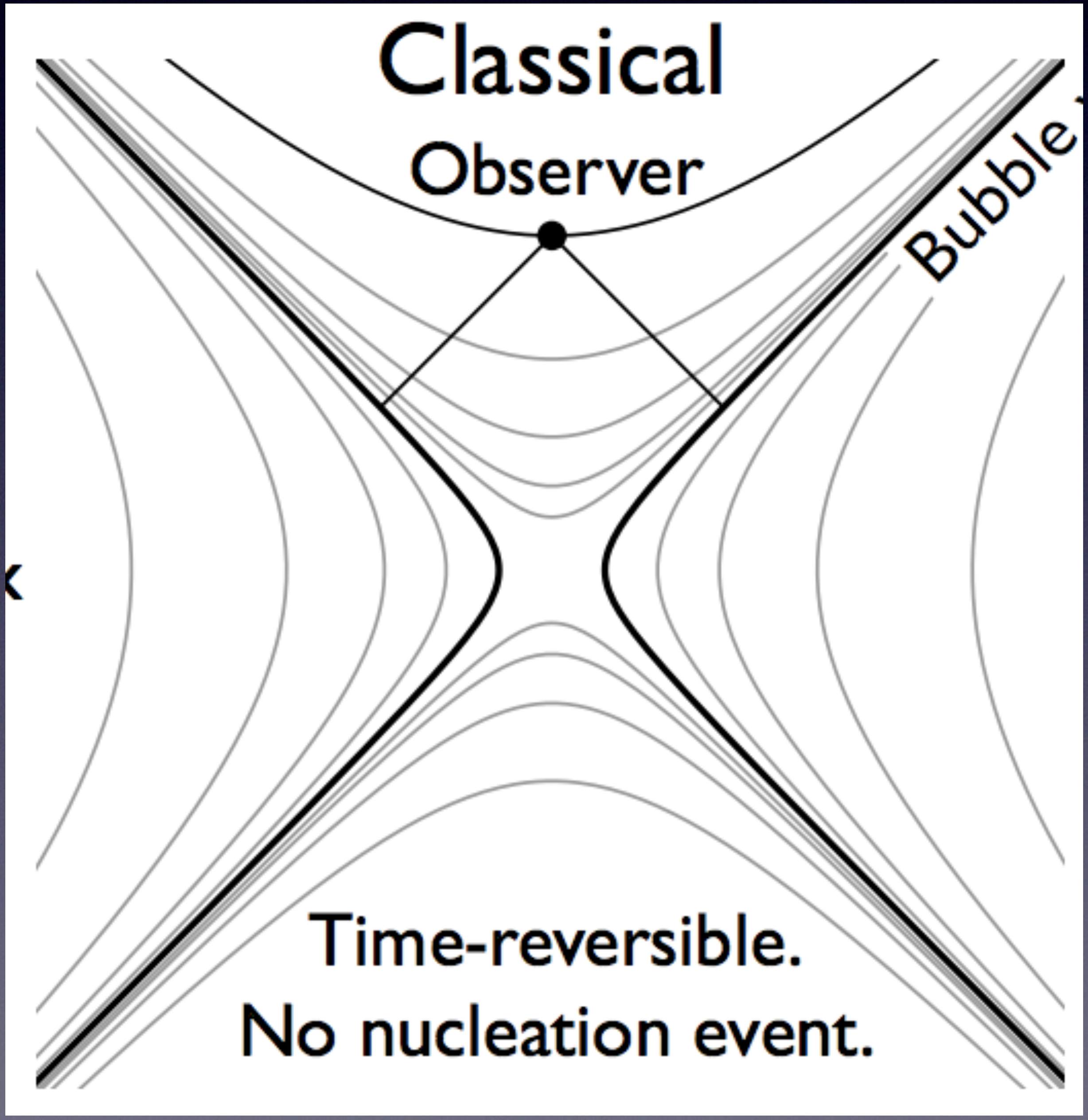
$$S_{\mathrm{I}} = 2\pi\phi_0^2 C(\lambda)$$



$$V(\phi) = V_0\left(-\cos\left(\frac{\phi}{\phi_0}\right) + \frac{\lambda^2}{2}\sin^2\left(\frac{\phi}{\phi_0}\right) + 1\right)$$

Real-Time Interpretation

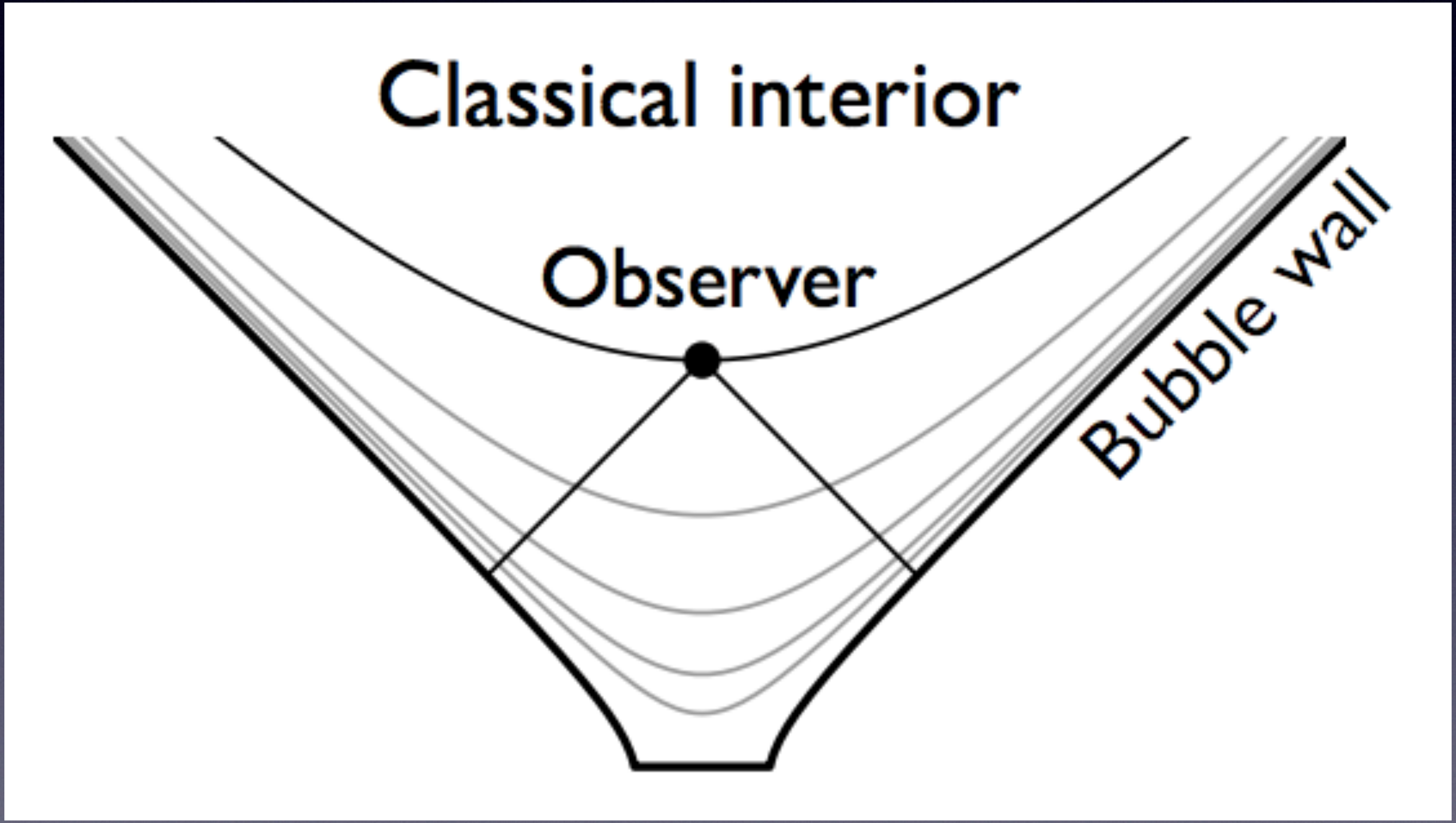
$$\phi(x,t) = \phi_{\rm I}(\sqrt{x^2 - t^2})$$



[Figure courtesy of Andrew Pontzen]

Ad-Hoc Nucleation

$$\phi(\mathbf{x}, t=0) = \phi_{\mathrm{I}}(|\mathbf{x}|)$$



No real-time classical description

[Figure courtesy of Andrew Pontzen]

Some Questions

Some Questions

- Time-dependent description of nucleation

Some Questions

- Time-dependent description of nucleation
  - Bubble precursor? Init. cond. at nucleation

Some Questions

- Time-dependent description of nucleation
  - Bubble precursor? Init. cond. at nucleation
- Bubble-bubble correlations

Some Questions

- Time-dependent description of nucleation
  - Bubble precursor? Init. cond. at nucleation
- Bubble-bubble correlations
- Fast decay/large fluctuation limit?

Some Questions

- Time-dependent description of nucleation
  - Bubble precursor? Init. cond. at nucleation
- Bubble-bubble correlations
- Fast decay/large fluctuation limit?
- Time evolving background/potential

Some Questions

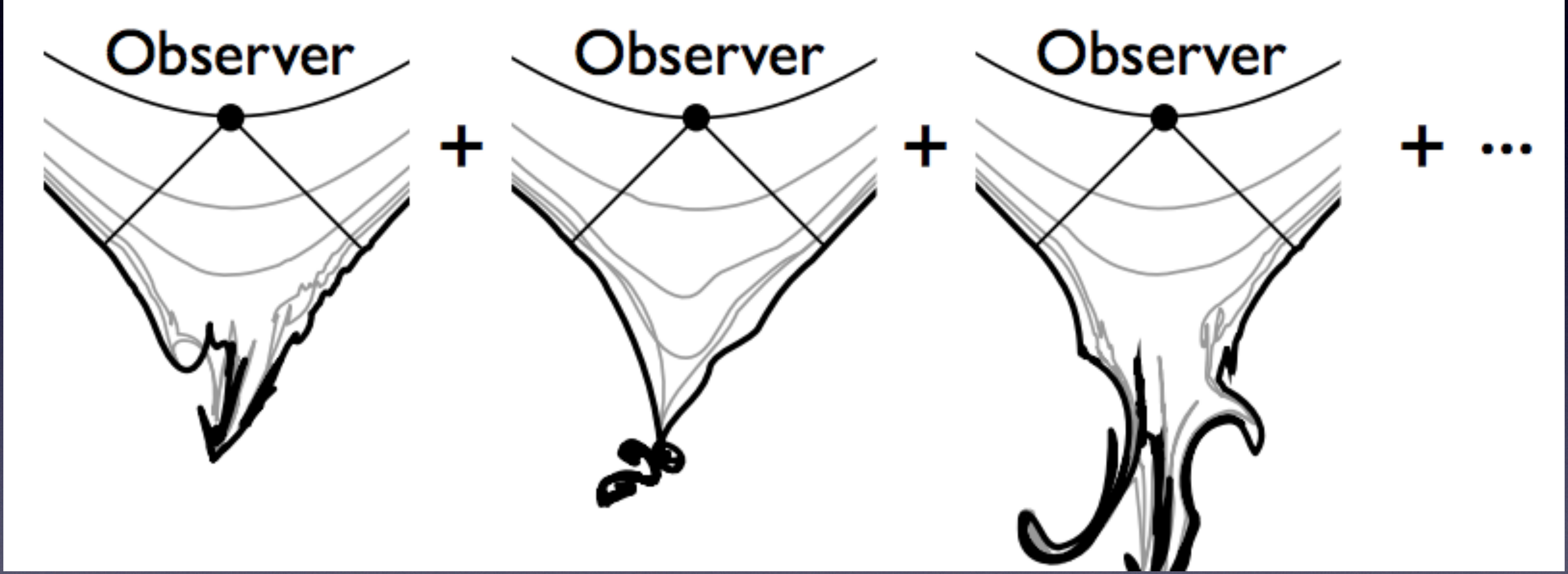
- Time-dependent description of nucleation
  - Bubble precursor? Init. cond. at nucleation
- Bubble-bubble correlations
- Fast decay/large fluctuation limit?
- Time evolving background/potential
- Nonvacuum state

Some Questions

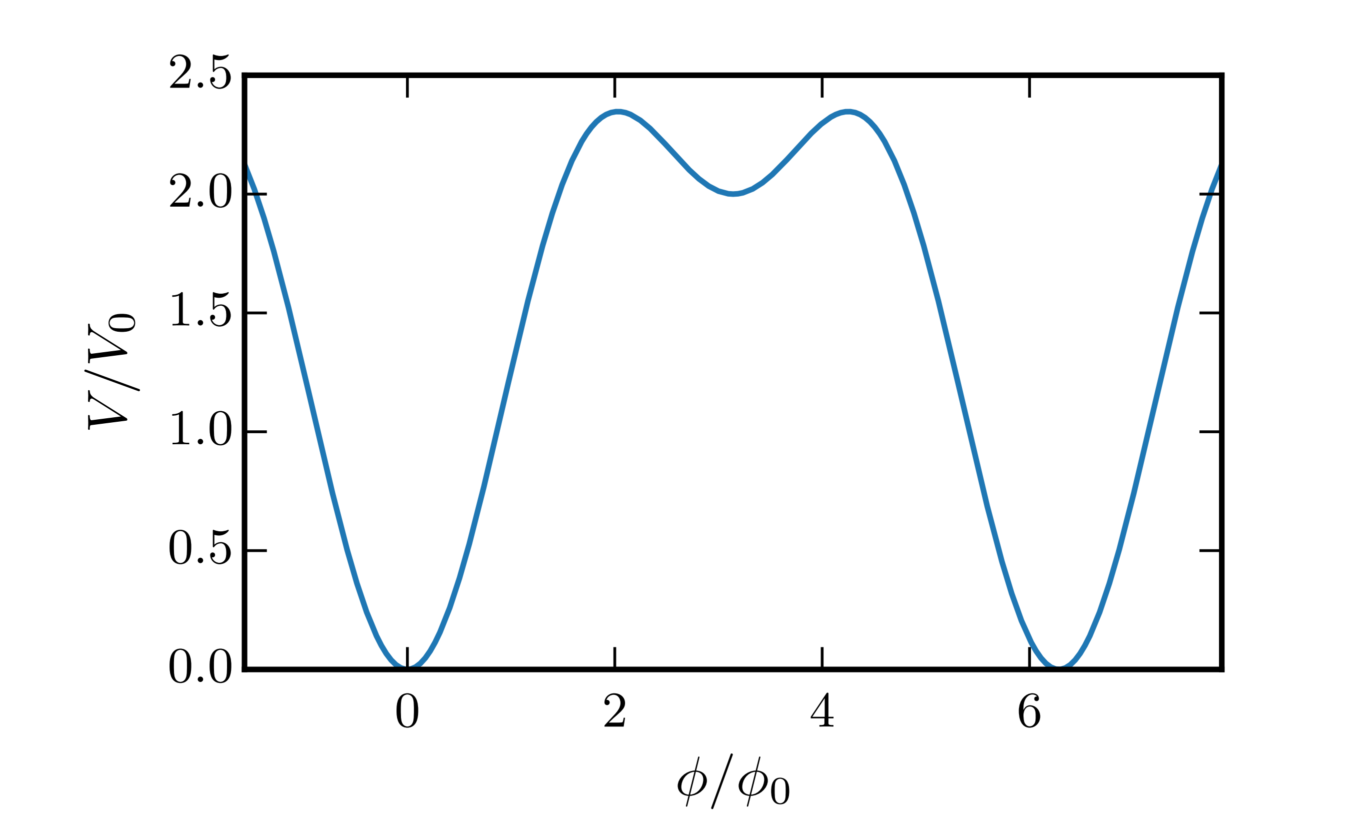
- Time-dependent description o… ion
  - Bubble precursor? Init… t nucleation
- Bubble-bubble corr…
- Fast decay/lar… ation limit?
- Time evol… kground/potential
- Nonvacu… state

QFT exponentially complex.
Need Approximations!

Full Evolution?

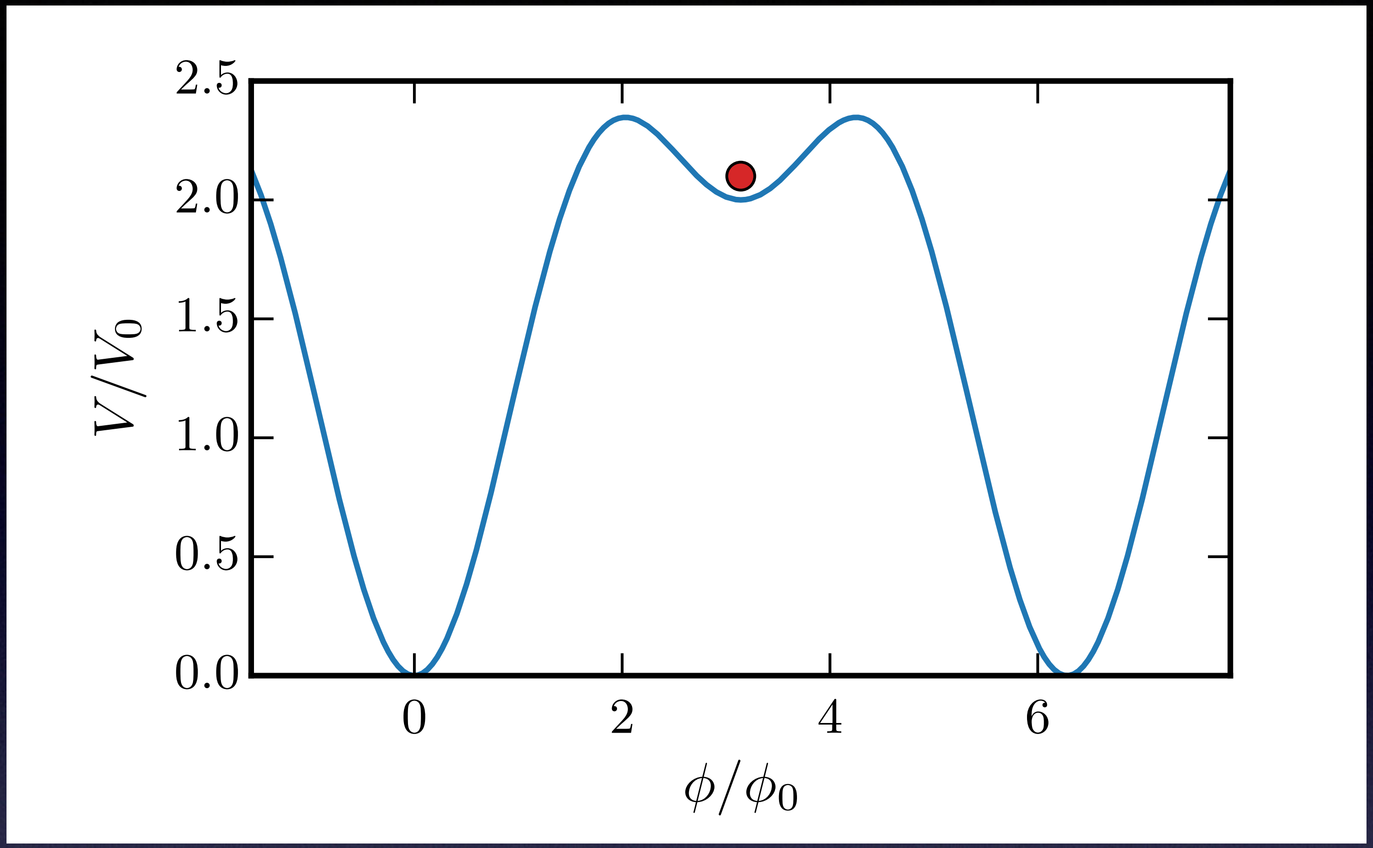


[Figure courtesy of Andrew Pontzen]



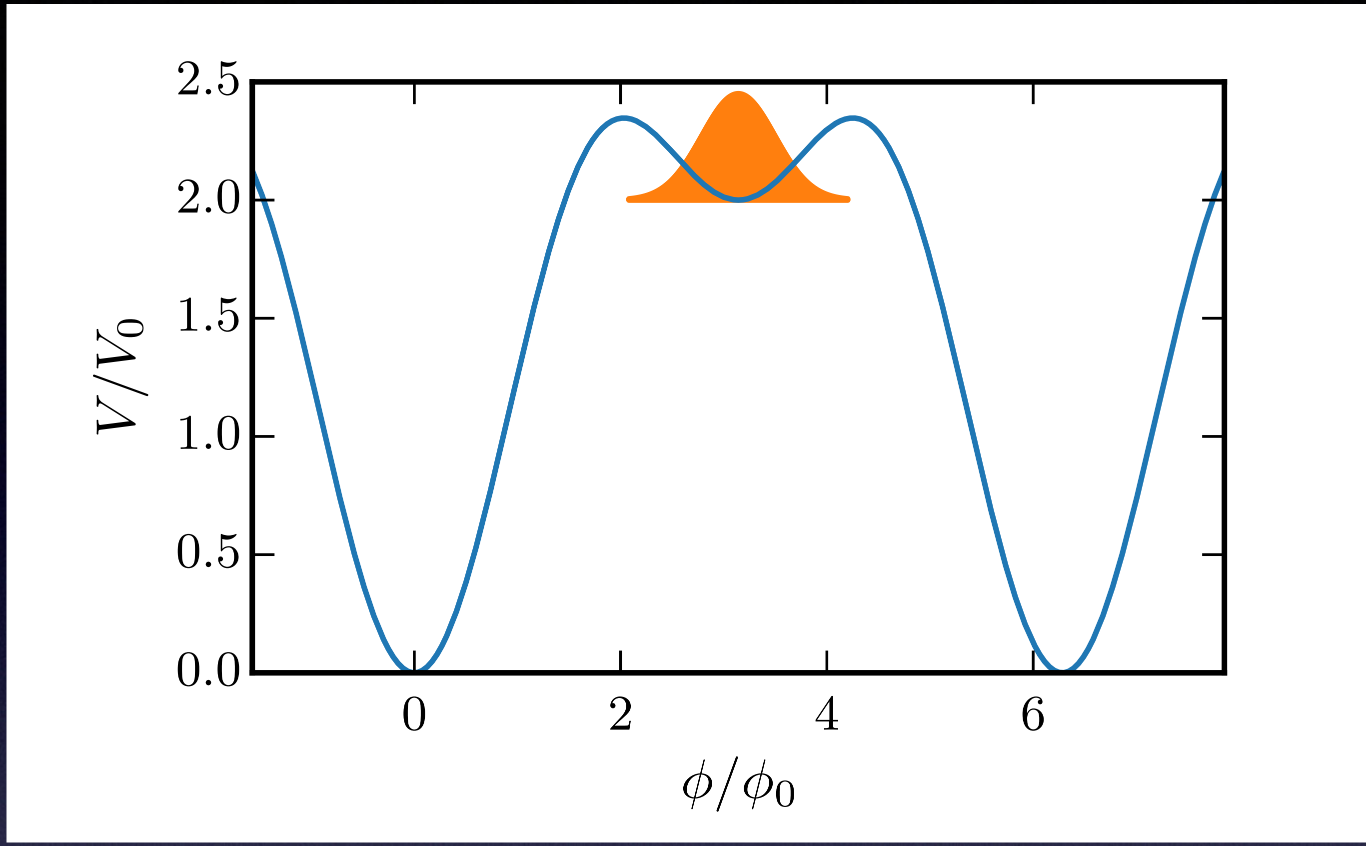
$$\phi =$$

$$\Pi =$$



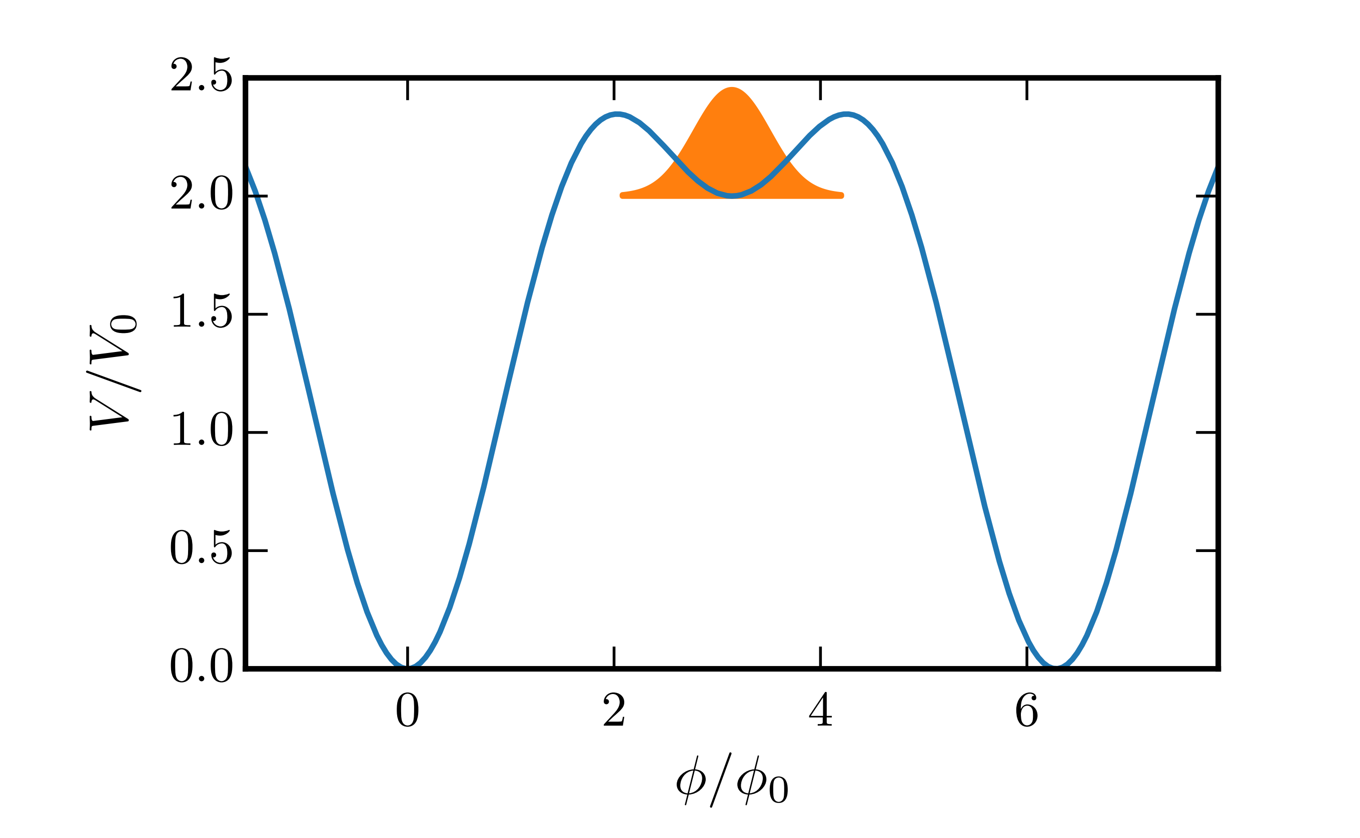
$$\phi = \phi_{\rm fv}$$

$$\Pi = 0$$



$$\phi = \phi_{\rm fv} + \delta\hat{\phi}(\mathbf{x}, t)$$

$$\Pi = 0 + \delta\hat{\Pi}(\mathbf{x}, t)$$

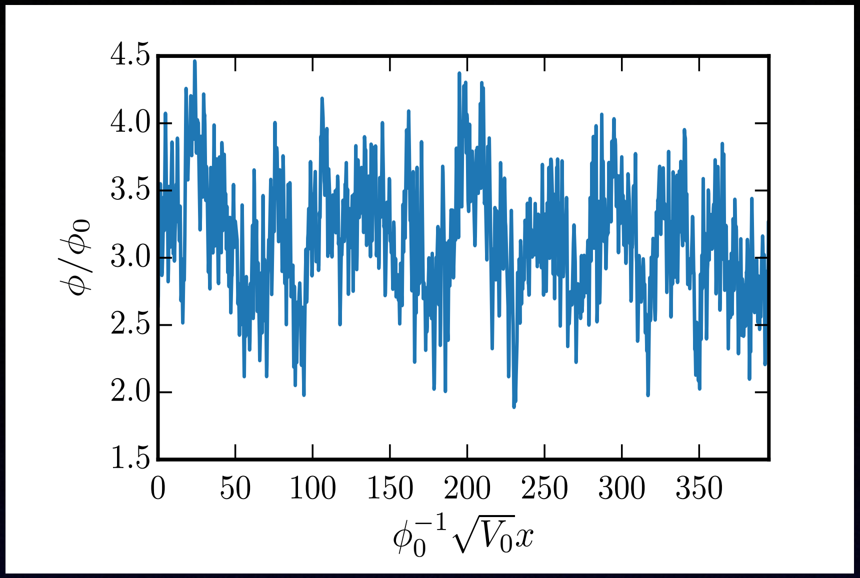


$$\phi = \phi_{\rm fv} + \delta\hat{\phi}(\mathbf{x}, t)$$

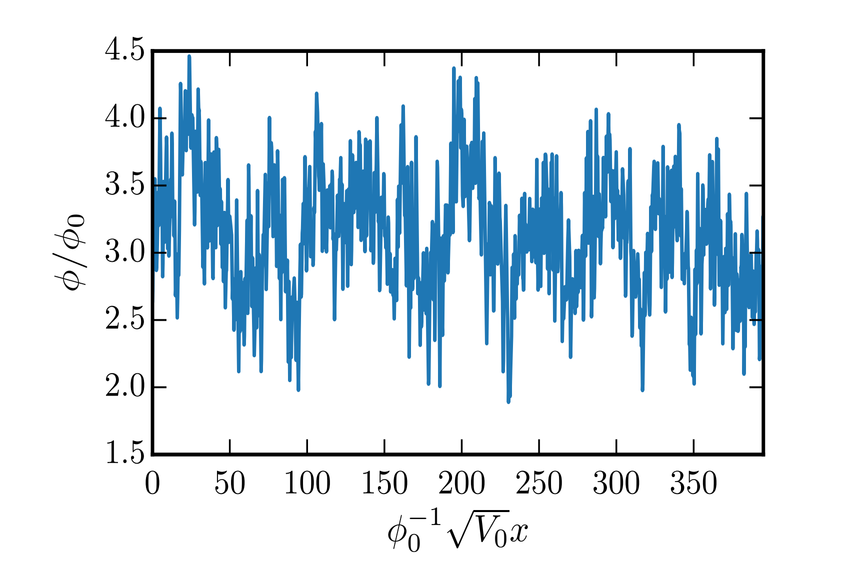
$$\Pi = 0 + \delta\hat{\Pi}(\mathbf{x}, t)$$

$$\langle \delta\tilde{\phi}_k \delta\tilde{\phi}_p^* \rangle = \frac{1}{2\omega_k}\delta(k-p) \qquad \langle \delta\tilde{\Pi}_k \delta\tilde{\Pi}_p^* \rangle = \frac{\omega_k}{2}\delta(k-p)$$

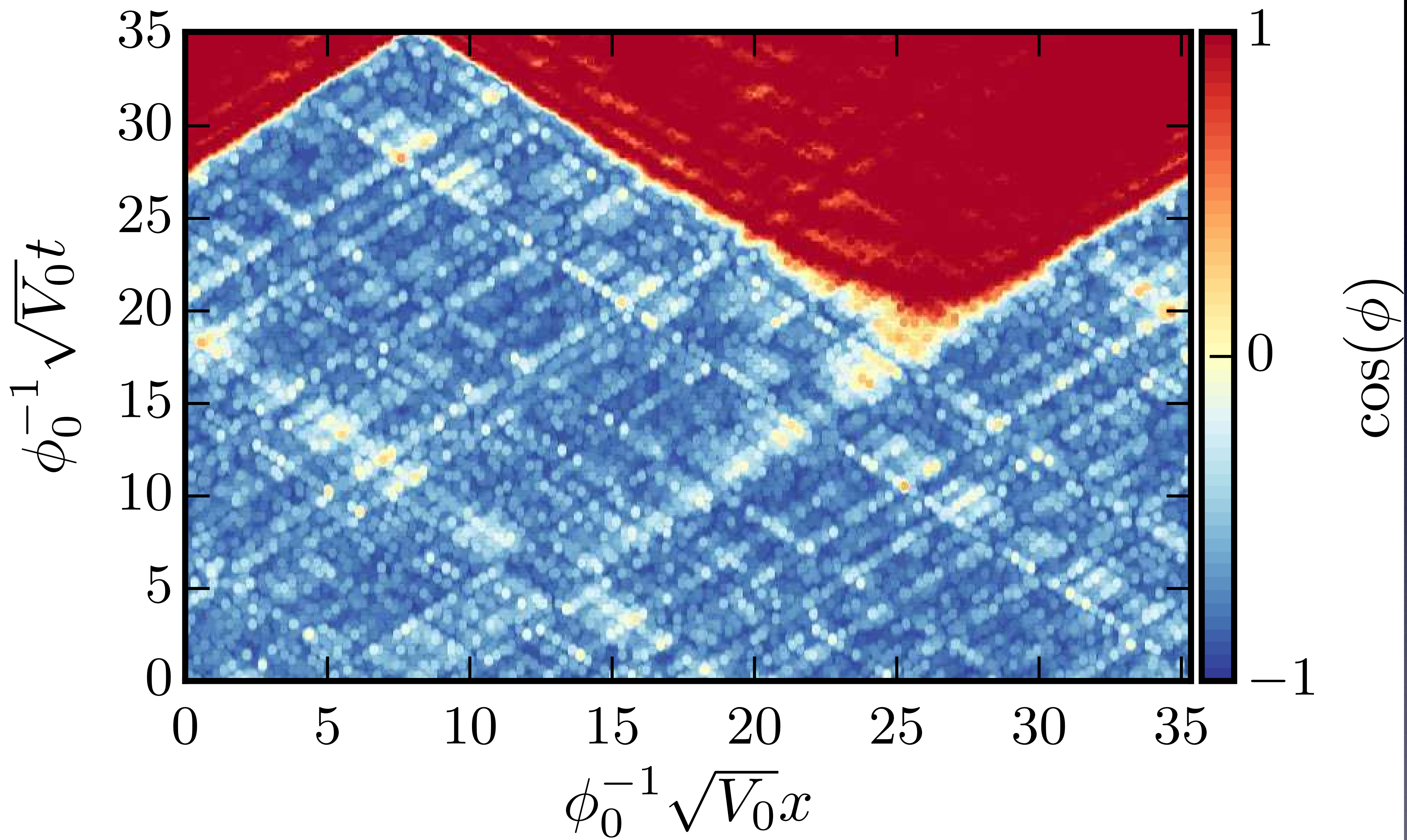
Quantum Commutators



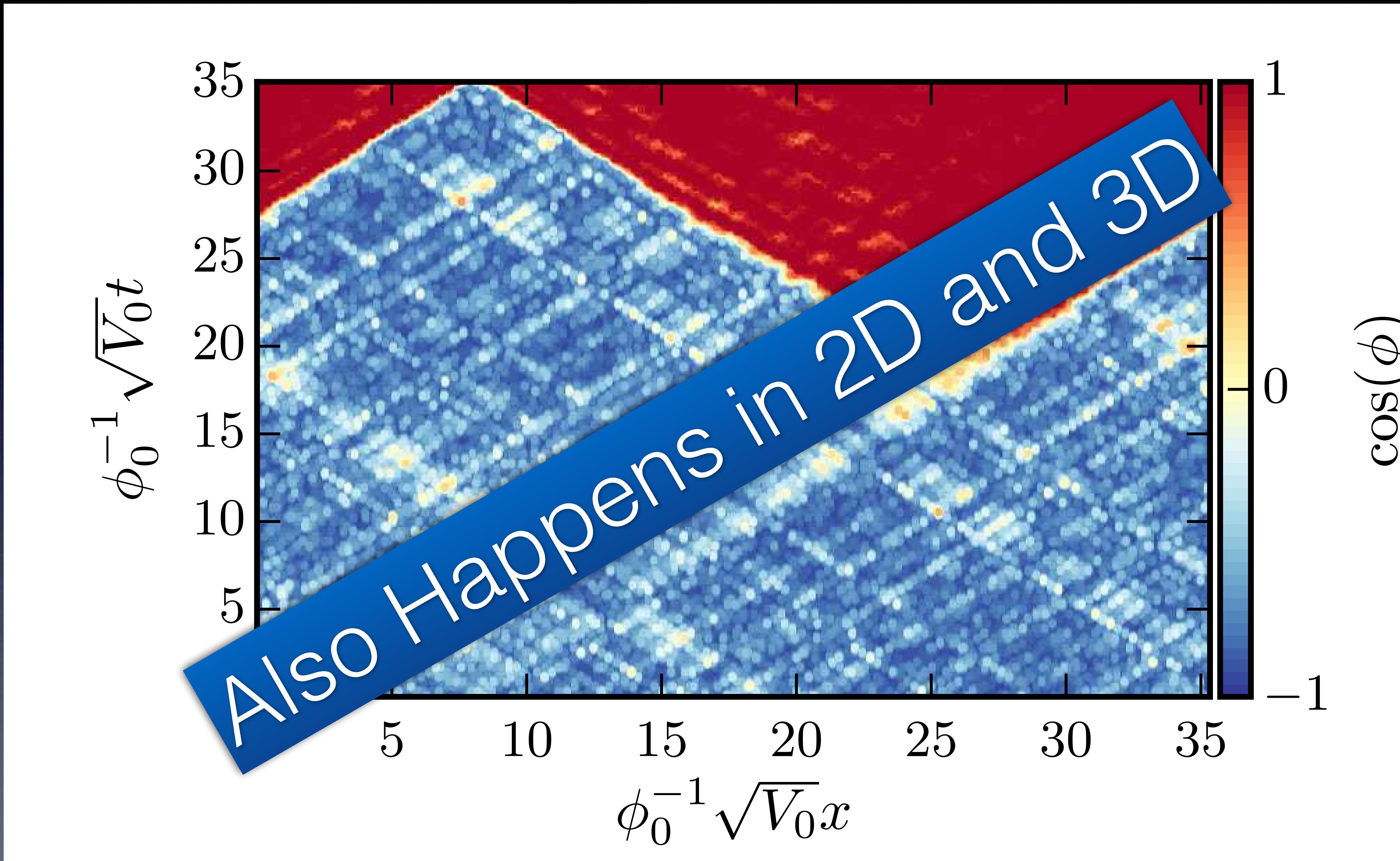
Quantum Commutators



$$\ddot{\phi} - \nabla^2 \phi + V'(\phi) = 0$$

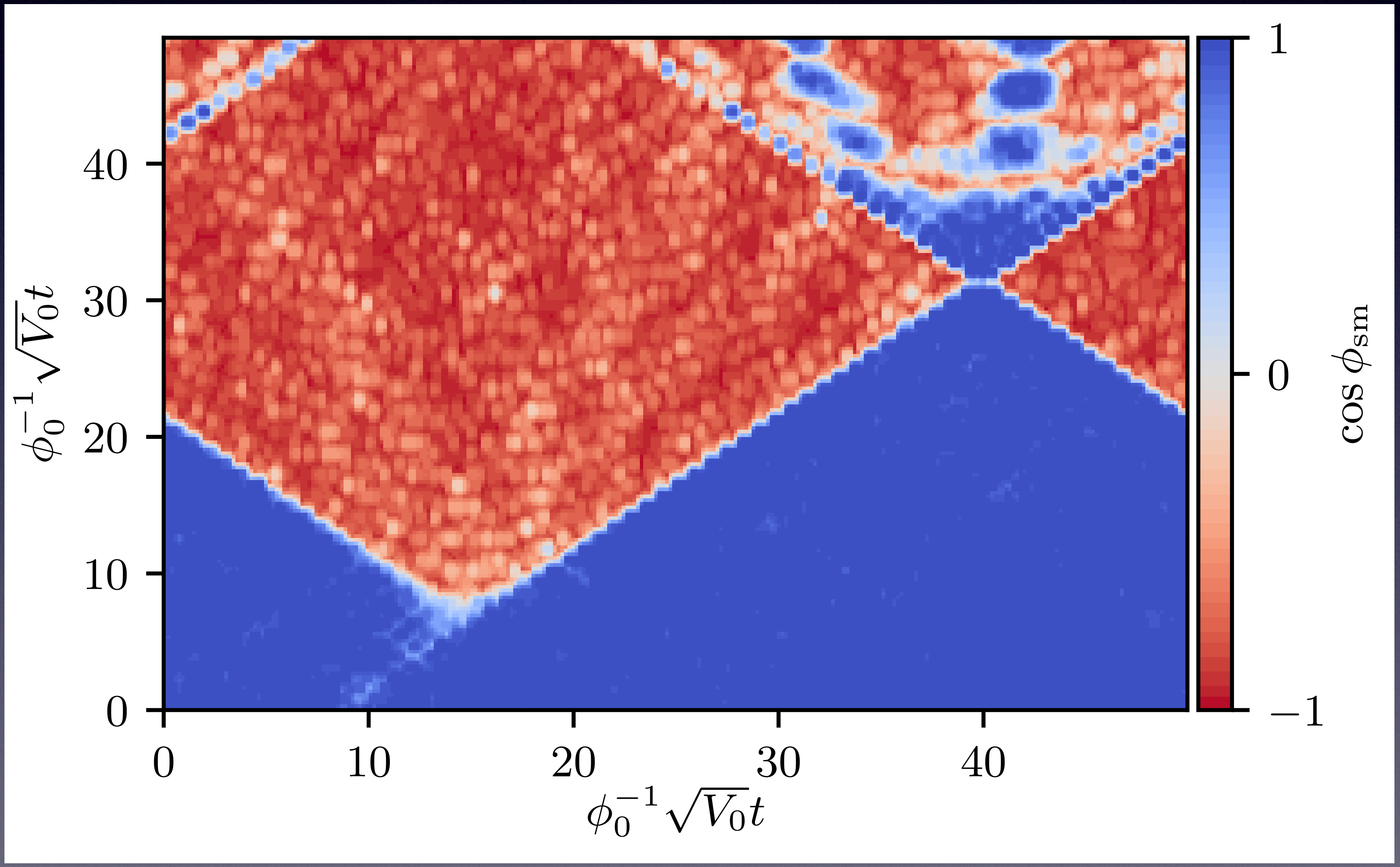


Classically-Allowed Vacuum Decay

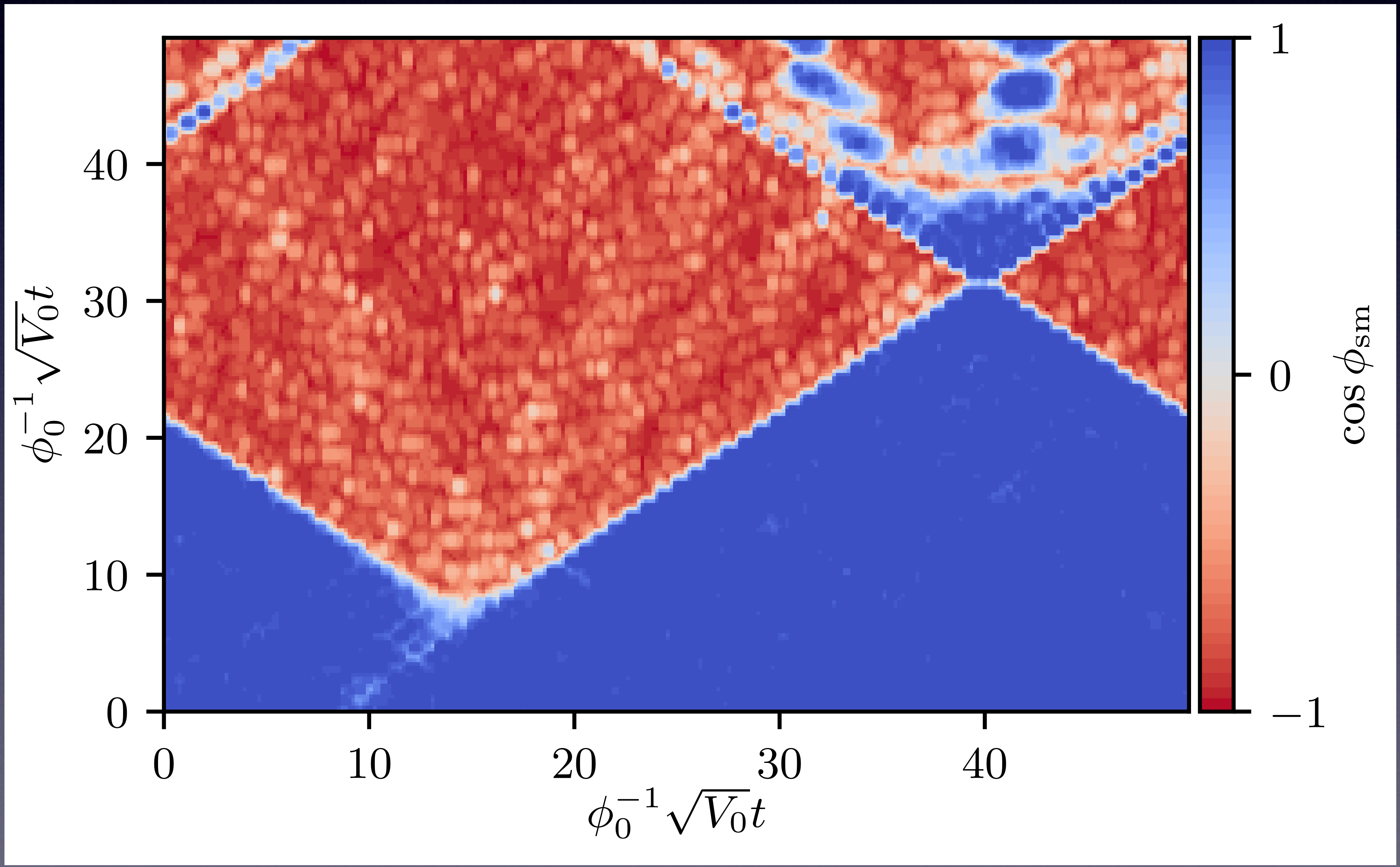


Classically-Allowed Vacuum Decay

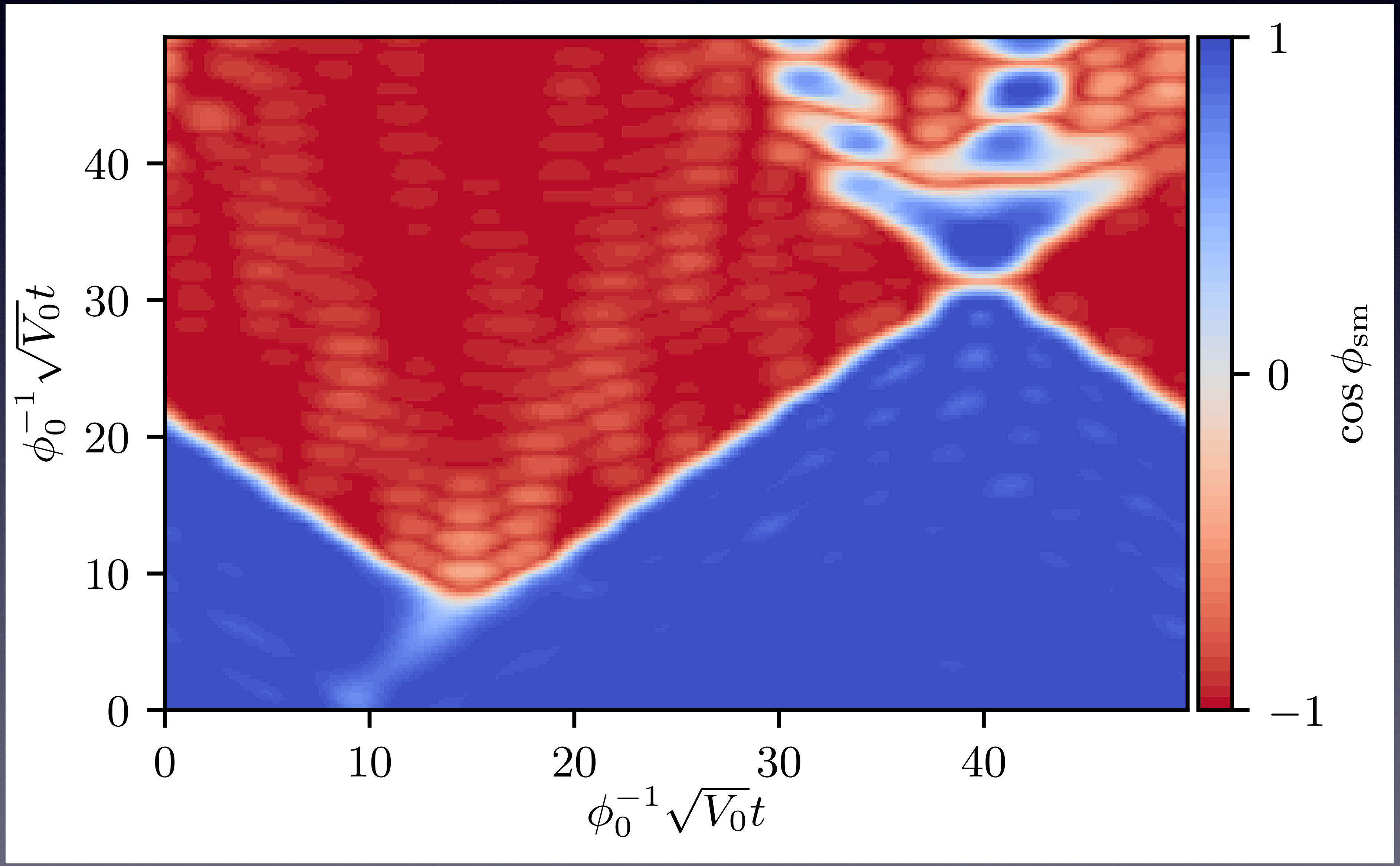
Energy of Bubble



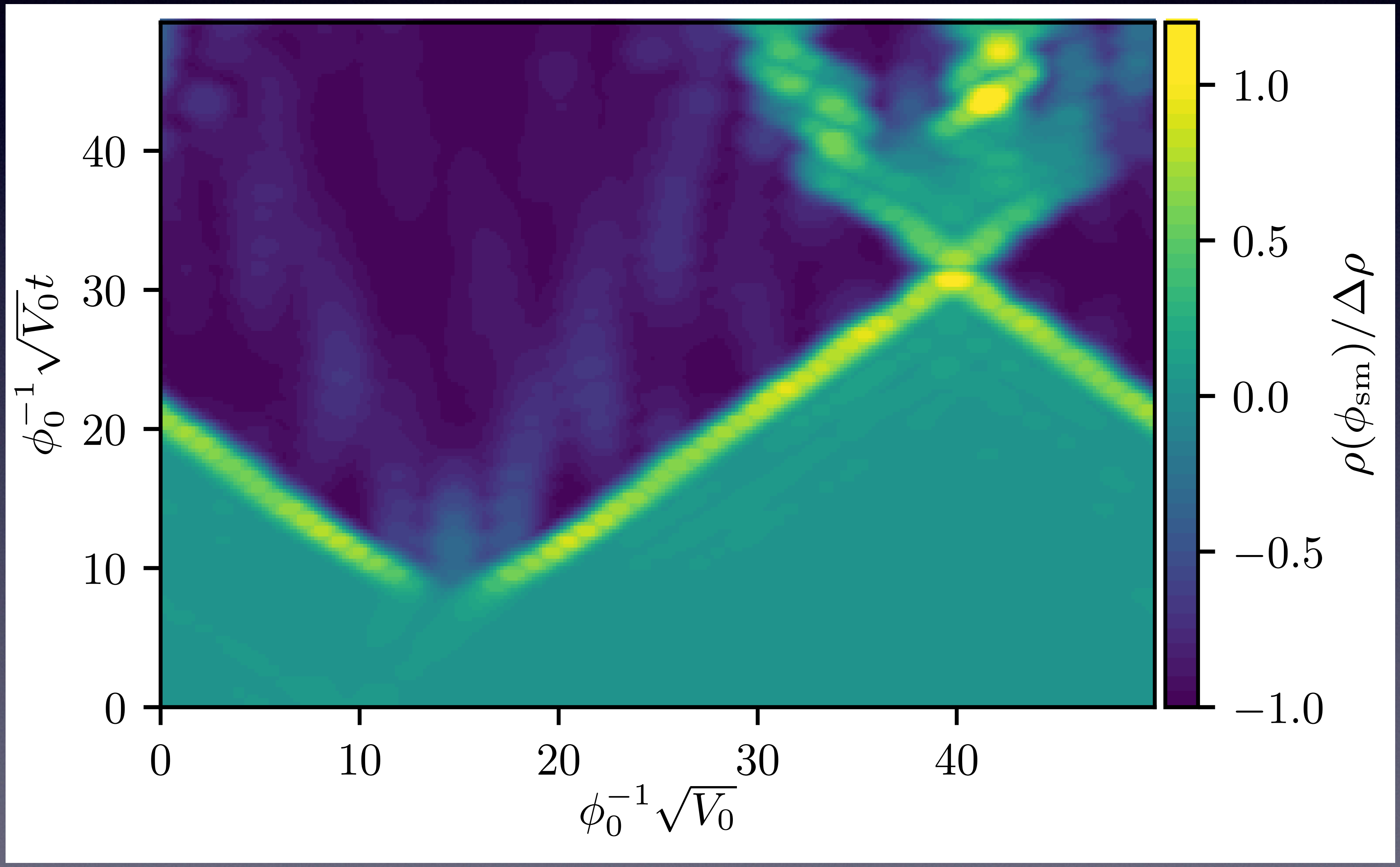
Restrict to long wavelength field modes instead



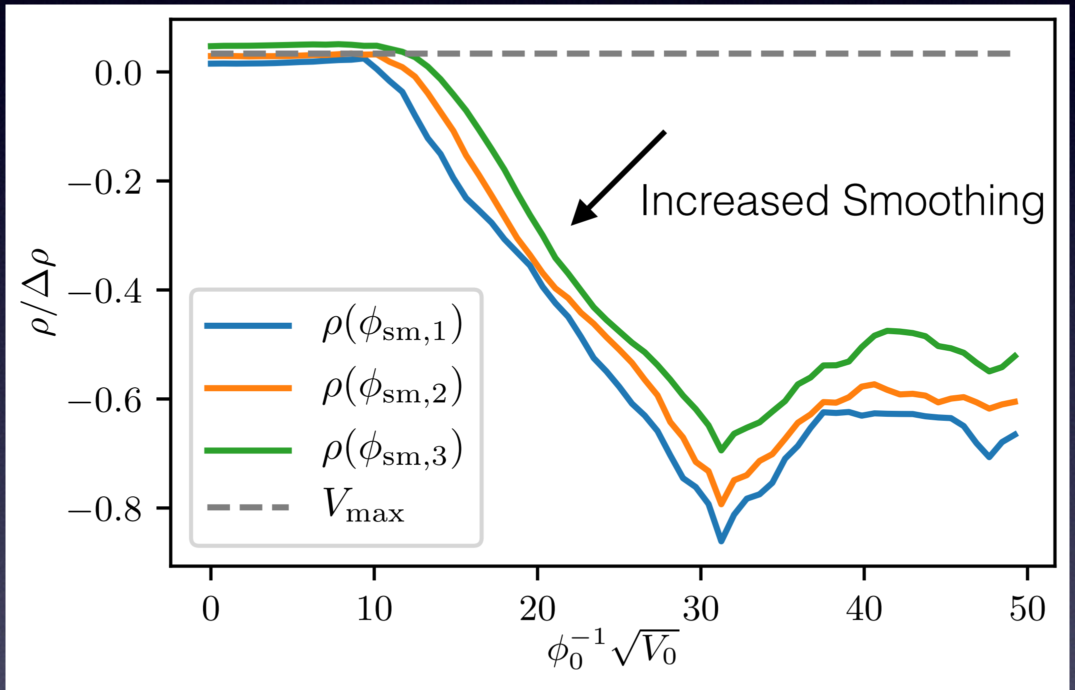
Restrict to long wavelength field modes instead



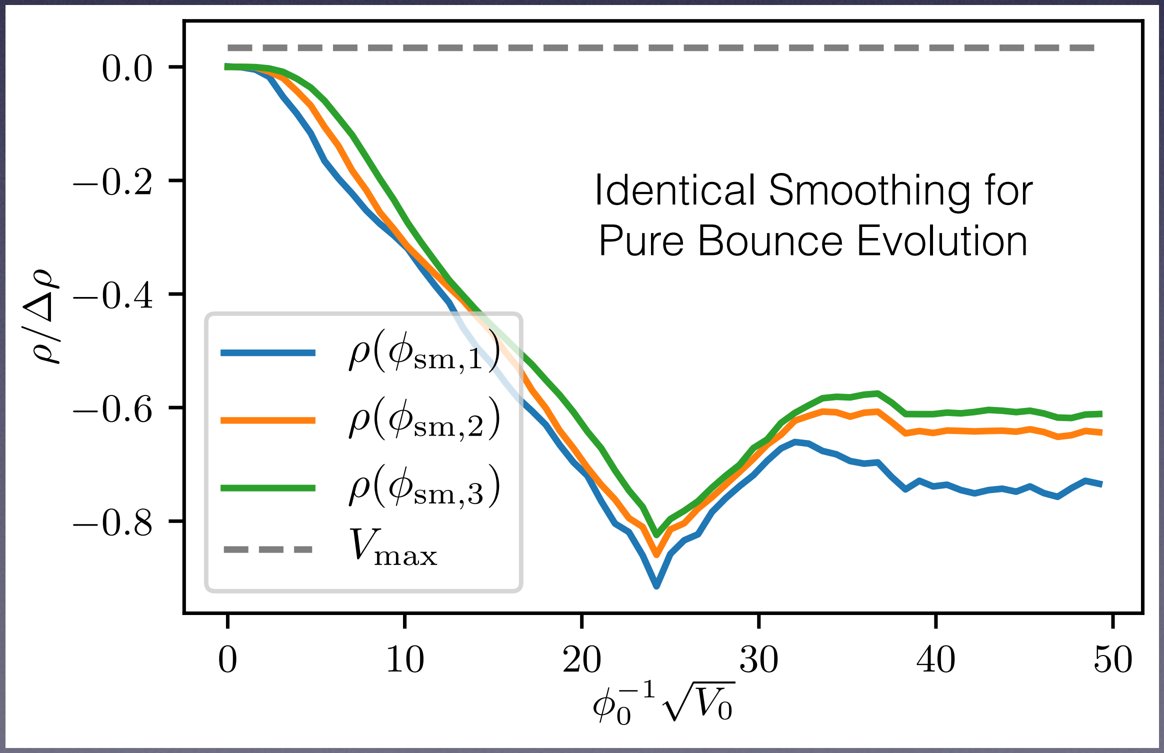
Energy of smoothed fields



Energy of long wave fields matches instanton



Compares well to bounce bubble initialized at t=0



Energy in long-wave modes as smoothing is varied

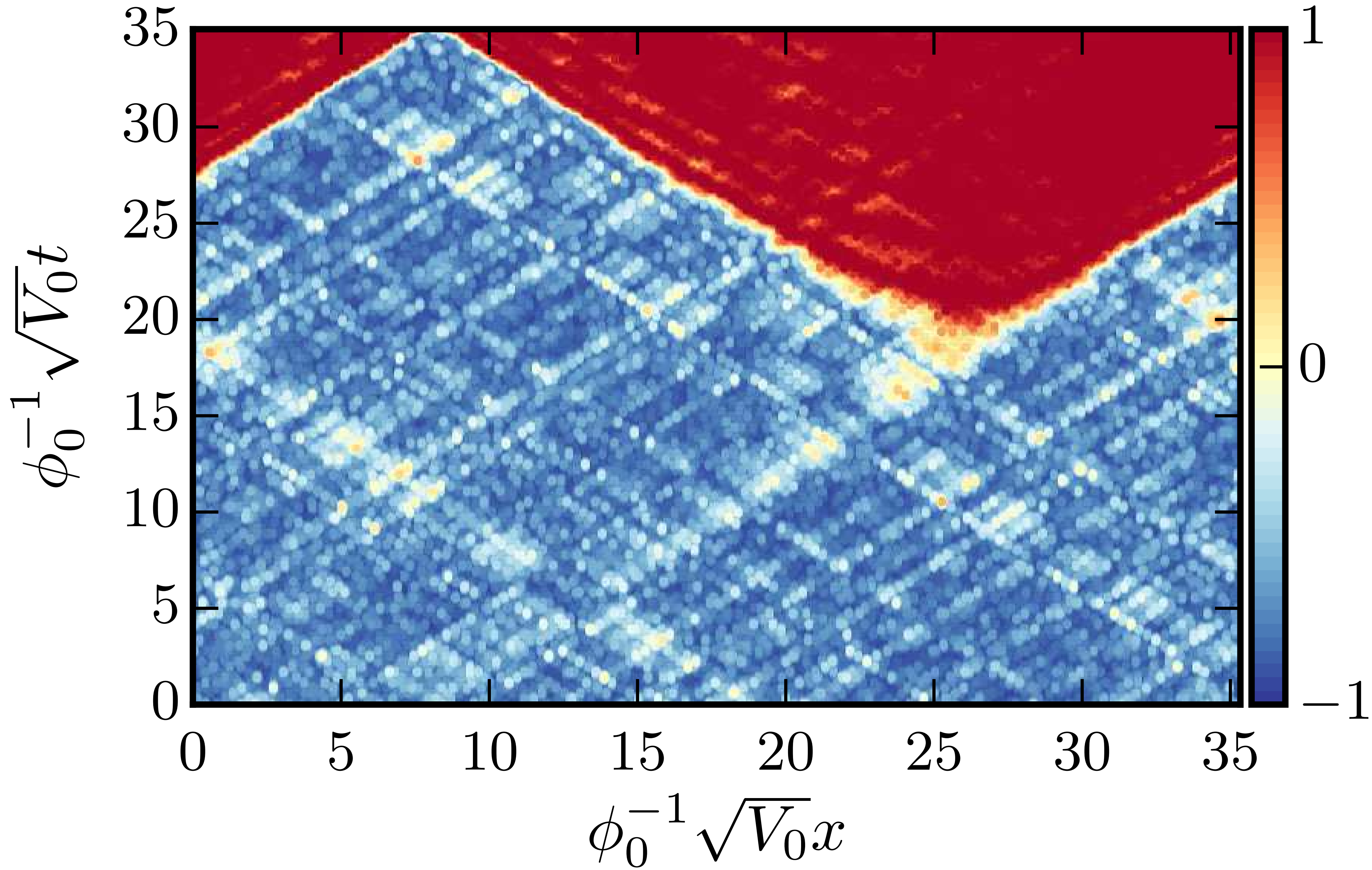
Decay Rates?

Prediction

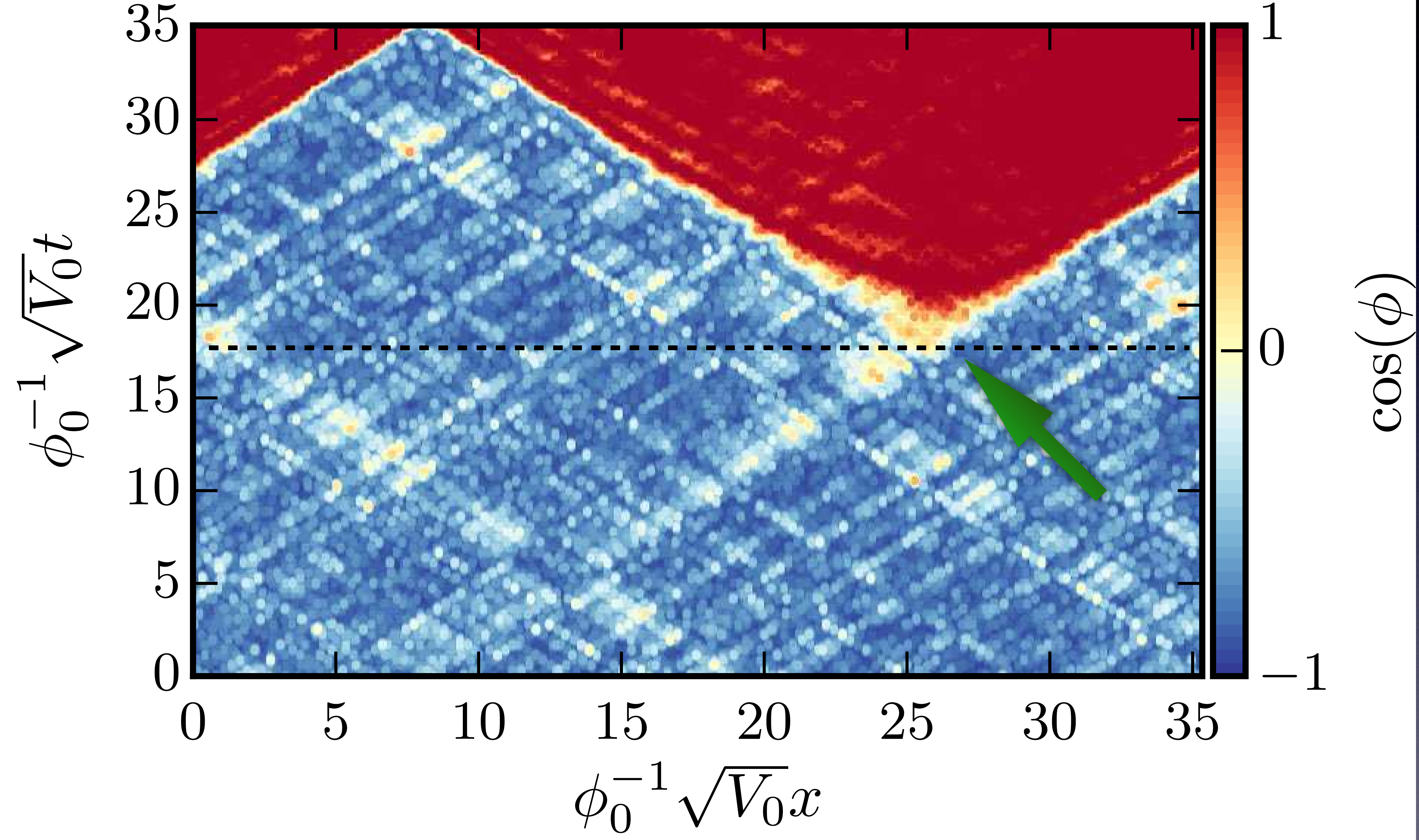
$$\frac{\Gamma_{\mathrm{I}}^{(1+1)}}{L} \approx g(\lambda, V_0, \phi_0) m_{\mathrm{eff}}^2 \phi_0^2 C(\lambda) e^{-2\pi\phi_0^2 C(\lambda)}$$

$\mathcal{O}(1)$ $\sim V''(\phi_{\mathrm{fv}})$ Instanton

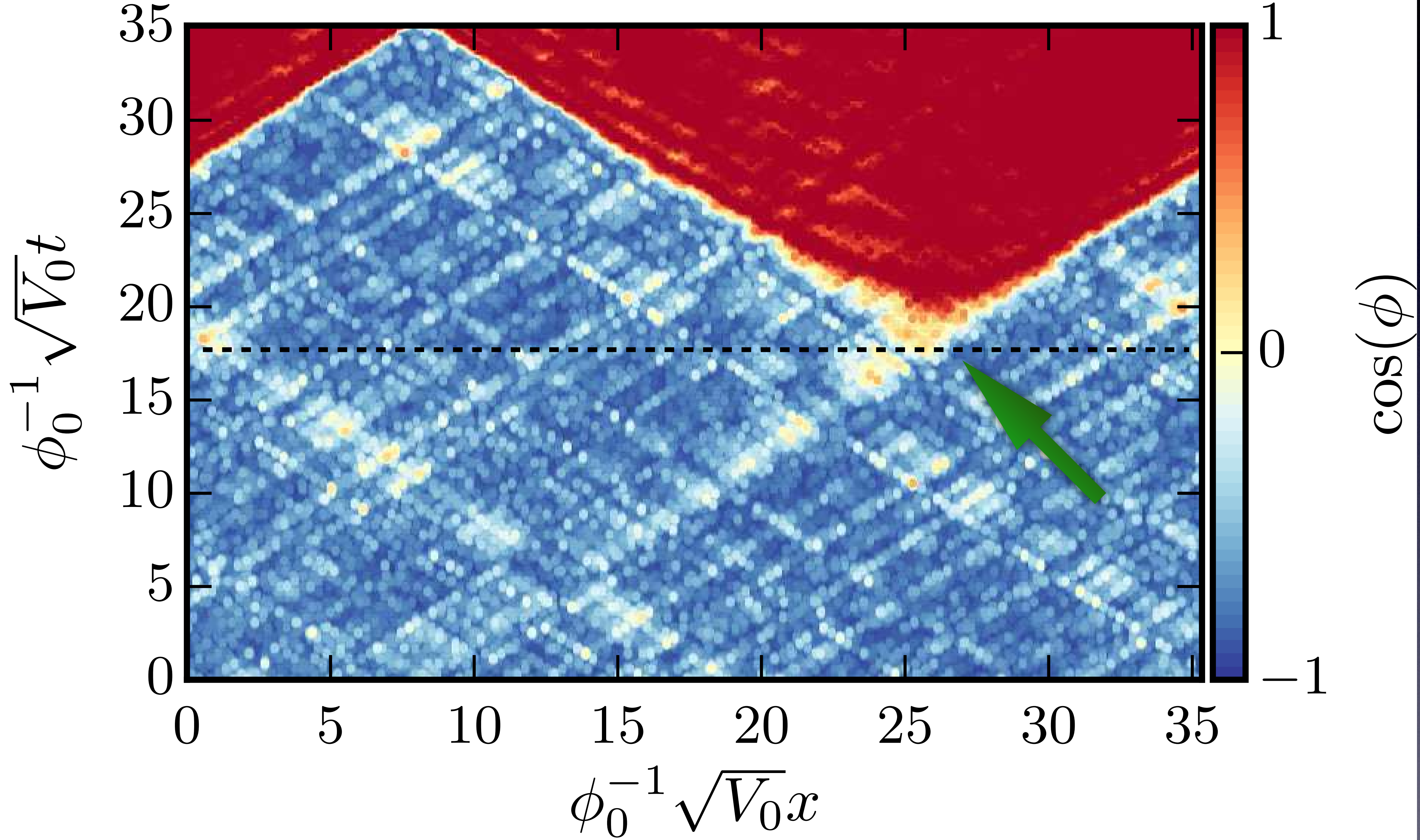
$$V(\phi) = V_0 \left(-\cos\left(\frac{\phi}{\phi_0}\right) + \frac{\lambda^2}{2} \sin^2\left(\frac{\phi}{\phi_0}\right) + 1 \right)$$



35
30
25
20
15
10
5
0
$\phi_0^{-1}\sqrt{V_0}t$
0
5
10
15
20
25
30
35
$\phi_0^{-1}\sqrt{V_0}x$
1
0
−1
$\cos(\phi)$

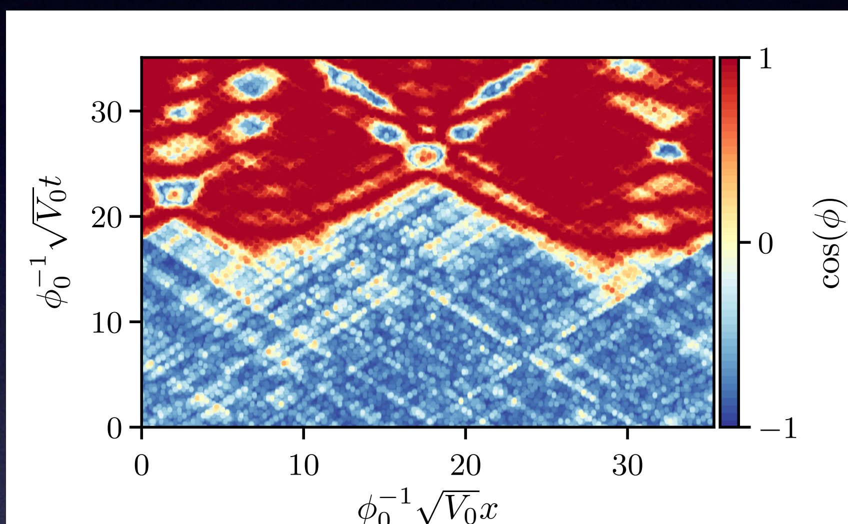


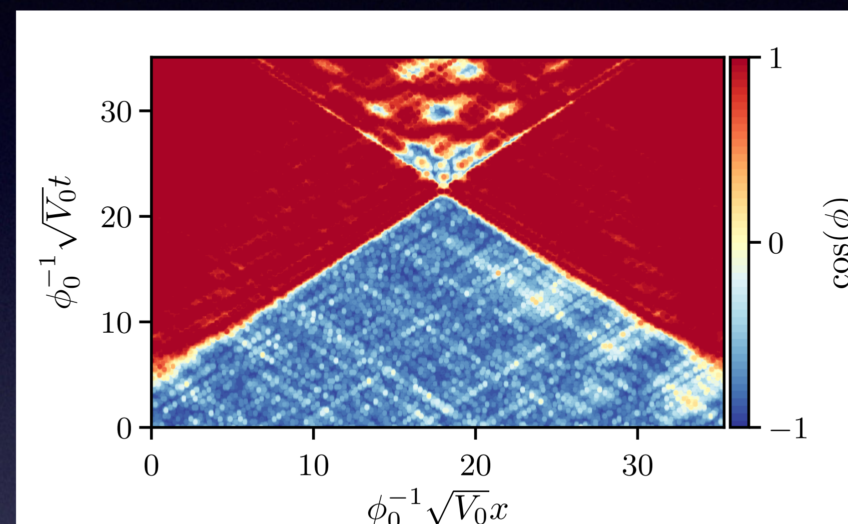
35
30
25
20
15
10
5
0
$\phi_0^{-1}\sqrt{V_0}t$
0
5
10
15
20
25
30
35
$\phi_0^{-1}\sqrt{V_0}x$
1
0
−1
$\cos(\phi)$

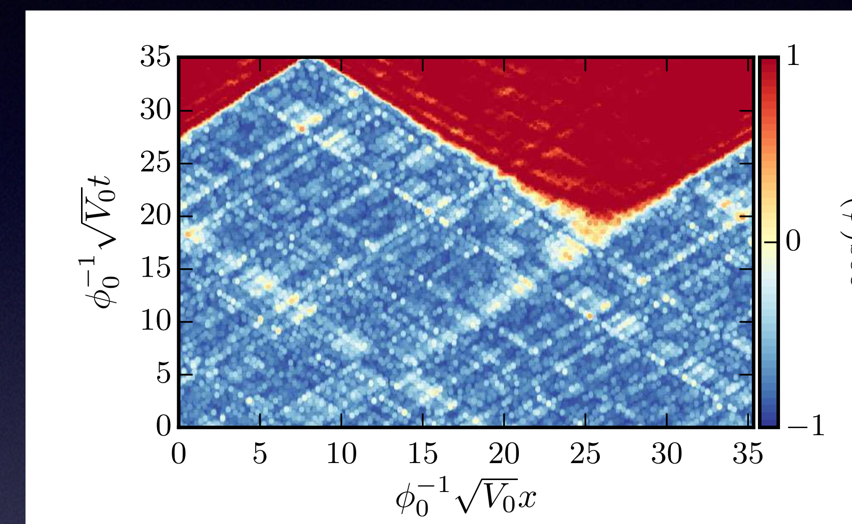


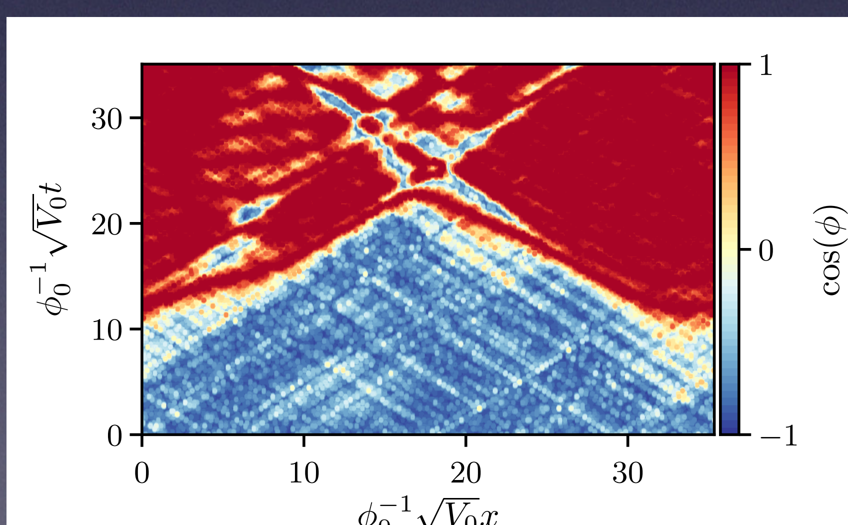
$t_{\text{decay}}^{(i)}$

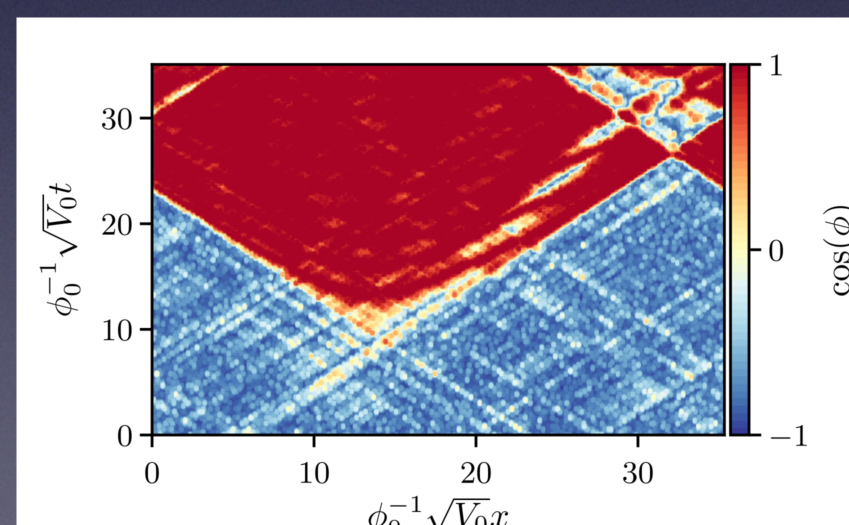
Monte Carlo Over ICs

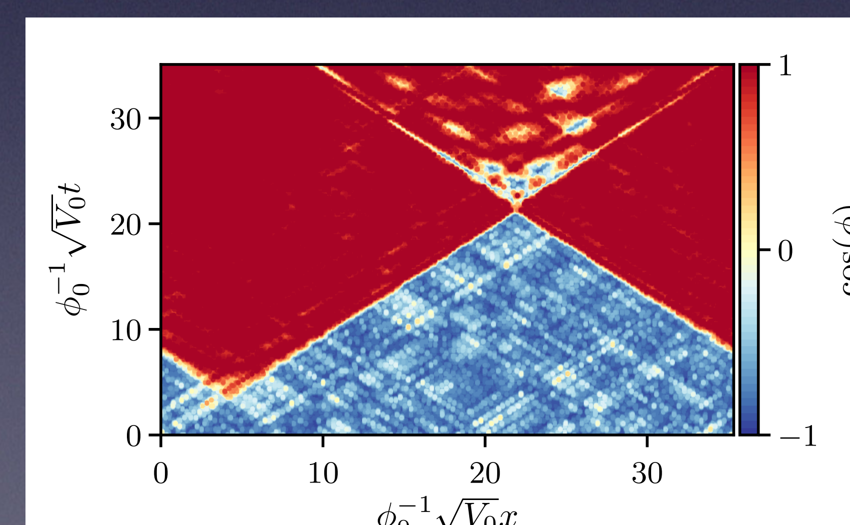






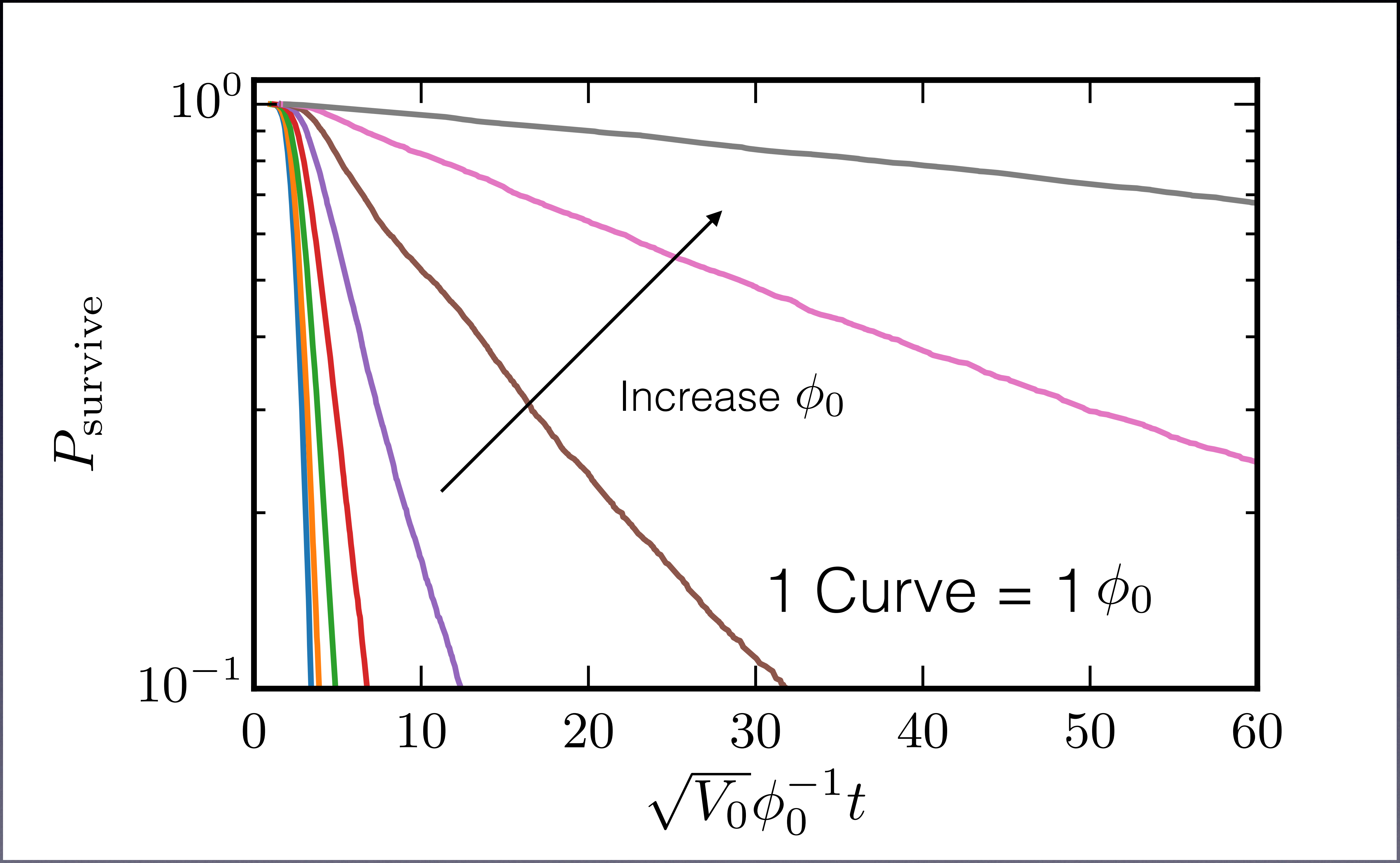






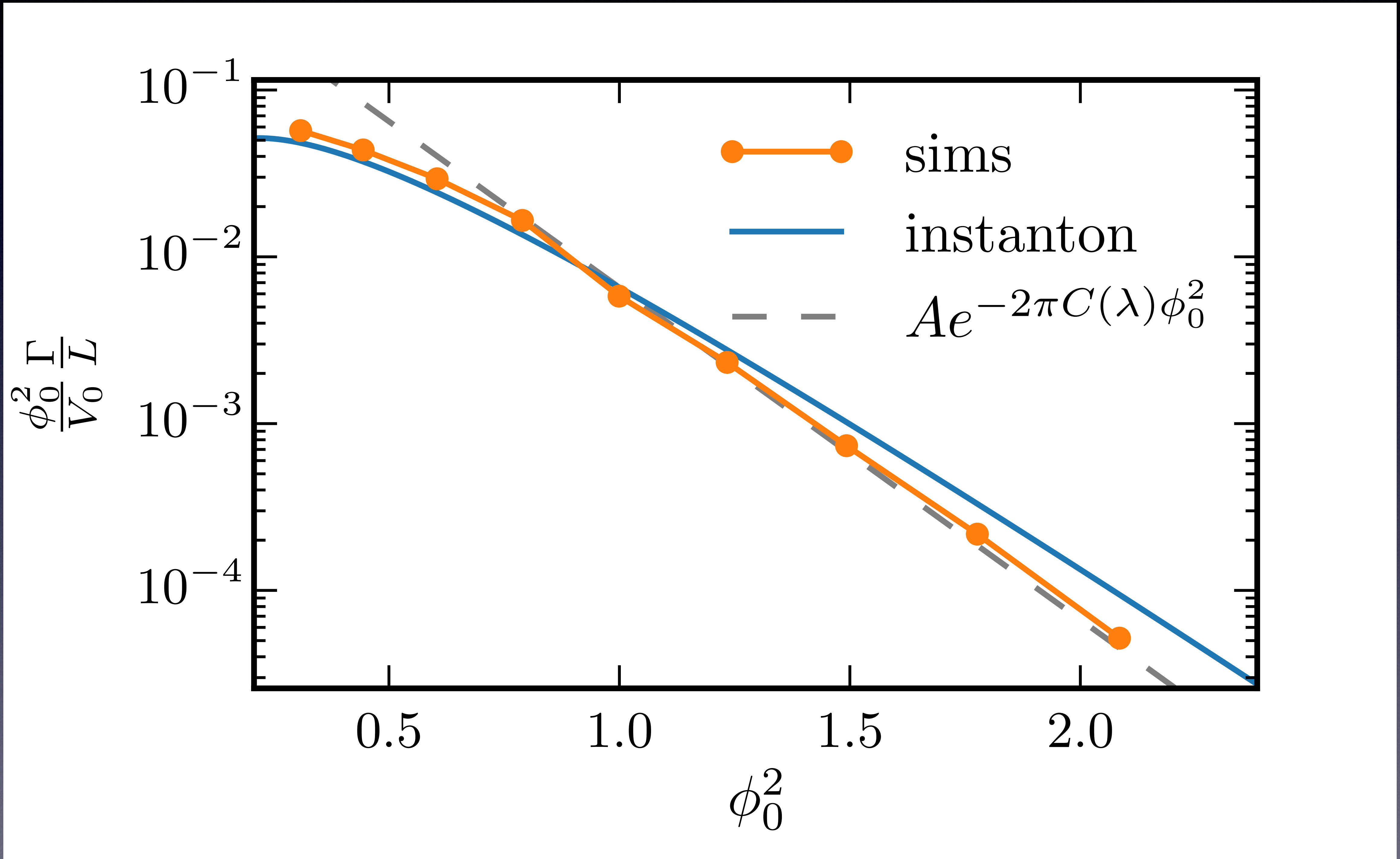
10000 sims / model x 1000 models ~ 10 million sims

$$P_{\text{survive}} \sim e^{-\Gamma(t-t_0)}$$



Sanity Check : $\Gamma \propto L$

$$\frac{\Gamma_{\rm I}^{(1+1)}}{L} = g(\lambda, \phi_0) m_{\rm eff}^2 \phi_0^2 e^{-2\pi\phi_0^2 C(\lambda)}$$

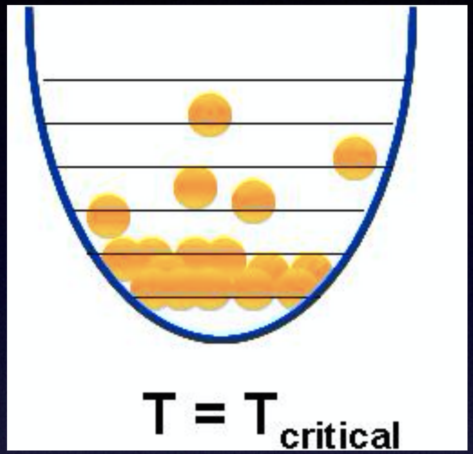


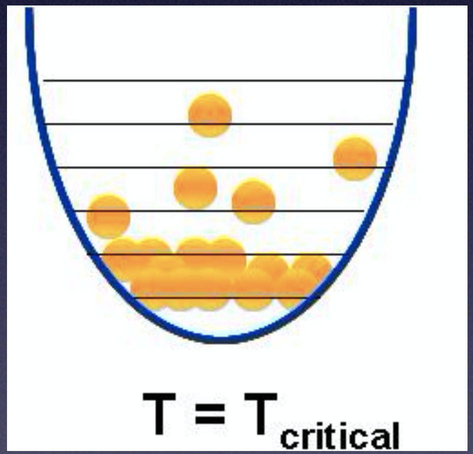
[JB, Johnson, Peiris, Pontzen, Weinfurtner, 1806.06069]

Can We Test This?

Analog Cold Atom BEC

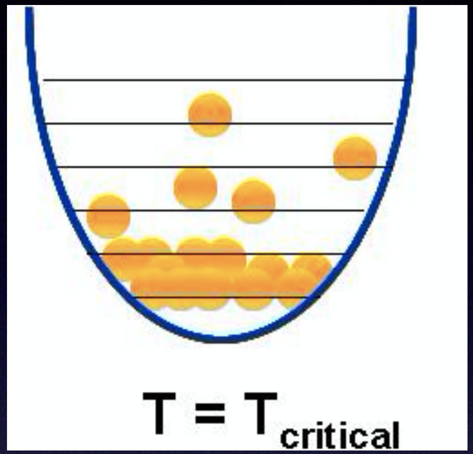
[JB, Johnson, Peiris, Weinfurtner, 1712.02356, 1904.07873
Fialko et al 1408.1163, 1607.01460]

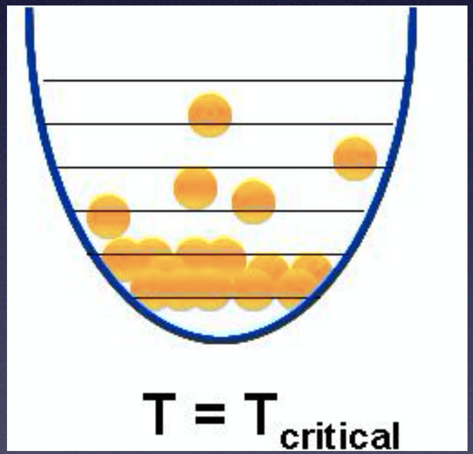


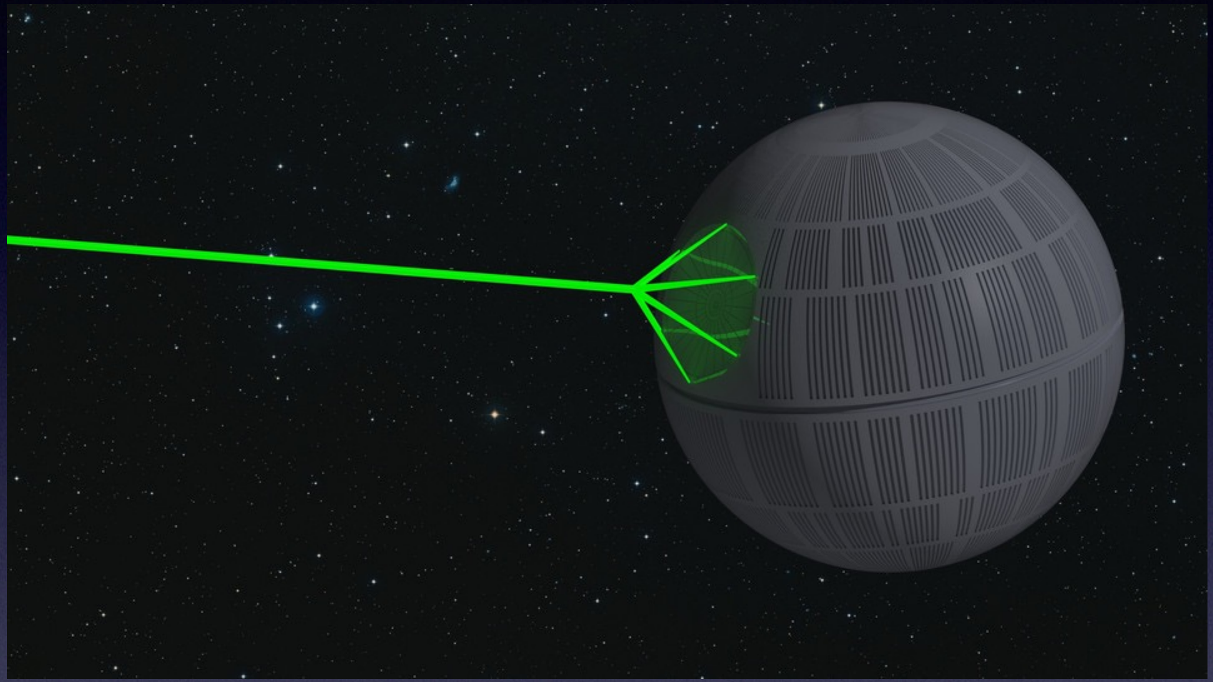


Analog Cold Atom BEC

[JB, Johnson, Peiris, Weinfurtner, 1712.02356, 1904.07873
Fialko et al 1408.1163, 1607.01460]

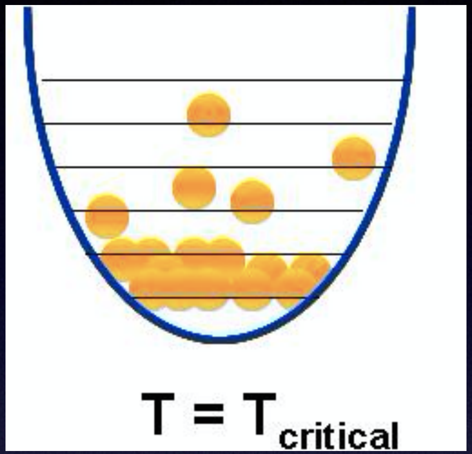


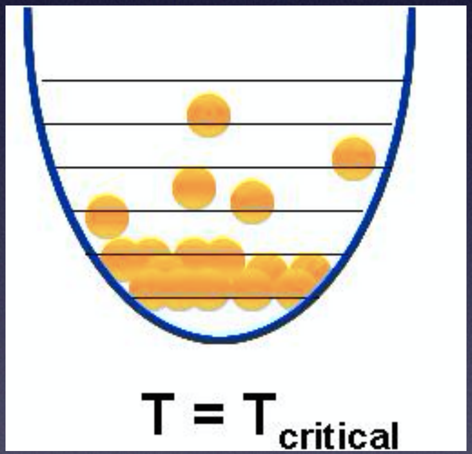


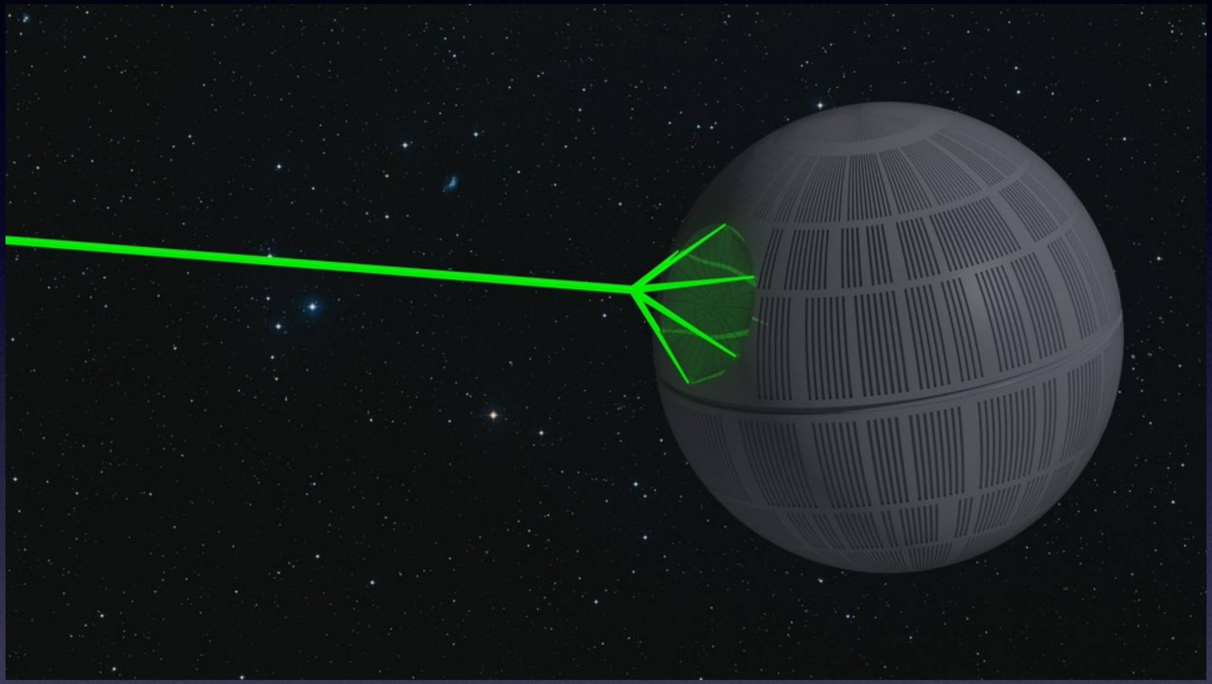


Analog Cold Atom BEC

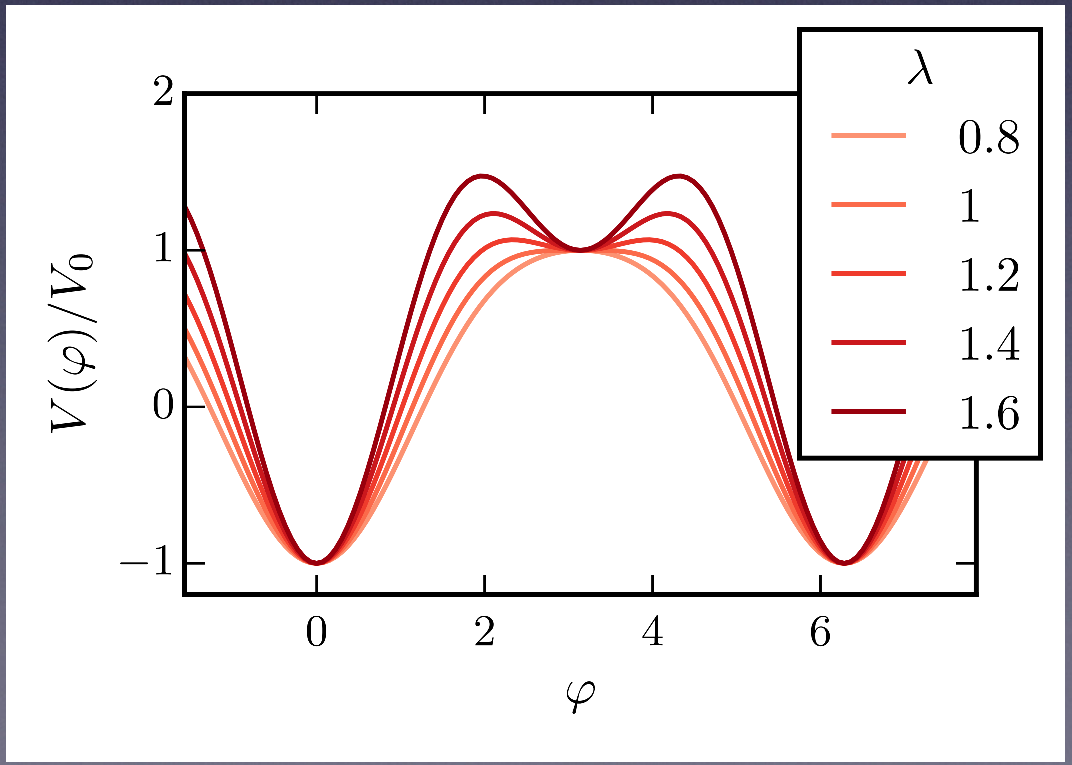
[JB, Johnson, Peiris, Weinfurtner, 1712.02356, 1904.07873
Fialko et al 1408.1163, 1607.01460]







Dynamics of relative phase
is a relativistic field
with periodic potential

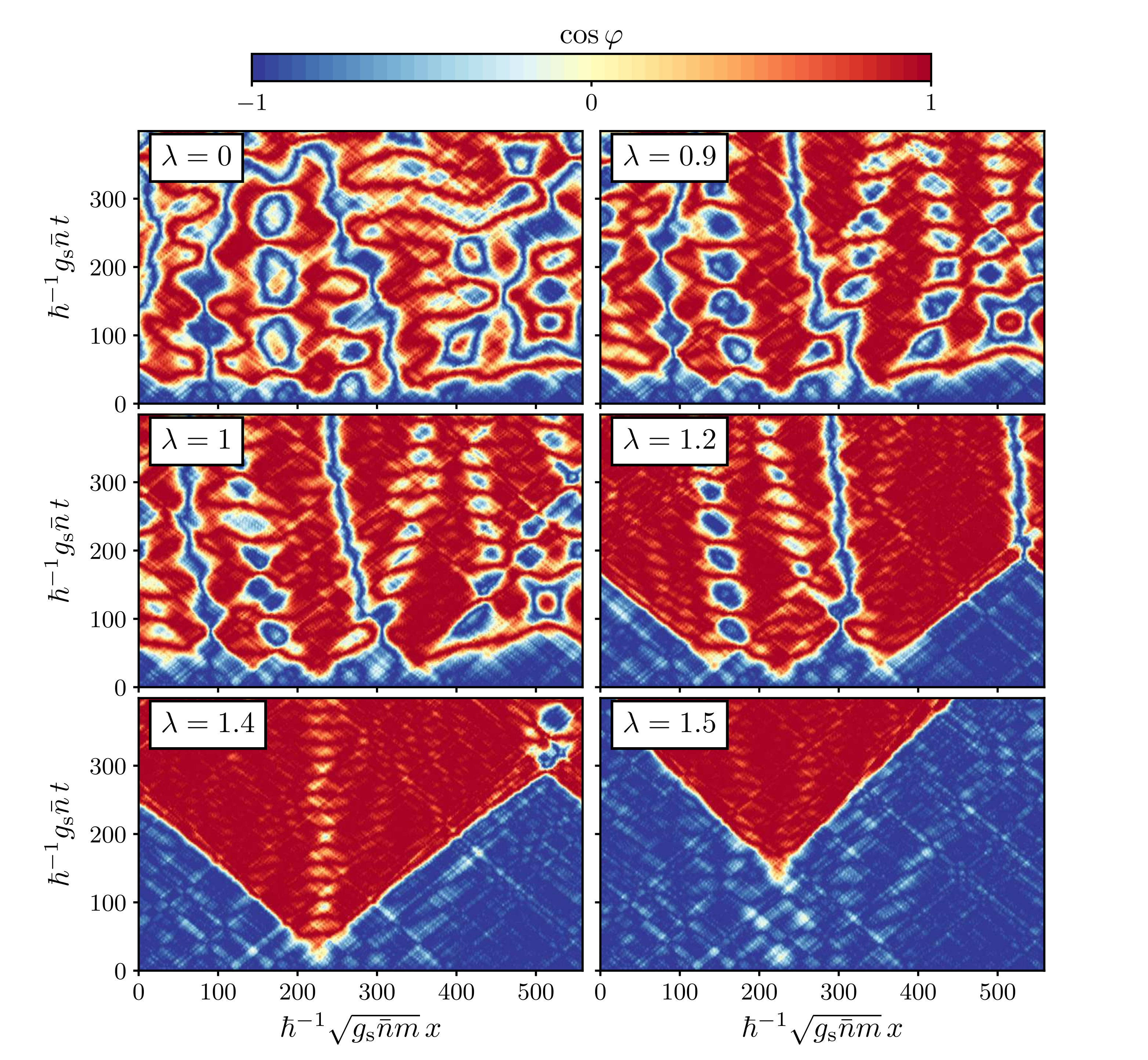


BEC Simulations

2nd-Order
Phase Transition

Rapid 1st-Order
Phase Transition

Slower 1st-Order
Phase Transition



Increase
Depth of
Minimum

QSimFP

Quantum Simulators for Fundamental Physics

Team

World-leading researchers in the following STFC and EPSRC areas:

Experimentalist Theorist Experimentalist/Theorist

Cosmology, Gravity and non-equilibrium Field Thoery

- Jonathan Braden (Canada, CITA)
- Hiranya Peiris (UK, UCL)
- Andrew Pontzen (UK, UCL)
- Mathew Johnson (Canada, Perimter Institute)
- Ian Moss (UK, Newcastle)
- Ruth Gregory (UK, Durham)
- Jorma Louko (UK, Nottingham)
- Ralf Schuetzhold (Germany, Helmholz Centre)
- Bill Unruh (Canada, Vancouver)
- Silke Weinfurtner (UK, Nottingham)

Ultracold Atoms

- Thomas Billam (UK, Newcastel)
- Zoran Hadzibabic (UK, Cambridge)
- Joerg Schmiedmayer (Austria, Vienna)

Superfluid 4He

- Carlo Barenghi (UK, Newcastle)
- John Owers-Bradley (UK, Nottingham)

Superfluid Nanofabrication

- Gregoire Ithier (UK, Royal Holloway London)
- Xavier Rojas (UK, Royal Holloway London)

Opto-mechanics

- Pierre Verlot (UK, Nottingham University)

Qunatum Optics

- Friedrich Koenig (UK, St. Andrews University)

More info at
www.qsimfp.org

Conclusions

Physical Process: False Vacuum Decay

Conclusions

Physical Process: False Vacuum Decay

- False Vacuum decay **can** occur via classical time-evolution (quantum is in initial state)

Conclusions

Physical Process: False Vacuum Decay

- False Vacuum decay **can** occur via classical time-evolution (quantum is in initial state)

- Decay rates ~ Euclidean Calculations

Conclusions

Physical Process: False Vacuum Decay

- False Vacuum decay **can** occur via classical time-evolution (quantum is in initial state)
- Decay rates ~ Euclidean Calculations
    - Alternative description of instanton (no tunnelling)

Conclusions

Physical Process: False Vacuum Decay

- False Vacuum decay **can** occur via classical time-evolution (quantum is in initial state)
- Decay rates ~ Euclidean Calculations
  - Alternative description of instanton (no tunnelling)
  - Complimentary to instanton (Euclidean rate wrong)

Conclusions

Physical Process: False Vacuum Decay

- False Vacuum decay **can** occur via classical time-evolution (quantum is in initial state)
- Decay rates ~ Euclidean Calculations
  - Alternative description of instanton (no tunnelling)
  - Complimentary to instanton (Euclidean rate wrong)
  - (Magic cancellation of amplitudes)

Conclusions

Physical Process: False Vacuum Decay

- False Vacuum decay **can** occur via classical time-evolution (quantum is in initial state)
- Decay rates ~ Euclidean Calculations
  - Alternative description of instanton (no tunnelling)
  - Complimentary to instanton (Euclidean rate wrong)
  - (Magic cancellation of amplitudes)
- Exciting possibility to test in the lab

Conclusions

Physical Process: False Vacuum Decay

- False Vacuum decay **can** occur via classical time-evolution (quantum is in initial state)
- Decay rates ~ Euclidean Calculations
  - **Alternative description of instanton (no tunnelling)**
  - Complimentary to instanton (Euclidean rate wrong)
  - (Magic cancellation of amplitudes)
- Exciting possibility to test in the lab

Current/Future Work

- Real-time ⟷ Instanton
  - Renormalisation, Fluc. Determinant, Wigner
- Mean bubble profile = instanton? (preliminary yes)
- Bubble-bubble correlations?
- Time-dependent background or potential
- Non-vacuum initial states (pure or mixed)
- Application to many fields (more robust than instanton)

THANK YOU!

QFT in Phase Space

[See Braden et al, also Hertzberg]

Consider the Wigner functional

$$W[\phi, \Pi] \equiv \int \mathcal{D}\eta e^{-\frac{i}{\hbar}\int d^d x \Pi(\mathbf{x})\eta(\mathbf{x})} \left\langle \phi + \frac{\eta}{2} \middle| \Psi \right\rangle \left\langle \Psi \middle| \phi - \frac{\eta}{2} \right\rangle$$

Important Properties

$$\int \mathcal{D}\phi\mathcal{D}\Pi \; W[\phi, \Pi] = 1$$

$$\langle \hat{\mathcal{O}}(\hat{\phi}, \hat{\Pi}) \rangle = \int \mathcal{D}\phi\mathcal{D}\Pi \, W(\phi, \Pi)\mathcal{O}_{\mathrm{W}}(\phi, \Pi)$$

W ~ quantum probability distribution

(caveat: Not postive definite in general,
but is for Gaussian states)

Wigner Approach

[see also Hertzberg and Yamada]

$$W[\phi,\Pi] \equiv \int \mathcal{D}\eta e^{-\frac{i}{\hbar}\int d^dx \Pi(\mathbf{x})\eta(\mathbf{x})} \left\langle \phi + \frac{\eta}{2} \middle| \Psi \right\rangle \left\langle \Psi \middle| \phi - \frac{\eta}{2} \right\rangle$$

$$\left[\frac{\partial}{\partial t} + \int d^dx \left(\Pi\frac{\delta}{\delta\phi} + \nabla^2\phi\frac{\delta}{\delta\Pi} - \frac{2}{i\hbar}V(\phi)\sin\left(\overleftarrow{\nabla}_\phi \frac{i\hbar}{2}\overrightarrow{\frac{\partial}{\partial\Pi}}\right)\right)\right] W[\phi(x),\Pi(x);t] = 0$$

Wigner Approach

[see also Hertzberg and Yamada]

$$W[\phi, \Pi] \equiv \int \mathcal{D}\eta e^{-\frac{i}{\hbar}\int d^d x \Pi(\mathbf{x})\eta(\mathbf{x})} \left\langle \phi + \frac{\eta}{2} \middle| \Psi \right\rangle \left\langle \Psi \middle| \phi - \frac{\eta}{2} \right\rangle$$

$$\left[\frac{\partial}{\partial t} + \int d^d x \left(\dot{\phi}\frac{\delta}{\delta\phi} + \dot{\Pi}\frac{\delta}{\delta\Pi}\right) + \mathcal{O}\left(\hbar^2 V'''(\phi)\frac{\delta^3}{\delta\Pi^3}\right)\right] W[\phi(x), \Pi(x); t] = 0$$

Wigner Approach

[see also Hertzberg and Yamada]

$$W[\phi,\Pi] \equiv \int \mathcal{D}\eta e^{-\frac{i}{\hbar}\int d^d x \Pi(\mathbf{x})\eta(\mathbf{x})} \left\langle \phi + \frac{\eta}{2} \middle| \Psi \right\rangle \left\langle \Psi \middle| \phi - \frac{\eta}{2} \right\rangle$$

$$\left[\frac{\partial}{\partial t} + \int d^d x \left(\dot{\phi}\frac{\delta}{\delta\phi} + \dot{\Pi}\frac{\delta}{\delta\Pi}\right) + \mathcal{O}\left(\hbar^2 V'''(\phi)\frac{\delta^3}{\delta\Pi^3}\right)\right] W[\phi(x),\Pi(x);t] = 0$$

Initial State (t=0)

Wigner Approach

[see also Hertzberg and Yamada]

$$W[\phi, \Pi] \equiv \int \mathcal{D}\eta e^{-\frac{i}{\hbar}\int d^d x \Pi(\mathbf{x})\eta(\mathbf{x})} \left\langle \phi + \frac{\eta}{2} \middle| \Psi \right\rangle \left\langle \Psi \middle| \phi - \frac{\eta}{2} \right\rangle$$

$$\left[\frac{\partial}{\partial t} + \int d^d x \left(\dot{\phi}\frac{\delta}{\delta\phi} + \dot{\Pi}\frac{\delta}{\delta\Pi}\right) + \mathcal{O}\left(\hbar^2 V'''(\phi)\frac{\delta^3}{\delta\Pi^3}\right)\right] W[\phi(x), \Pi(x); t] = 0$$

Classical Evolution

Initial State (t=0)

Wigner Approach

[see also Hertzberg and Yamada]

$$W[\phi,\Pi] \equiv \int \mathcal{D}\eta e^{-\frac{i}{\hbar}\int d^dx\Pi(\mathbf{x})\eta(\mathbf{x})} \left\langle \phi + \frac{\eta}{2}\middle| \Psi \right\rangle \left\langle \Psi \middle| \phi - \frac{\eta}{2} \right\rangle$$

$$\left[\frac{\partial}{\partial t} + \int d^dx \left(\dot{\phi}\frac{\delta}{\delta\phi} + \dot{\Pi}\frac{\delta}{\delta\Pi}\right) + \mathcal{O}\left(\hbar^2 V'''(\phi)\frac{\delta^3}{\delta\Pi^3}\right)\right] W[\phi(x),\Pi(x);t] = 0$$

Classical Evolution **Quantum “Noise”** **Initial State (t=0)**

Wigner Approach

[see also Hertzberg and Yamada]

$$W[\phi, \Pi] \equiv \int \mathcal{D}\eta e^{-\frac{i}{\hbar}\int d^d x \Pi(\mathbf{x})\eta(\mathbf{x})} \left\langle \phi + \frac{\eta}{2} \middle| \Psi \right\rangle \left\langle \Psi \middle| \phi - \frac{\eta}{2} \right\rangle$$

$$\left[\frac{\partial}{\partial t} + \int d^d x \left(\dot{\phi}\frac{\delta}{\delta\phi} + \dot{\Pi}\frac{\delta}{\delta\Pi}\right) + \mathcal{O}\left(\hbar^2 V'''(\phi)\frac{\delta^3}{\delta\Pi^3}\right)\right] W[\phi(x), \Pi(x); t] = 0$$

Classical Evolution **Quantum "Noise"** **Initial State (t=0)**