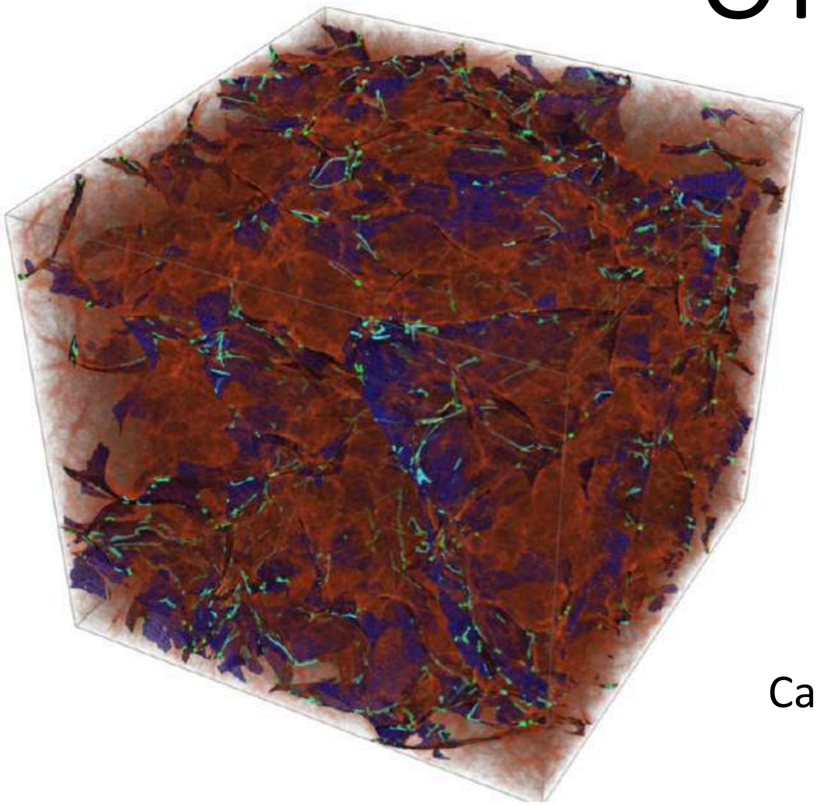
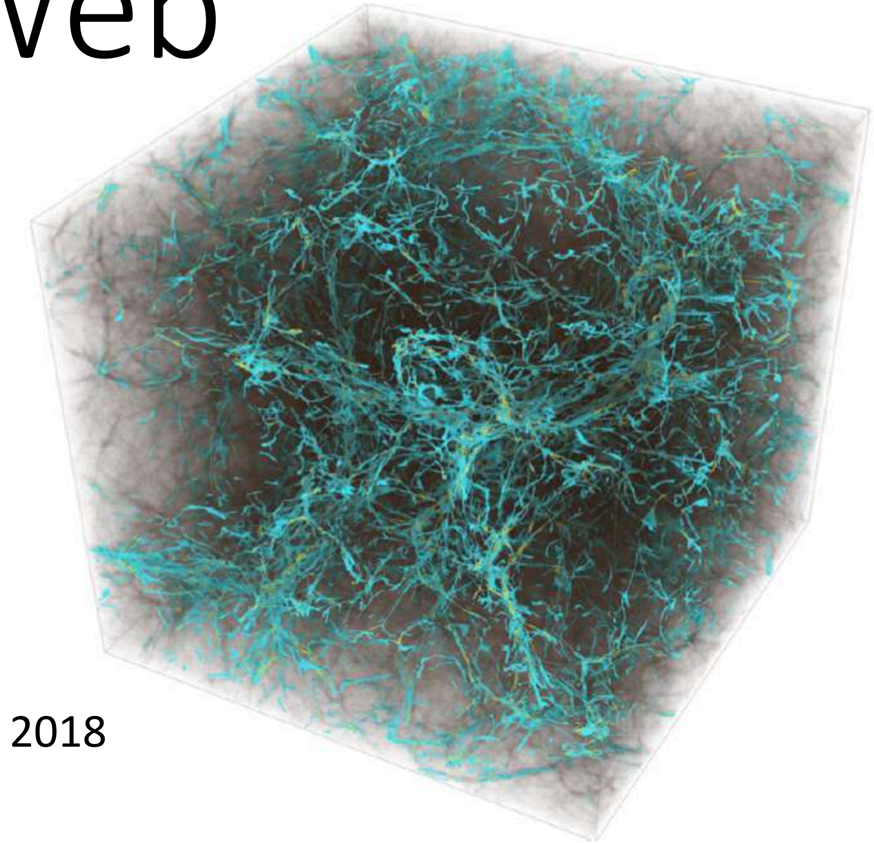
The Caustic Skeleton of the Cosmic Web



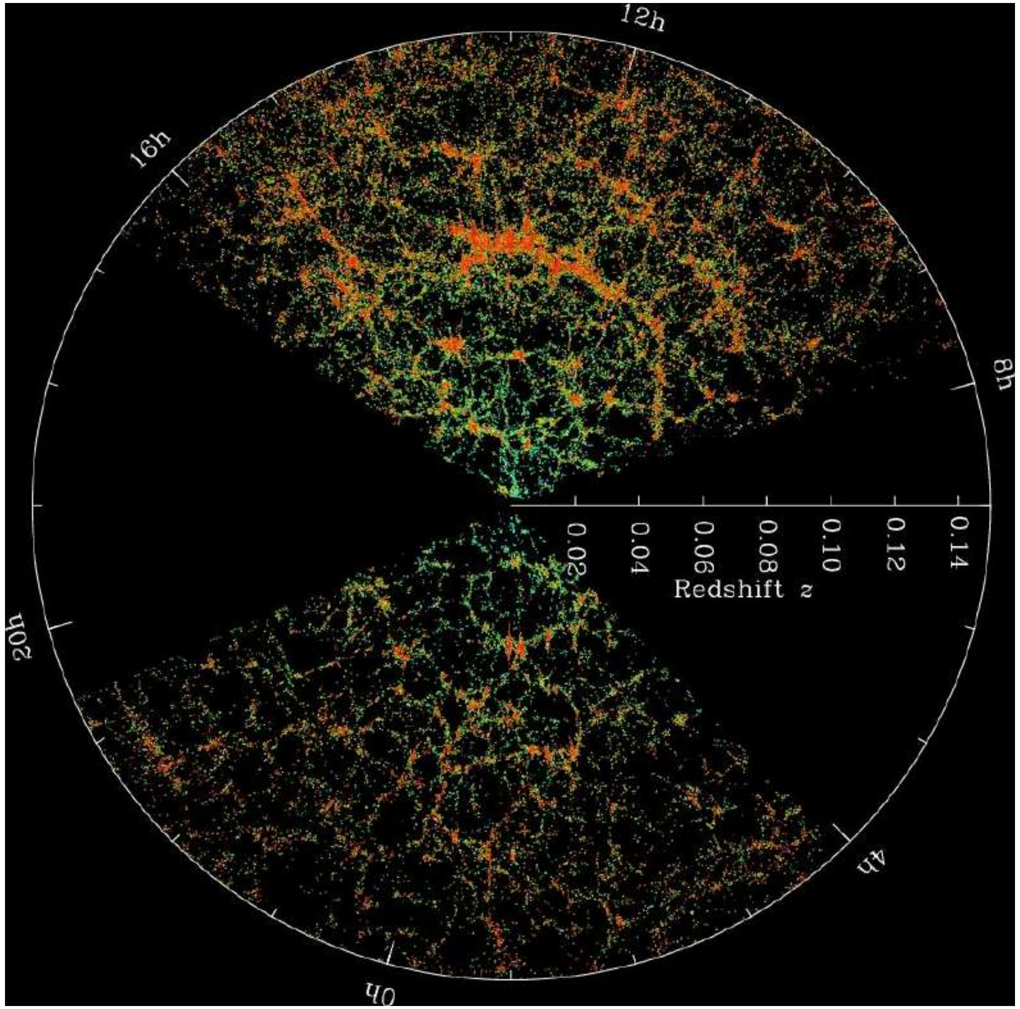
Job Feldbrugge
Perimeter Institute
Carnegie Mellon University



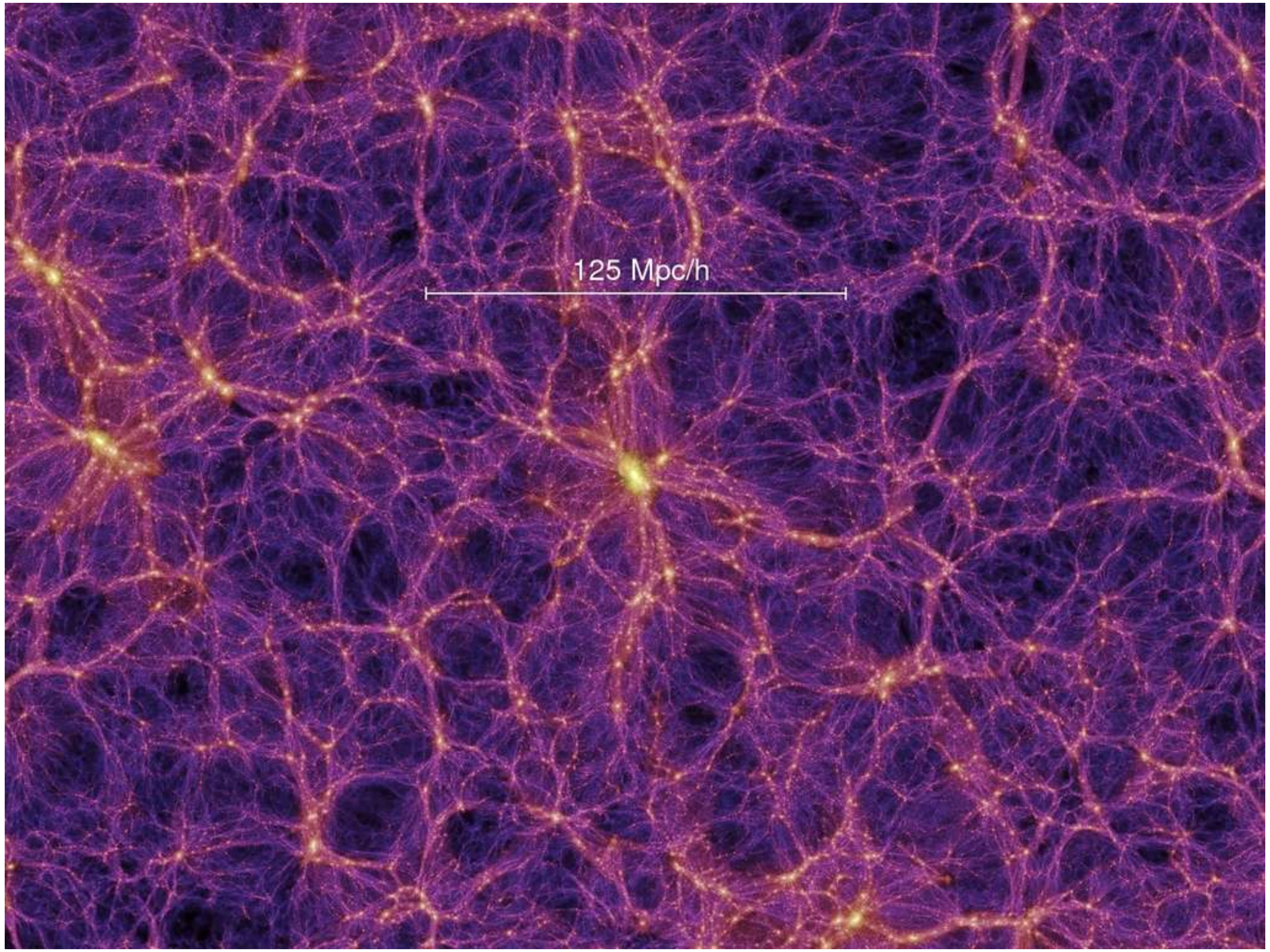
Caustic Skeleton & Cosmic Web, JCAP, Issue 05, 2018

Collaborators: R. van de Weygaert, J. Hidding, J. Feldbrugge

Cosmic Web



SDSS redshift survey



Millennium simulation

Arnol’d et al.

- *Vladimir Arnol’d* extended *René Thom* classification of stable degenerate critical points to **Lagrangian catastrophe theory**

- The **classification of caustics** was applied to *large-scale structure formation* to predict the geometric structure of the *cosmic web*

1972 NORMAL FORMS FOR FUNCTIONS NEAR DEGENERATE CRITICAL POINTS, THE WEYL GROUPS OF A_k, D_k, E_k AND LAGRANGIAN SINGULARITIES

V. I. Arnol'd

1980 EVOLUTION OF SINGULARITIES OF POTENTIAL FLOWS IN COLLISION-FREE MEDIA AND THE METAMORPHOSIS OF CAUSTICS IN THREE-DIMENSIONAL SPACE

V. I. Arnol'd

1982 The Large Scale Structure of the Universe I. General Properties. One- and Two-Dimensional Models

V. I. ARNOLD
Moscow State University, U.S.S.R.

and

S. F. SHANDARIN and YA. B. ZELDOVICH
Institute of Applied Mathematics, Moscow, U.S.S.R.

(*Received August* 11, 1981)

The classification of caustics

Singularity class	Singularity name	Feature in the 2D cosmic web	Feature in the 3D cosmic web
A_2	fold	collapsed region	collapsed region
A_3	cusp	filament	wall or membrane
A_4	swallowtail	cluster or knot	filament
A_5	butterfly	not stable	cluster or knot
D_4	hyperbolic/elliptic	cluster or knot	filament
D_5	parabolic	not stable	cluster or knot

The identification of the different caustics in the 2- and 3-dimensional cosmic web

Lagrangian fluid dynamics

- Describe the deformation of the fluid
- The density spikes at the caustics

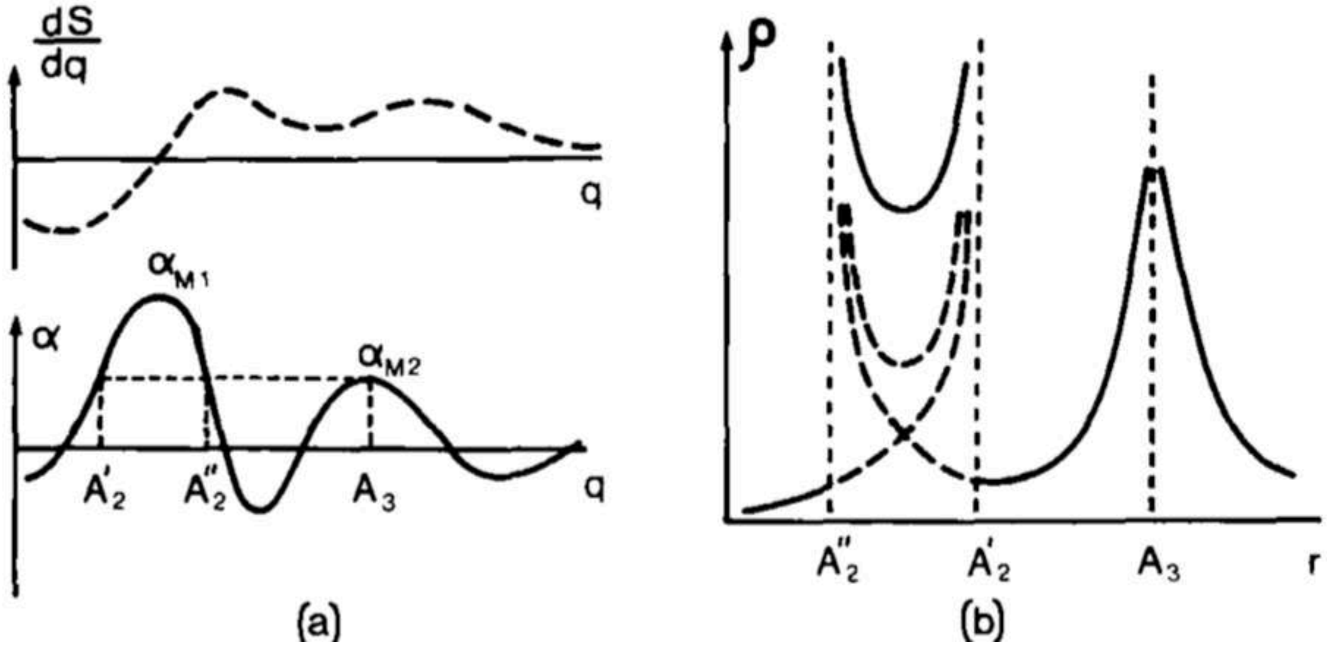
$$s_t(q) = x_t(q) - q$$

$$\mathcal{M} = \frac{\partial s_t}{\partial q} = \begin{pmatrix} M_{1,1} & M_{2,1} & M_{3,1} \\ M_{1,2} & M_{2,2} & M_{3,2} \\ M_{1,3} & M_{2,3} & M_{3,3} \end{pmatrix}$$

$$\mathcal{M} v_i = \mu_i v_i$$

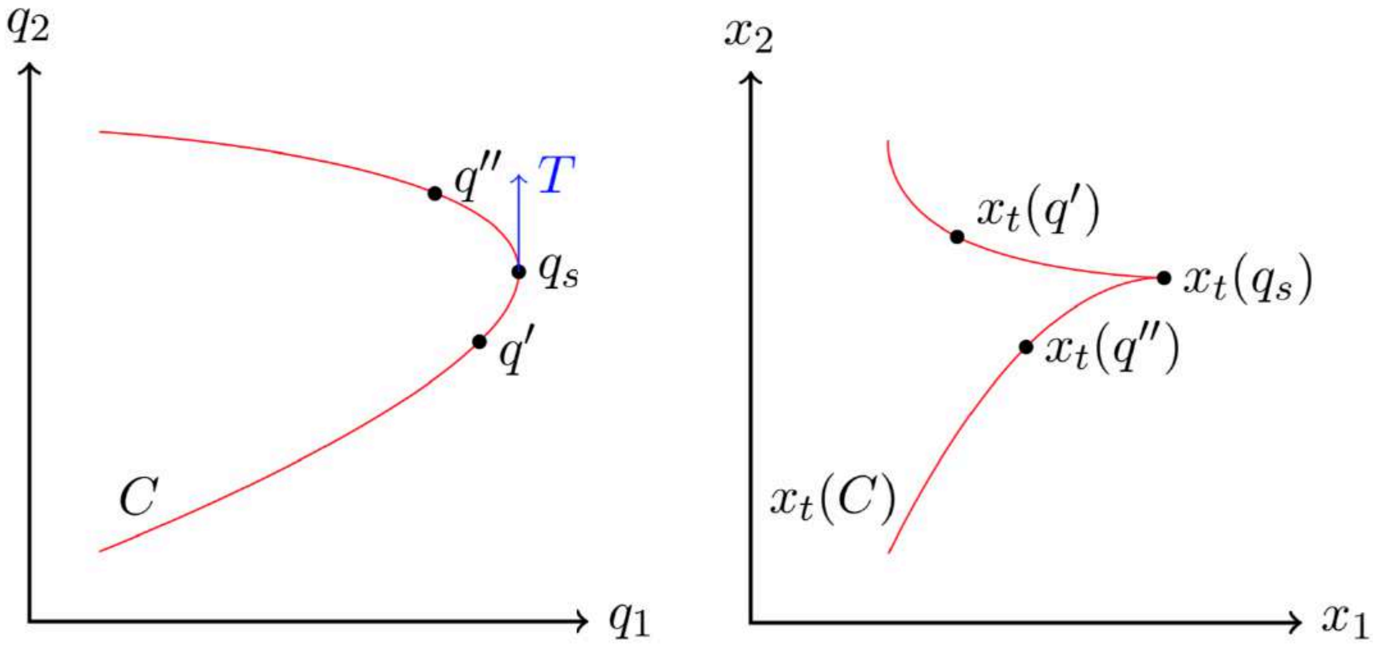
$$\rho(x', t) = \sum_{q \in A_t(x')} \frac{\rho_i(q)}{|1 + \mu_{t1}(q)||1 + \mu_{t2}(q)||1 + \mu_{t3}(q)|}$$

$$1 + \mu_i = 0$$



Arnol'd, Shandarin, Zel'dovich (1982)

Shell-crossing condition

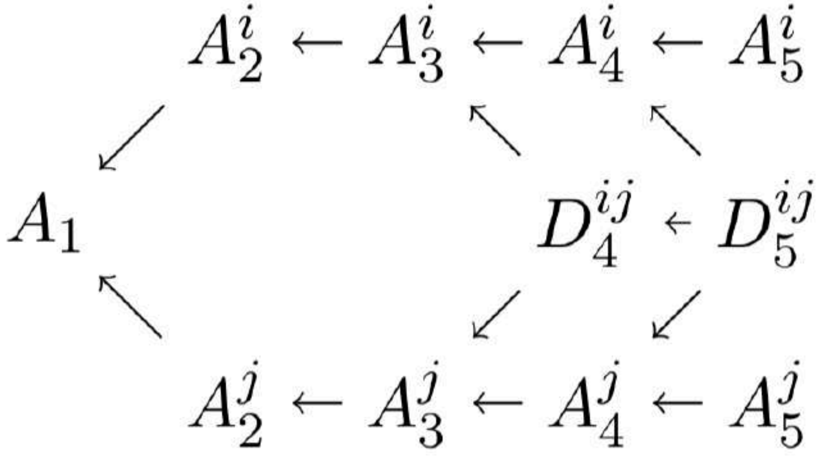


Theorem: *A manifold $M \subset L$ forms a singularity under the mapping x_t in the point $x_t(q_s) \in x_t(M) \subset E$ at time t, meaning that $x_t(M)$ is not smooth in $x_t(q_s)$, if and only if there exists at least one nonzero tangent vector $T \in T_{q_s}M$ satisfying*

$$(1 + \mu_{it}(q_s))v^*_{it}(q_s) \cdot T = 0$$

for all $i = 1, 2, \ldots, dim(L)$.

Caustic conditions



Iterative application of the shell-crossing condition

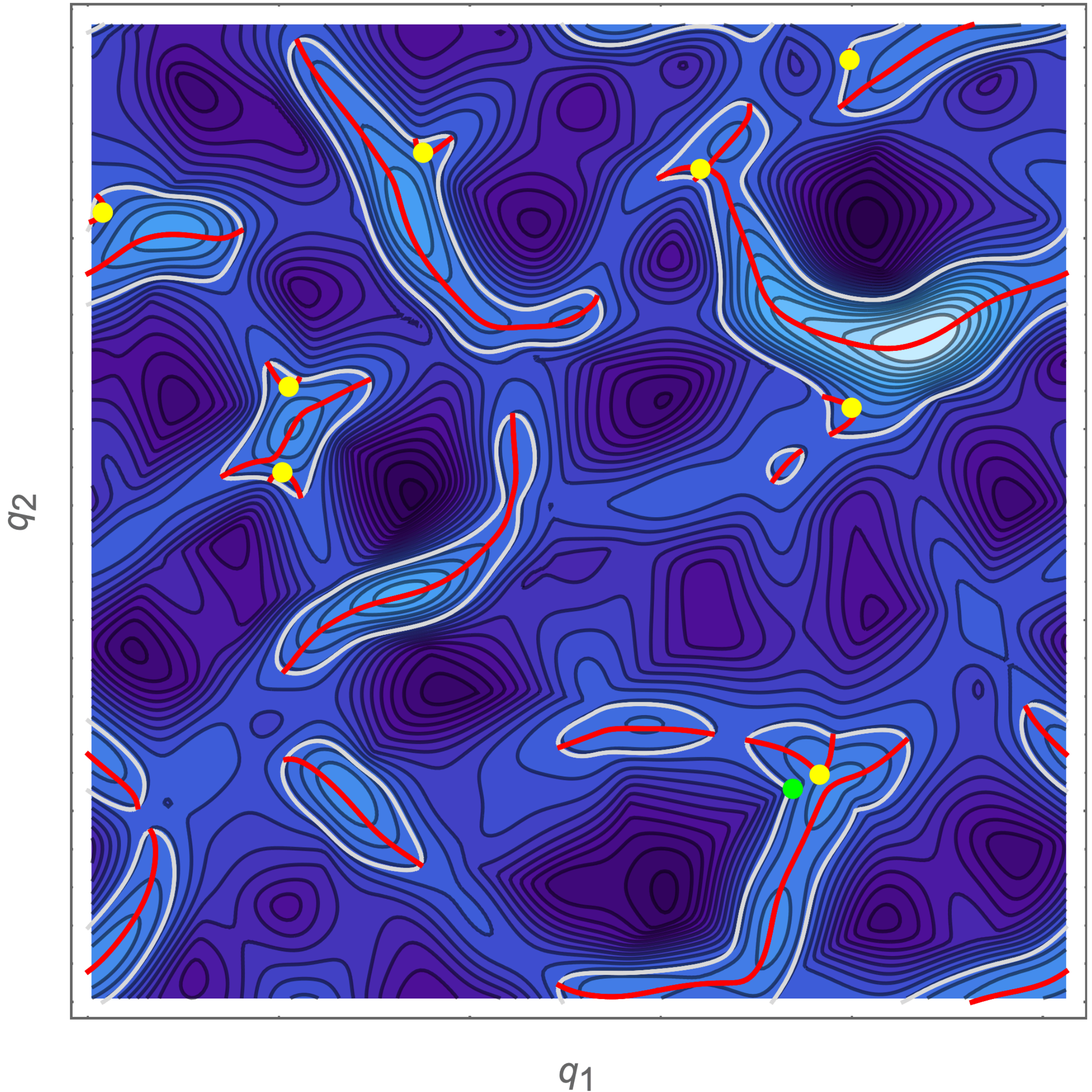
$$(1+\mu_{it}(q_s))v_{it}^*(q_s)\cdot T = 0$$

leads to the caustic conditions on both the eigenvalue and eigenvector fields:

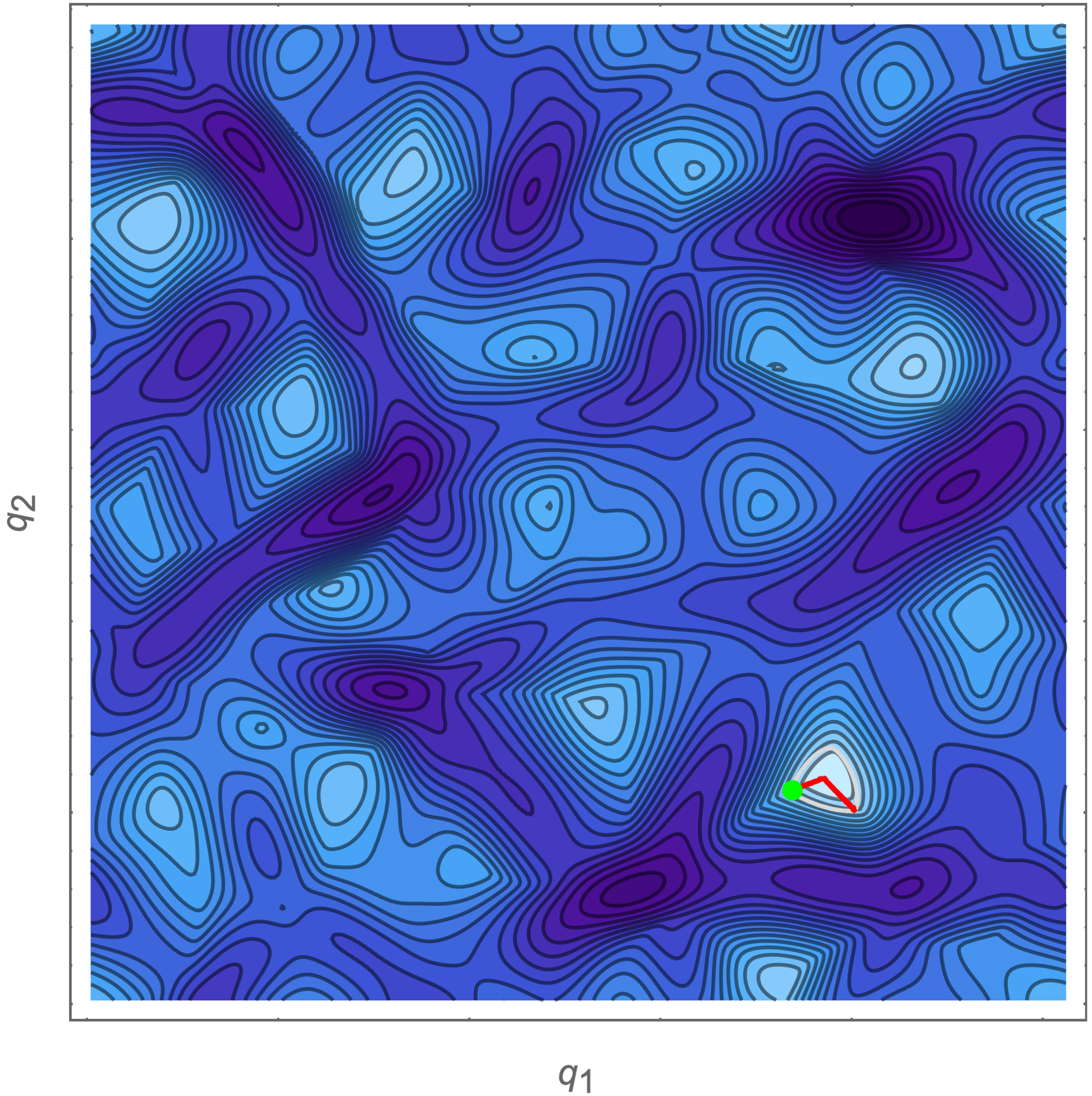
Fold:	$A_2^i(t) = \{\mathbf{q} \in L \mid 1 + \mu_{it}(\mathbf{q}) = 0\}$
Cusp:	$A_3^i(t) = \{\mathbf{q} \in L \mid \mathbf{q} \in A_2^i(t), \mathbf{v}_i \cdot \nabla \mu_{it} = 0\}$
Swallowtail:	$A_4^i(t) = \{\mathbf{q} \in L \mid \mathbf{q} \in A_3^i(t), \mathbf{v}_i \cdot \nabla(\mathbf{v}_i \cdot \nabla \mu_{it}) = 0\}$
Butterfly:	$A_5^i(t) = \{\mathbf{q} \in L \mid \mathbf{q} \in A_4^i(t), \mathbf{v}_i \cdot \nabla(\mathbf{v}_i \cdot \nabla(\mathbf{v}_i \cdot \nabla \mu_{it})) = 0\}$
Umbilic:	$D_4^{ij}(t) = \{\mathbf{q} \in L \mid 1 + \mu_{it}(\mathbf{q}) = 1 + \mu_{jt}(\mathbf{q}) = 0\}$
Parabolic:	$D_5^{ij}(t) = \{\mathbf{q} \in L \mid q \in D_4^{ij}(t), \mathbf{v}_i \cdot \nabla \mu_i = \mathbf{v}_j \cdot \nabla \mu_j = 0\}$

Morse-Smale theory of full deformation tensor field. No free parameters!

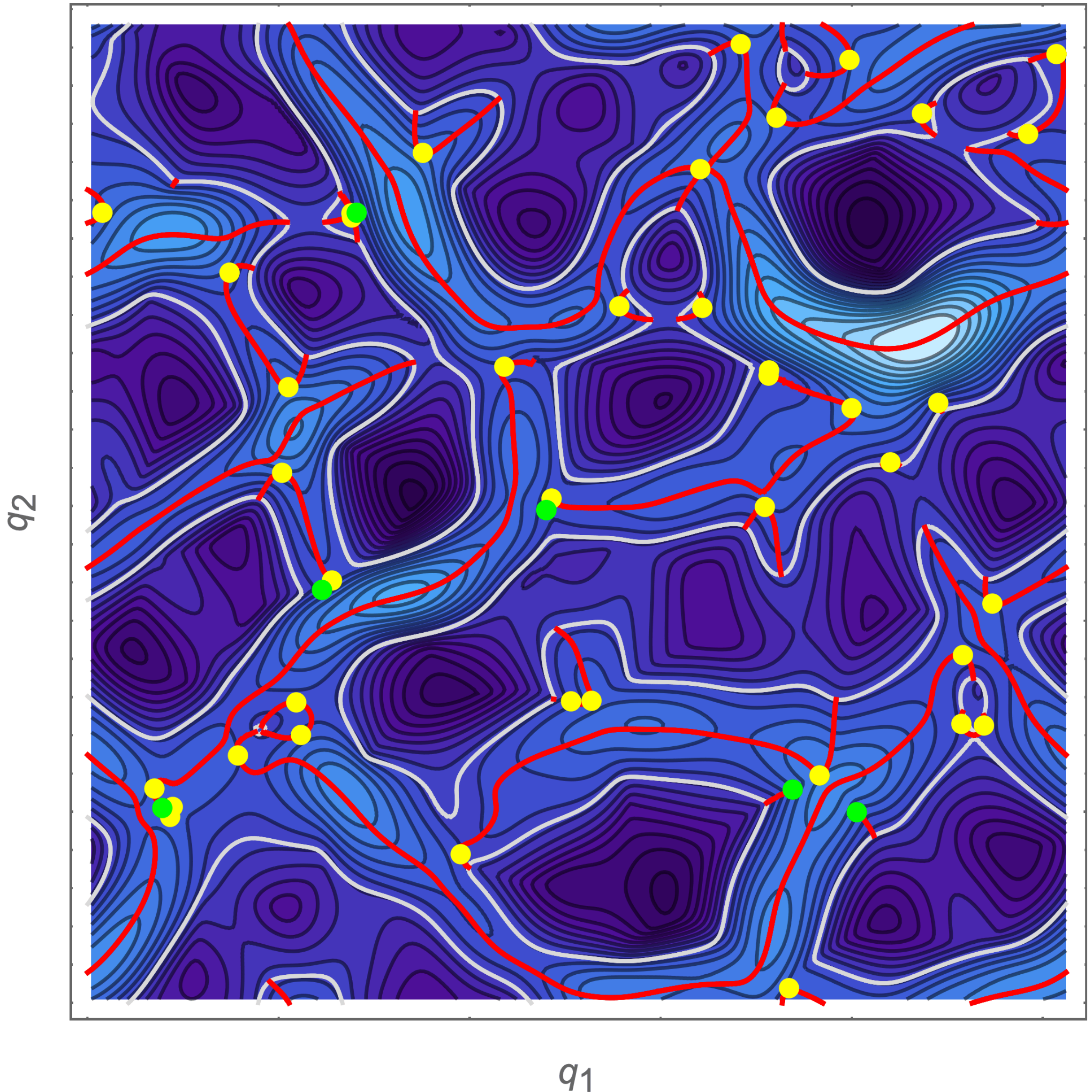
First eigenvalue



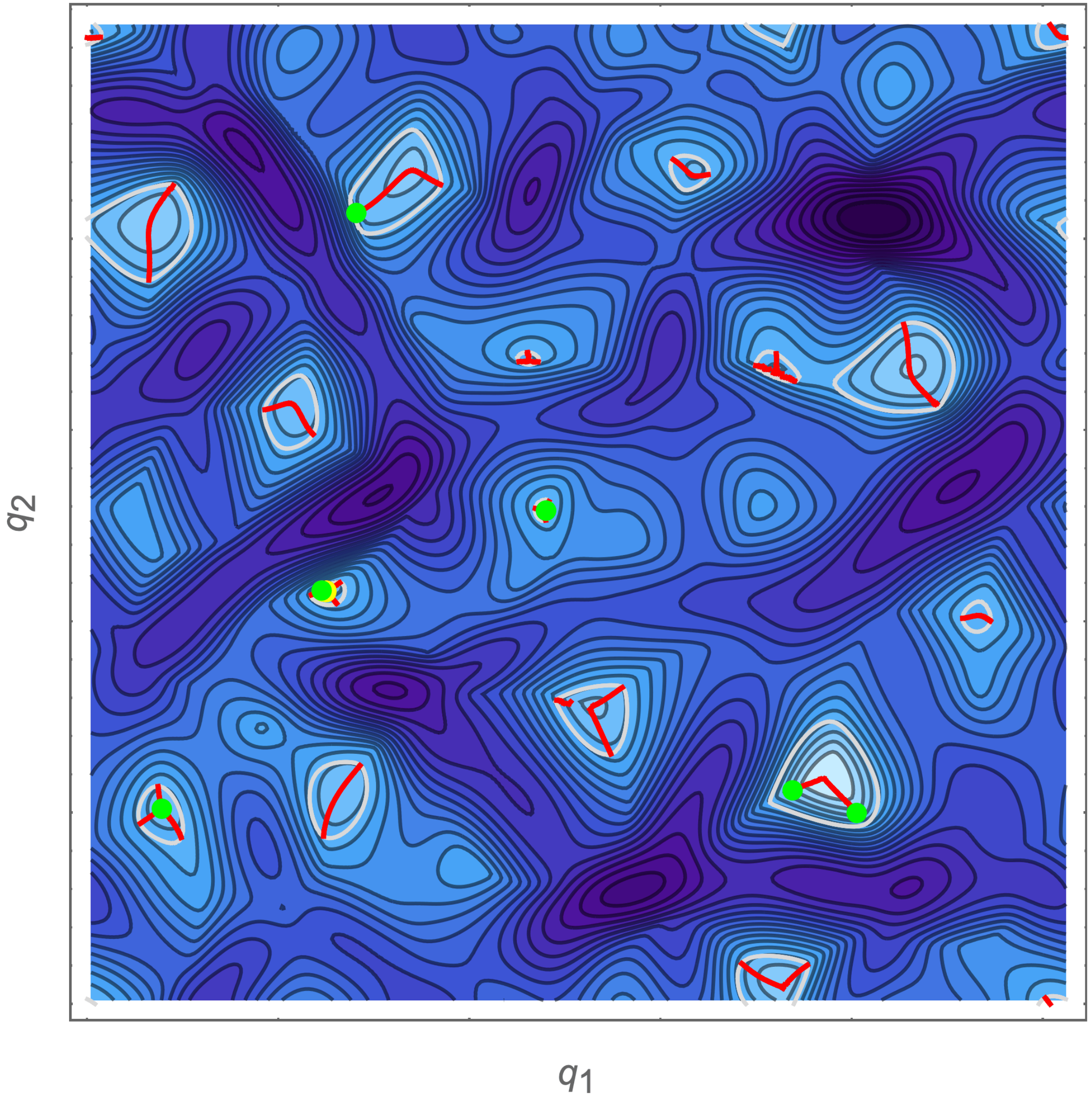
Second eigenvalue



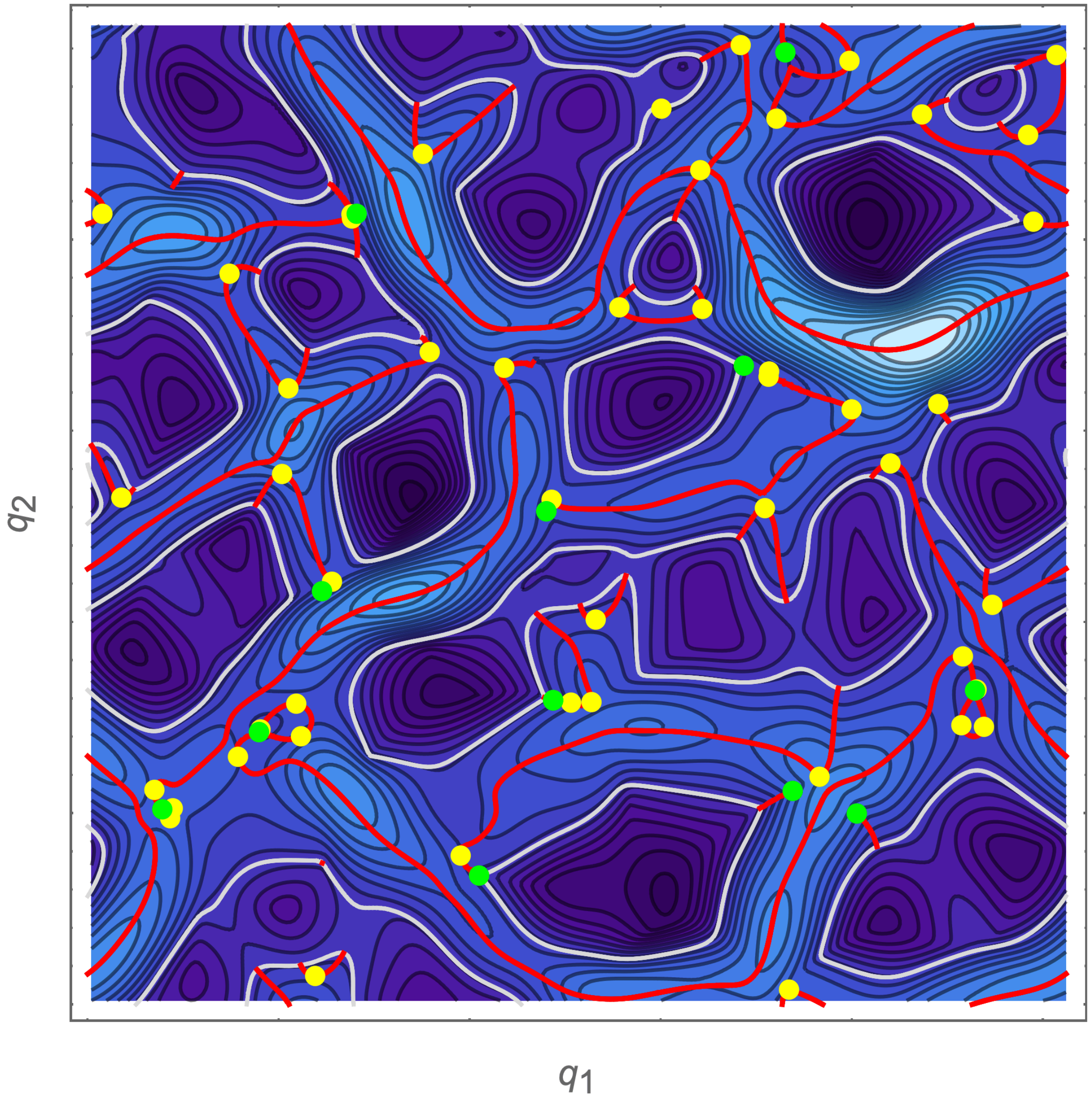
First eigenvalue



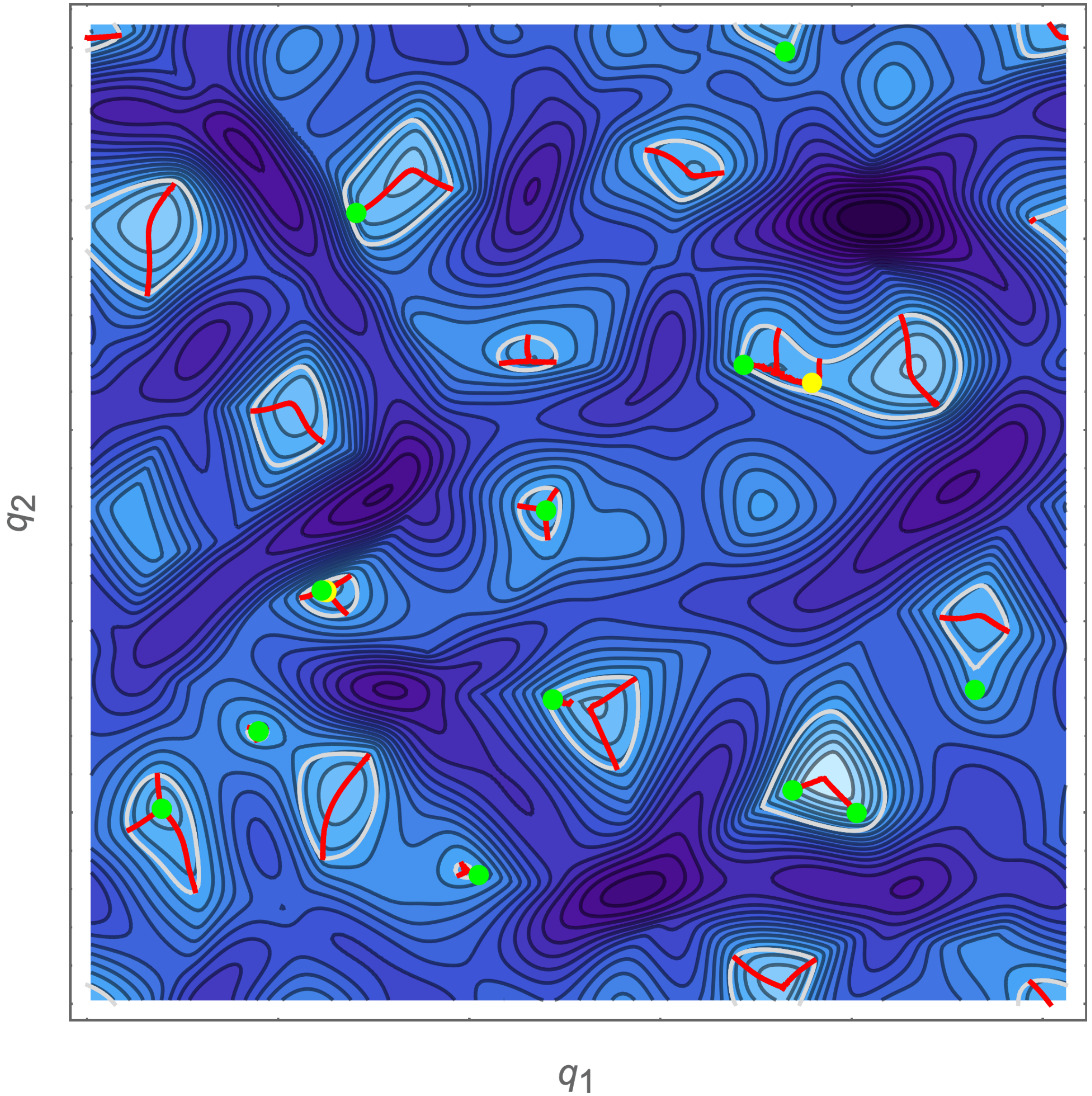
Second eigenvalue

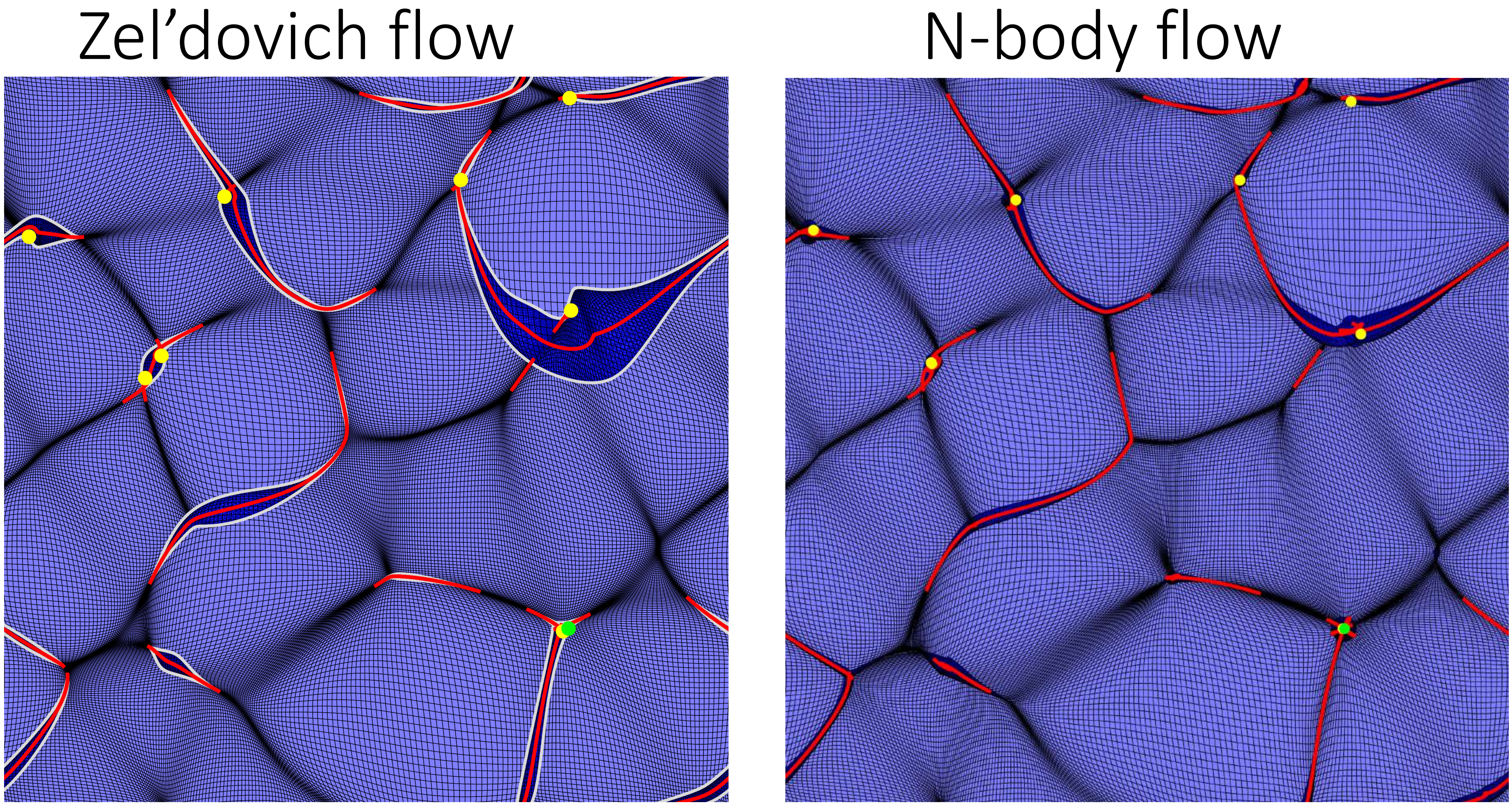


First eigenvalue

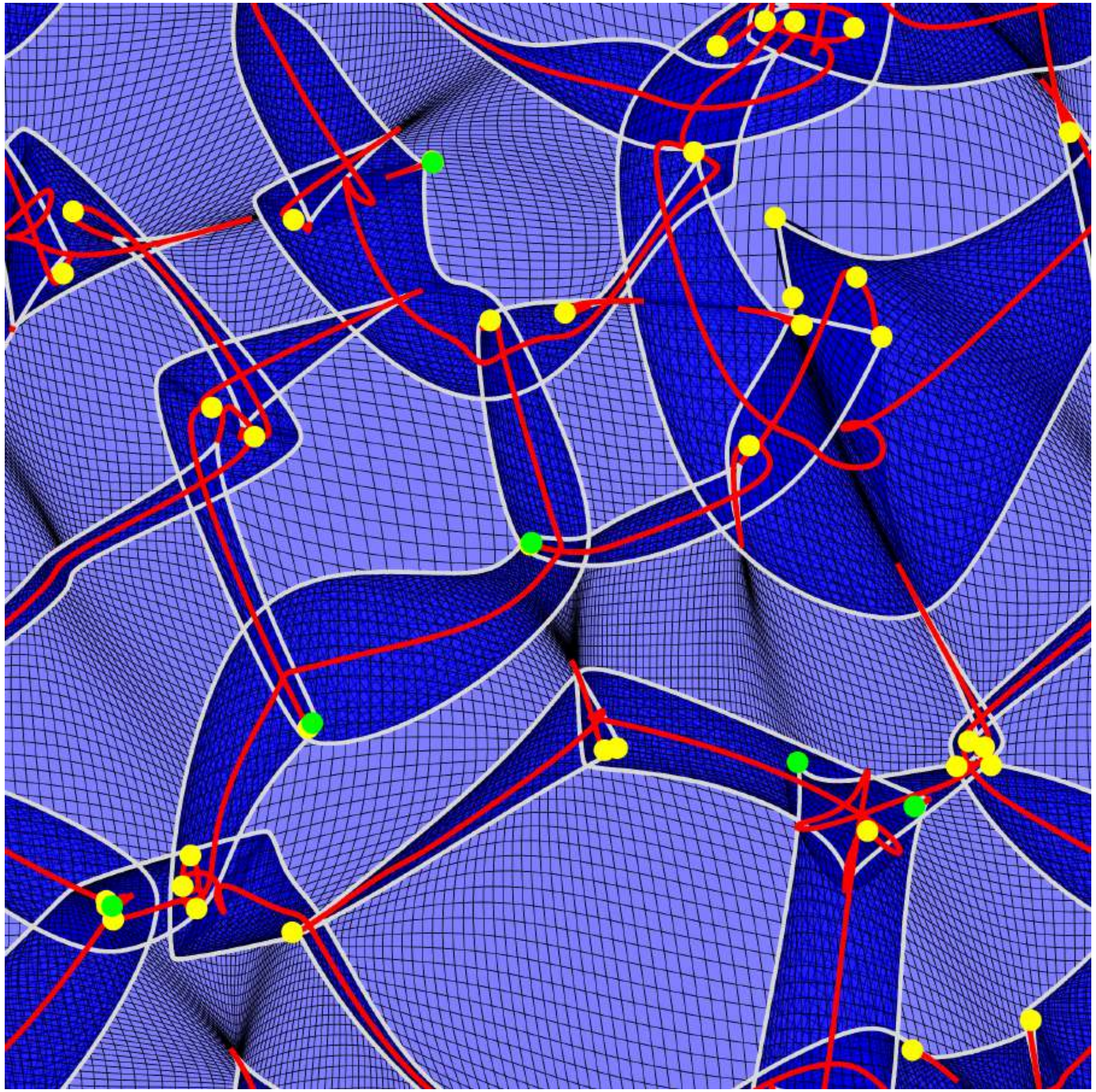


Second eigenvalue

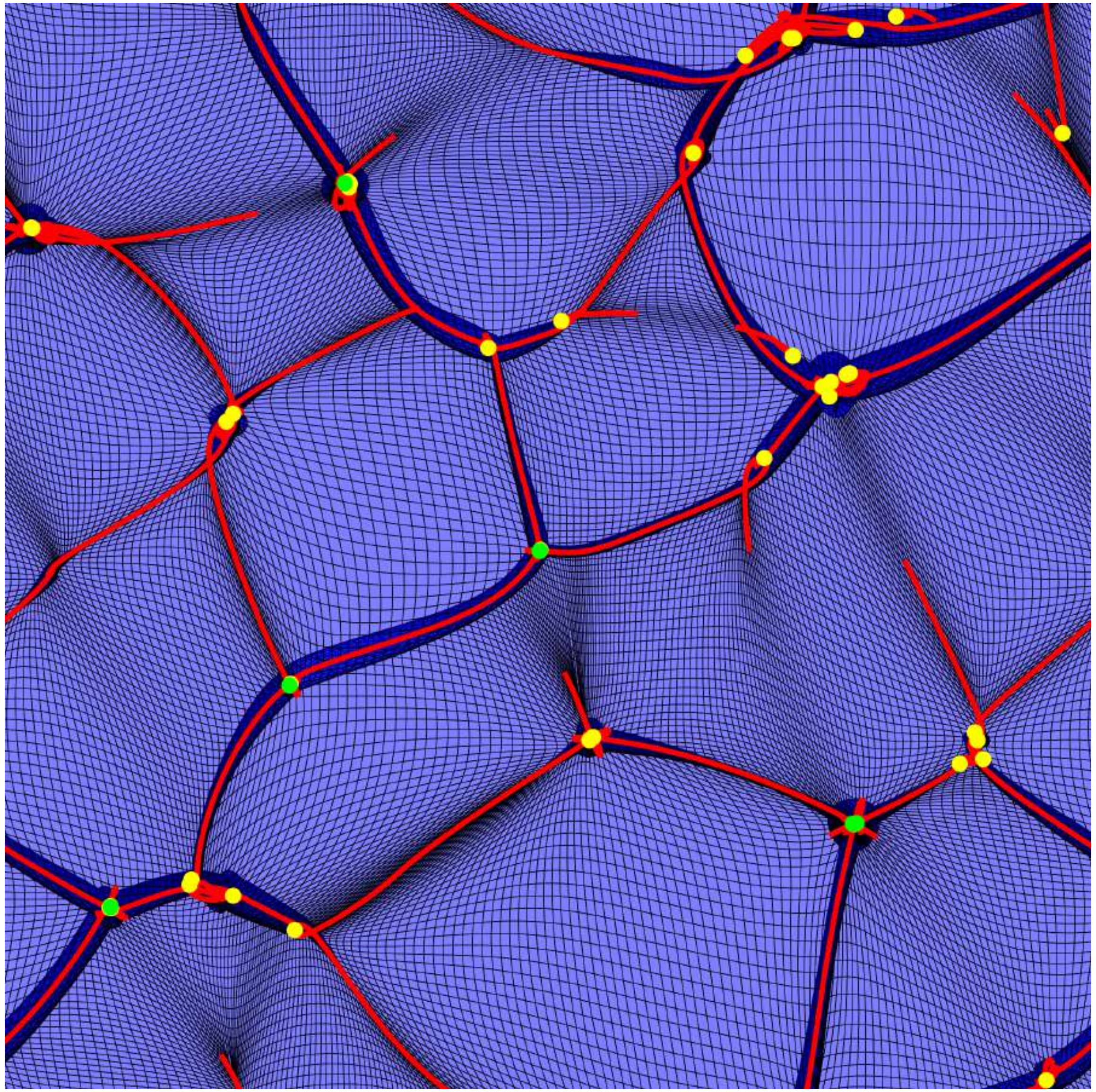



Zel'dovich flow
N-body flow

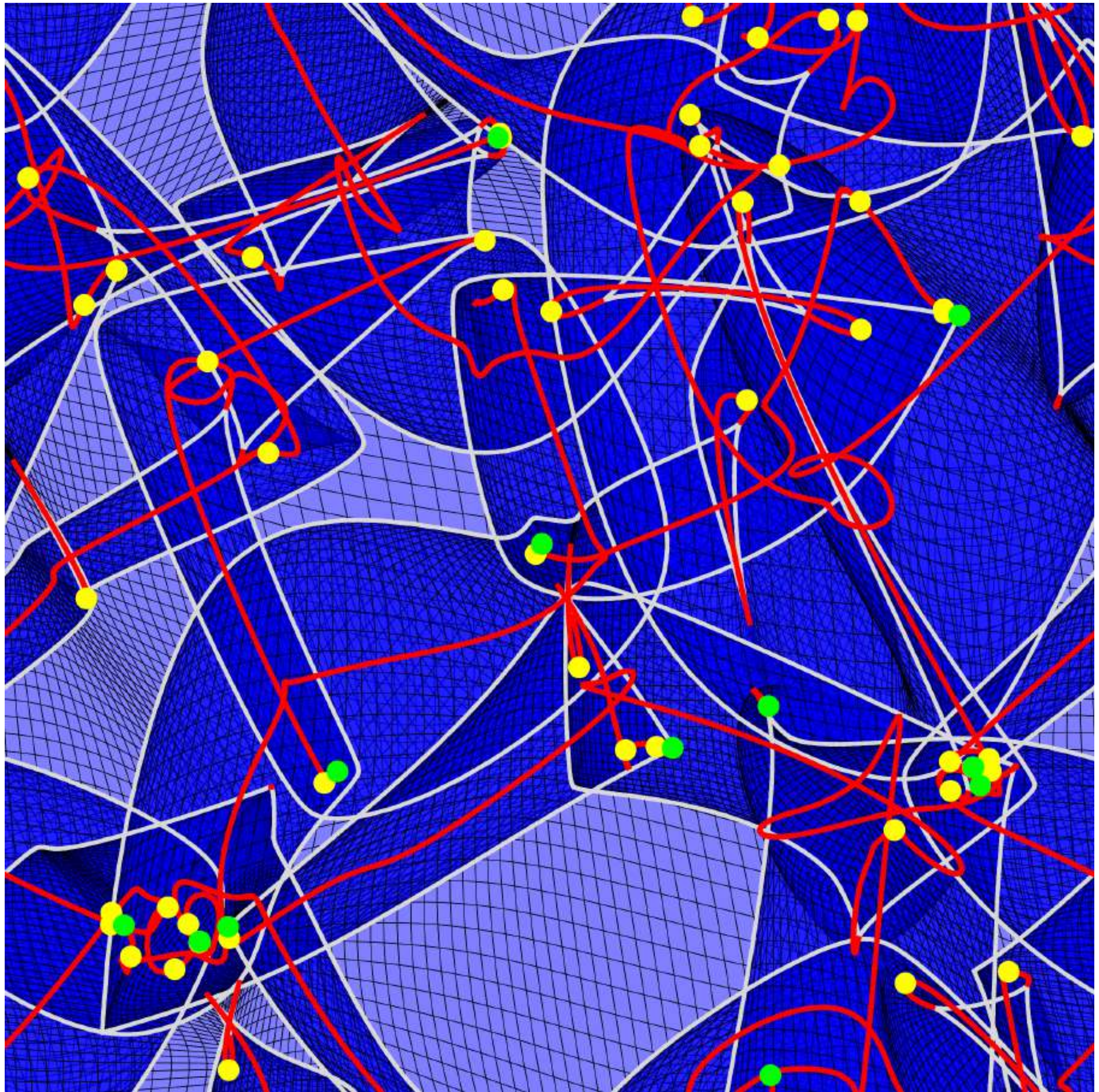
Zel'dovich flow



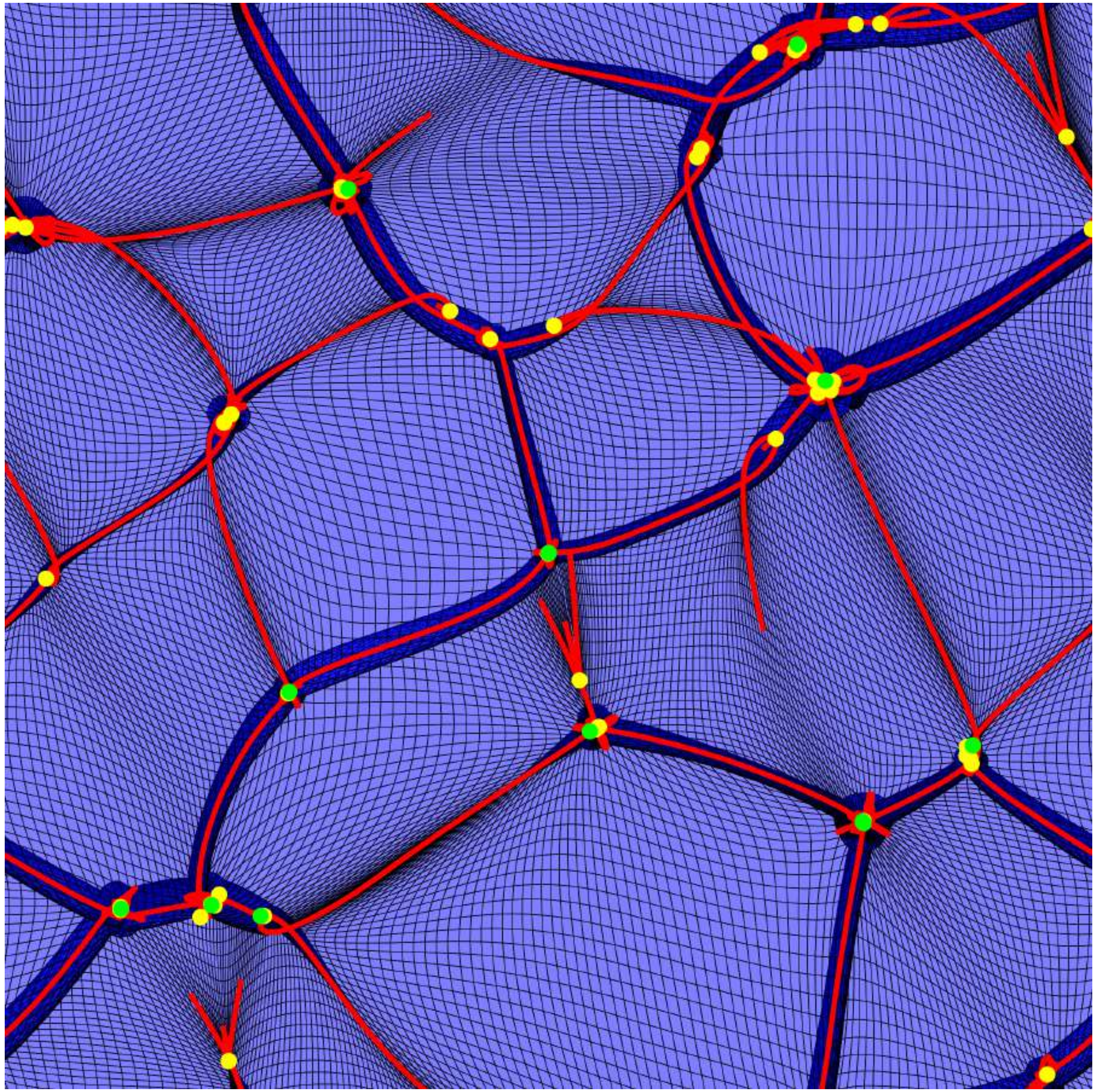
N-body flow



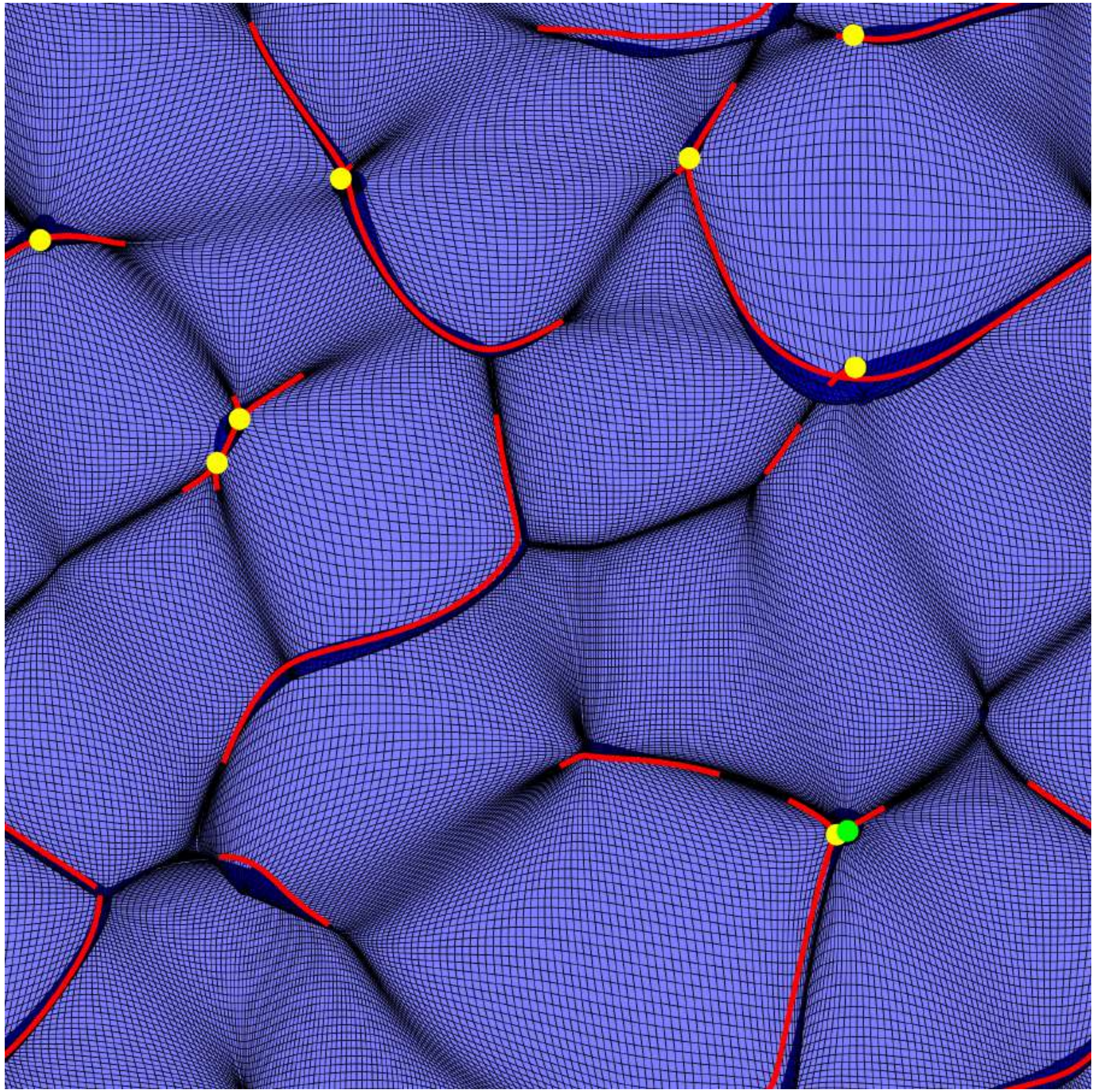
Zel'dovich flow



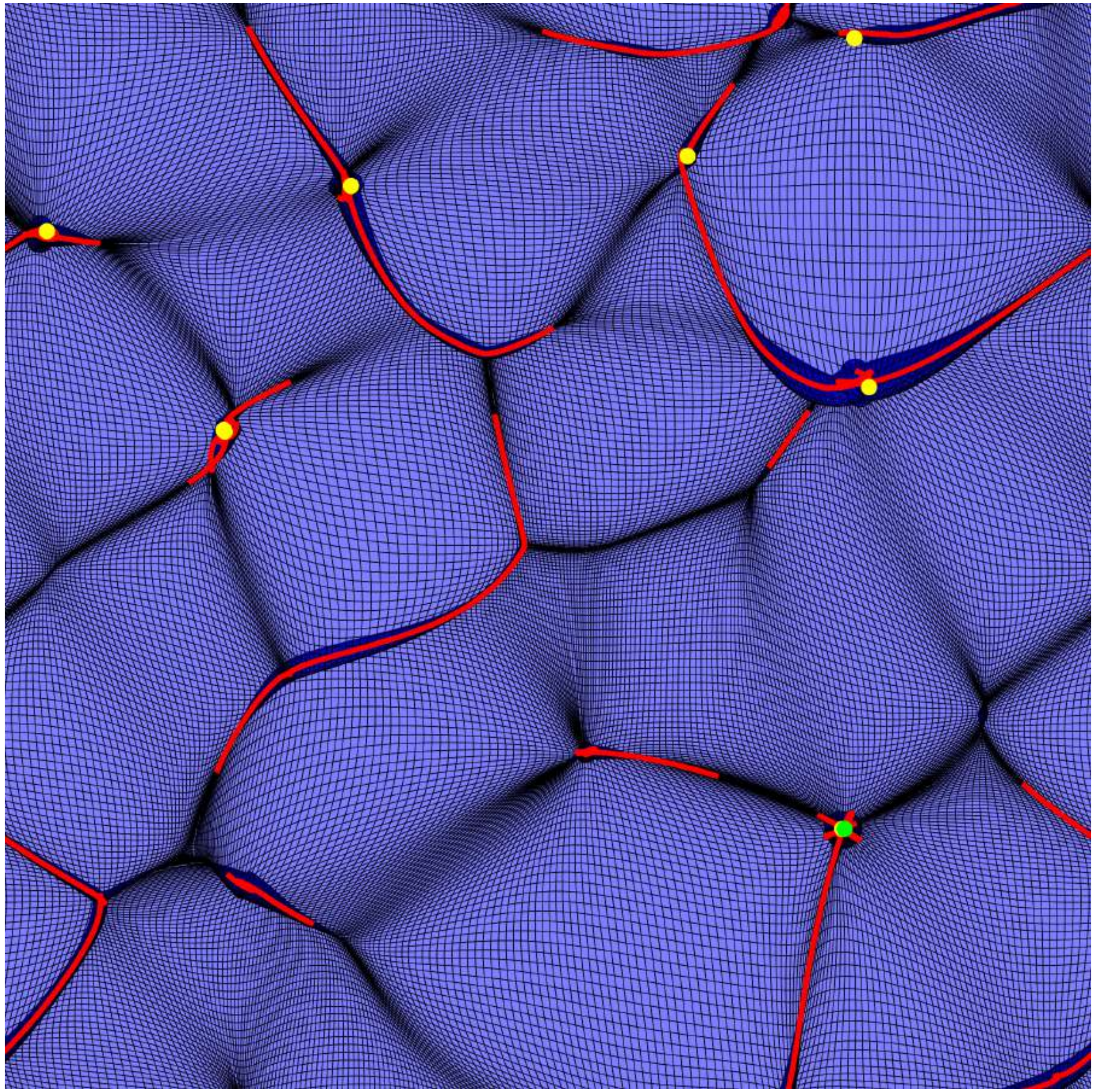
N-body flow



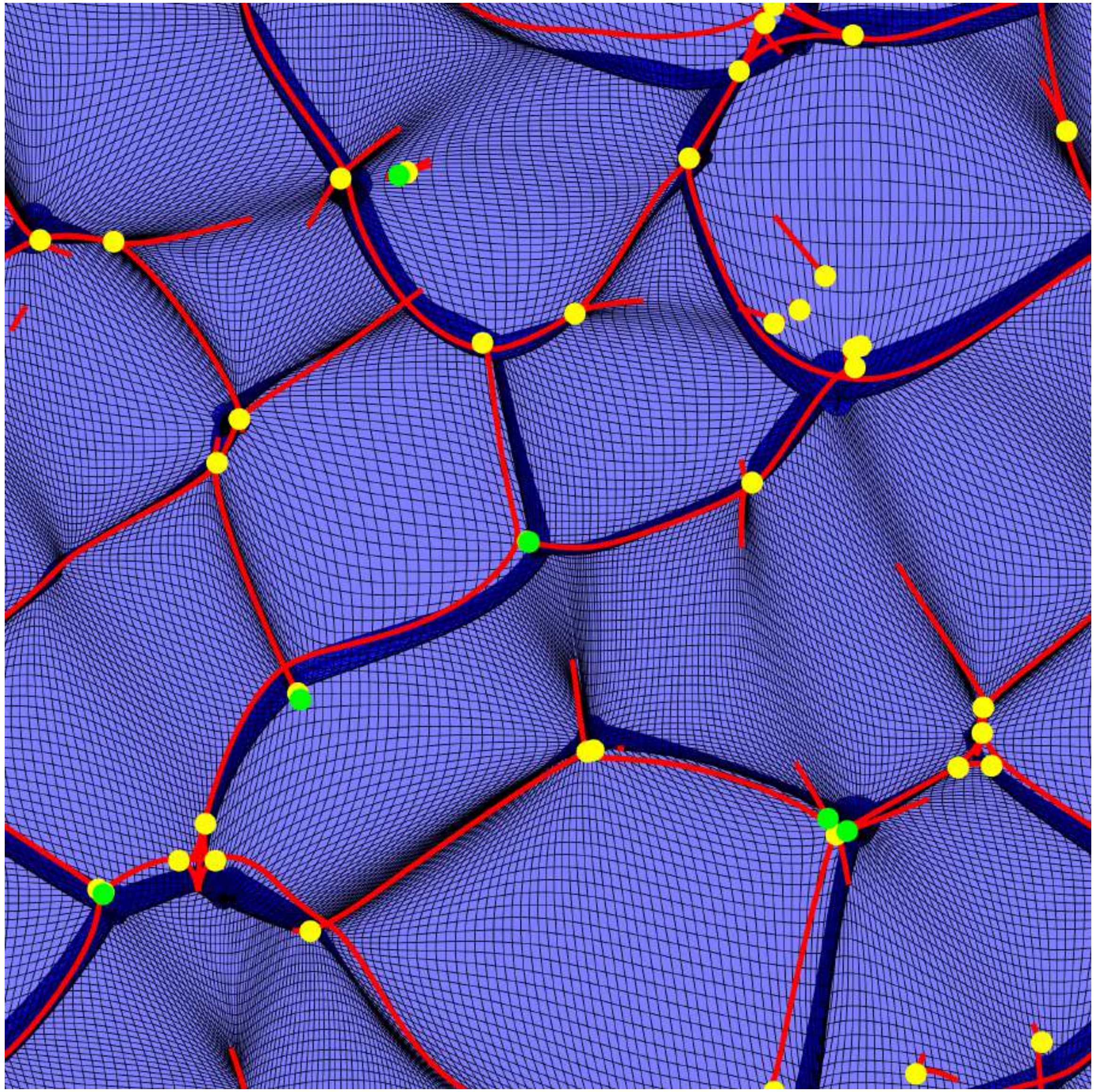
Scotch flow



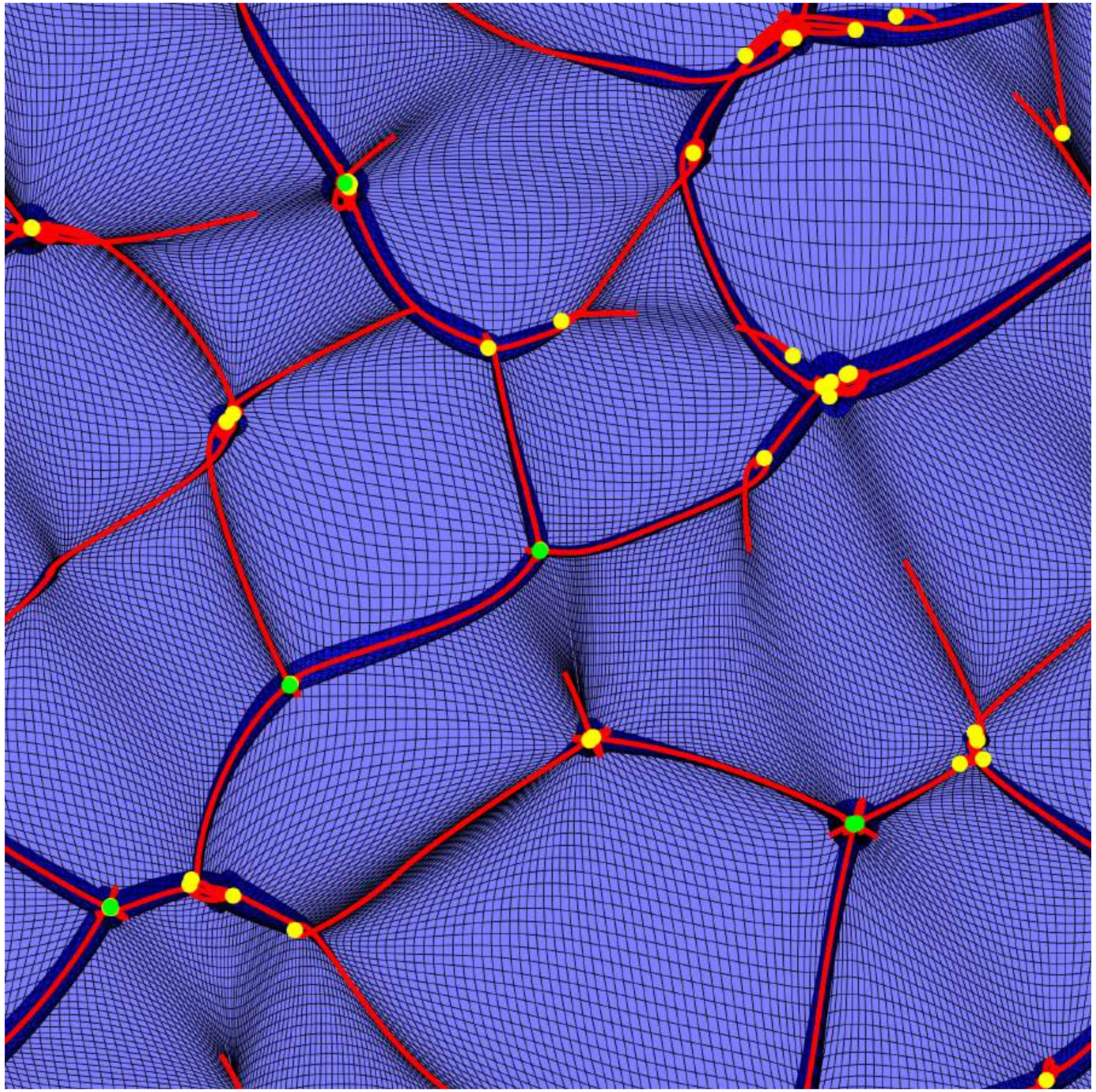
N-body flow



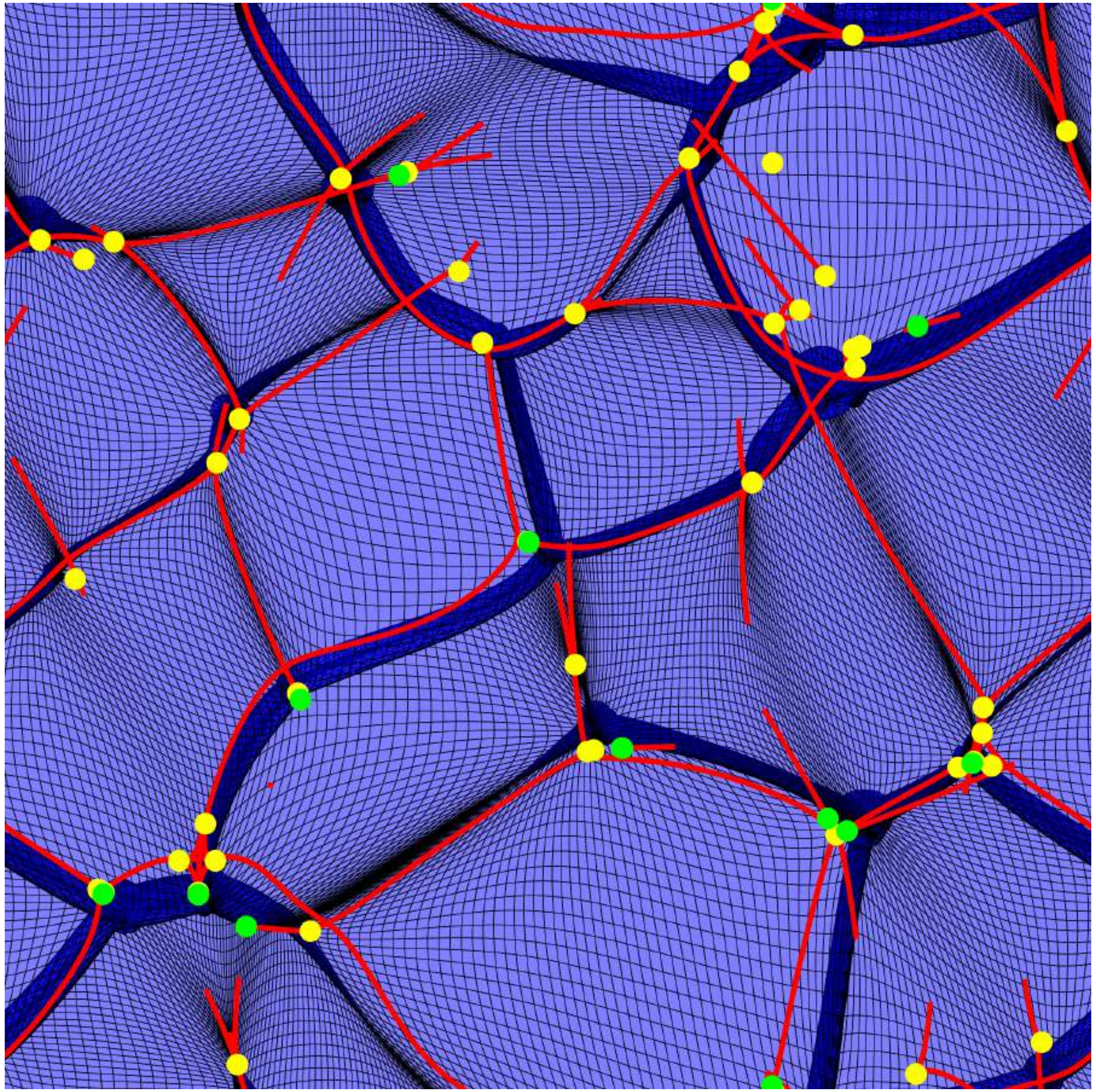
Scotch flow



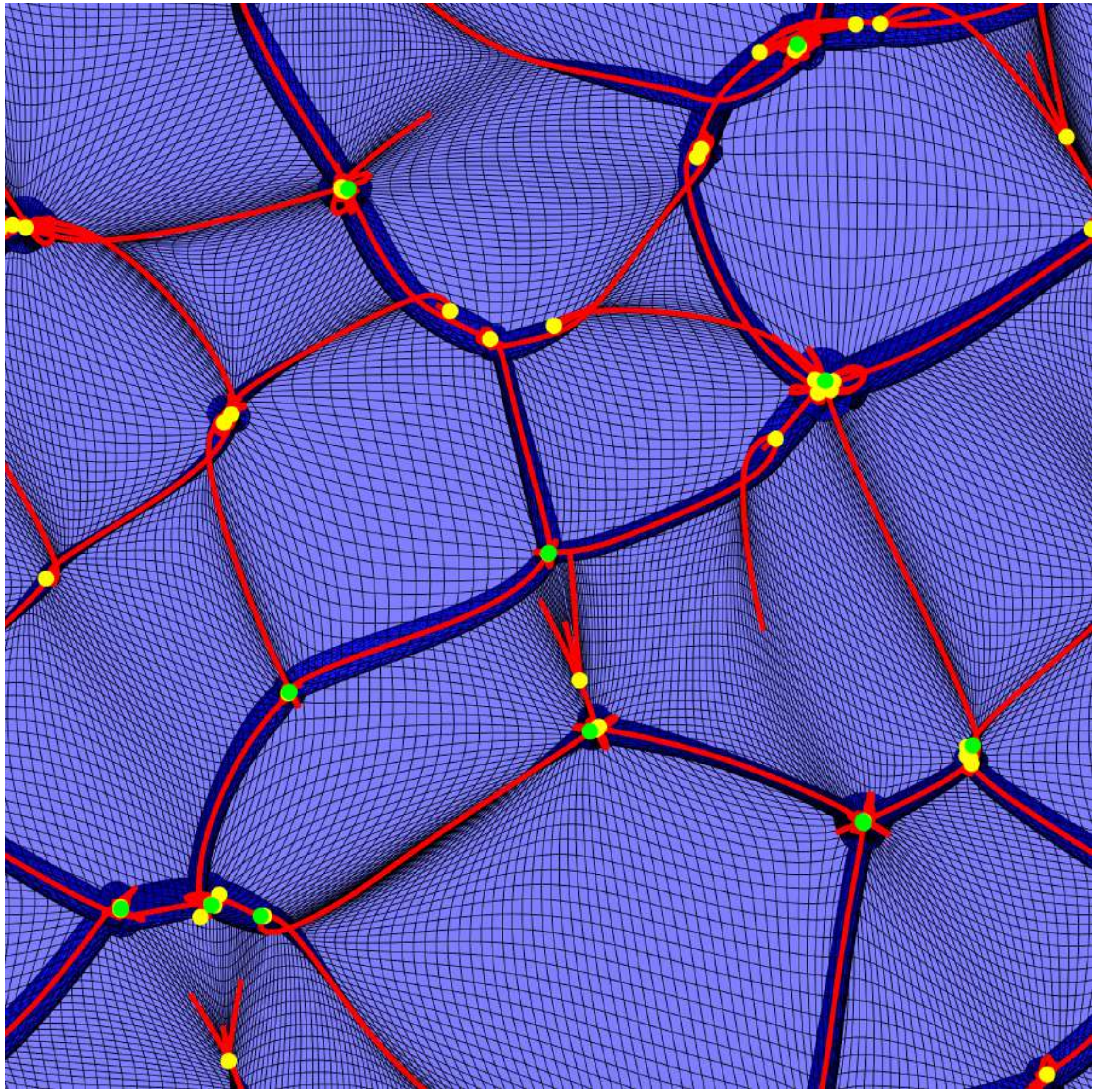
N-body flow



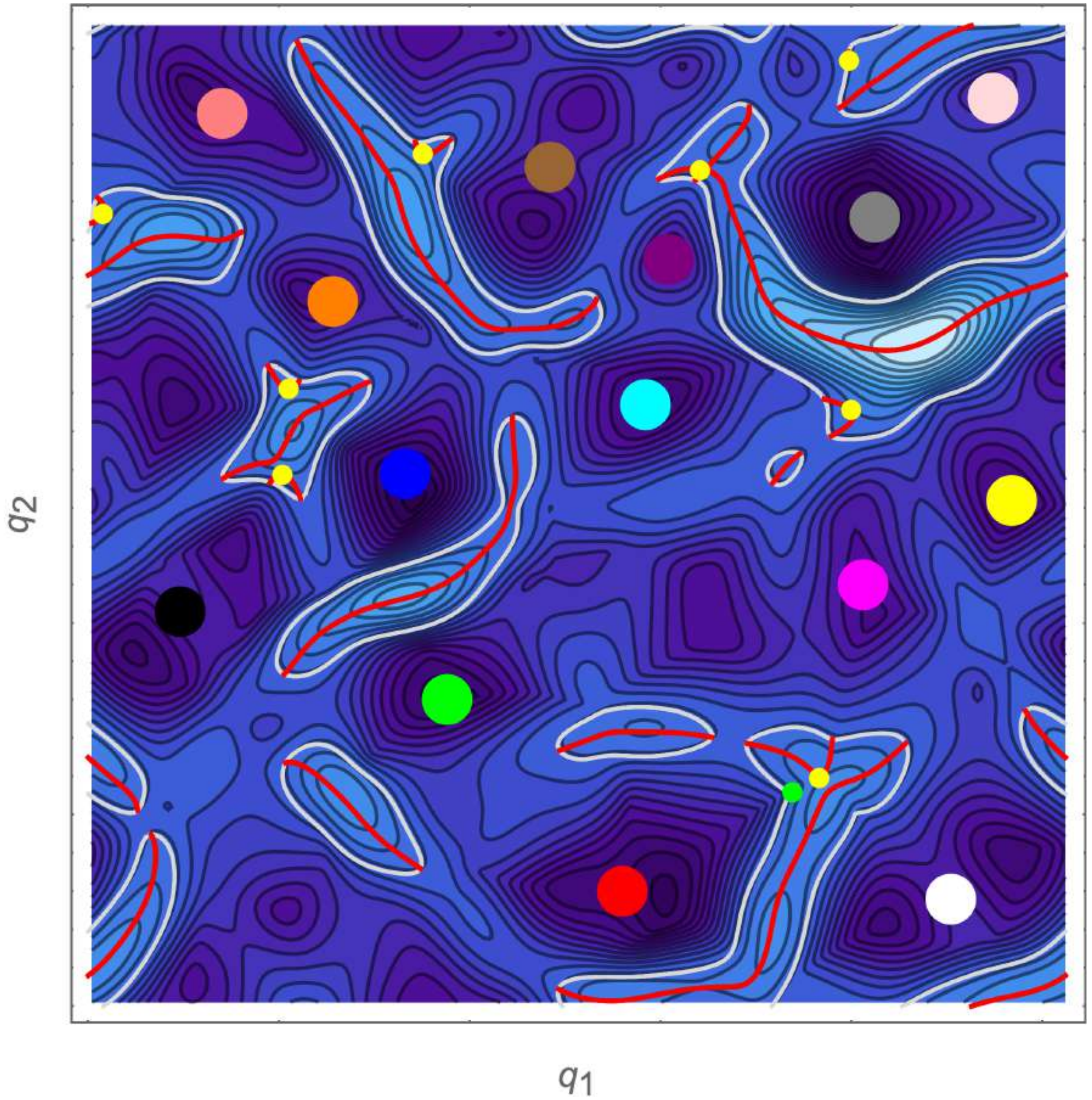
Scotch flow



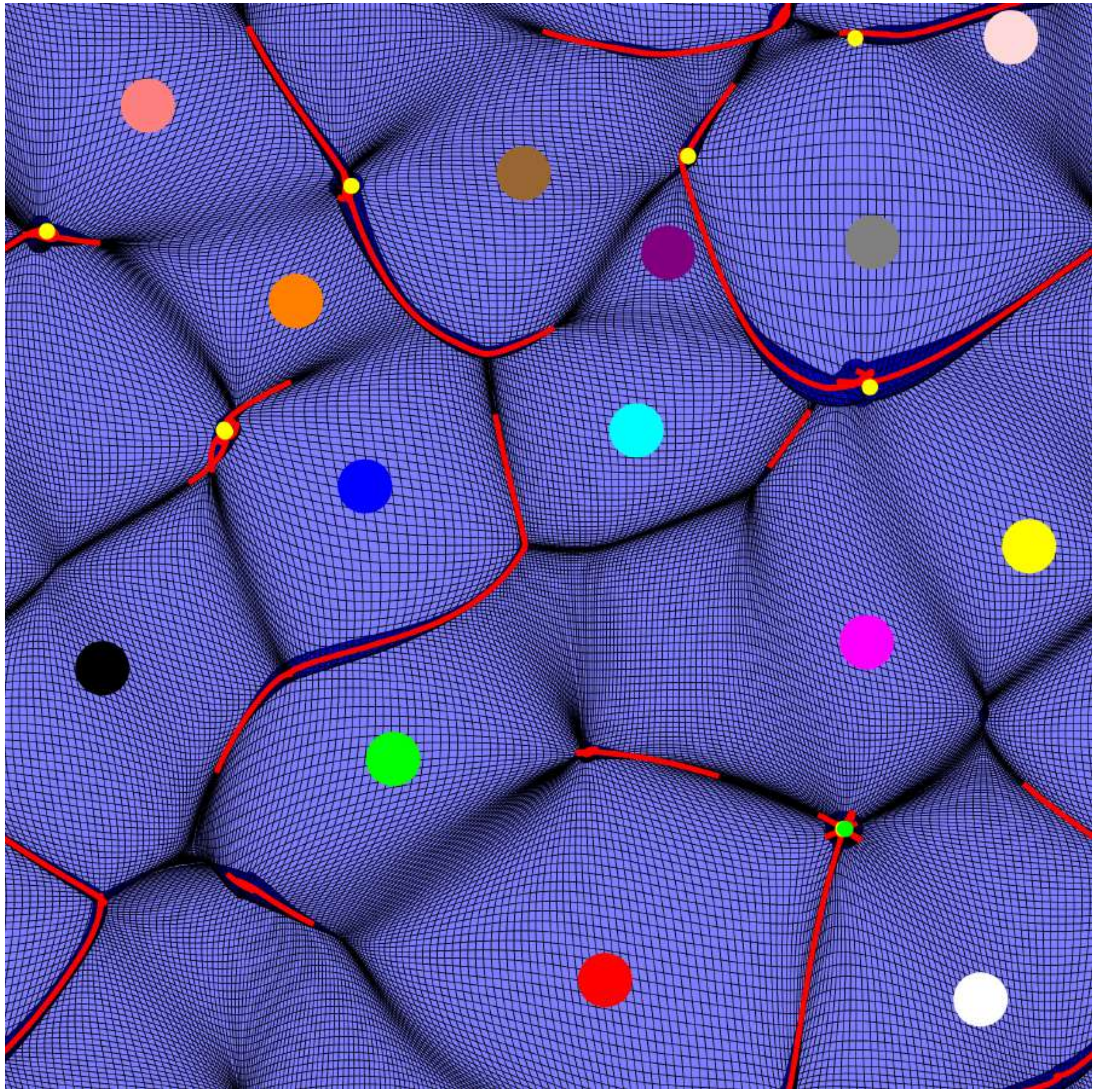
N-body flow



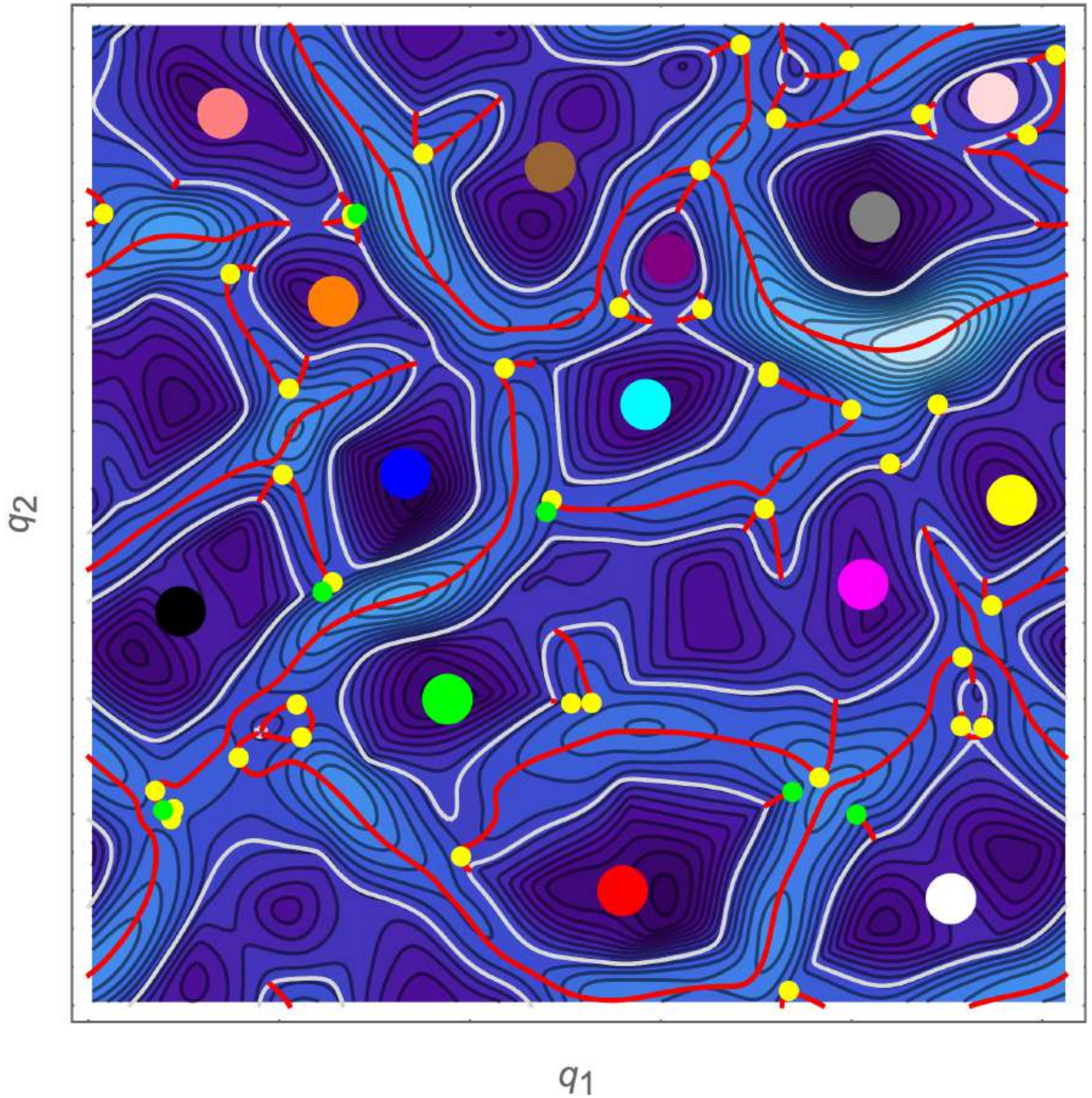
Lagrangian space



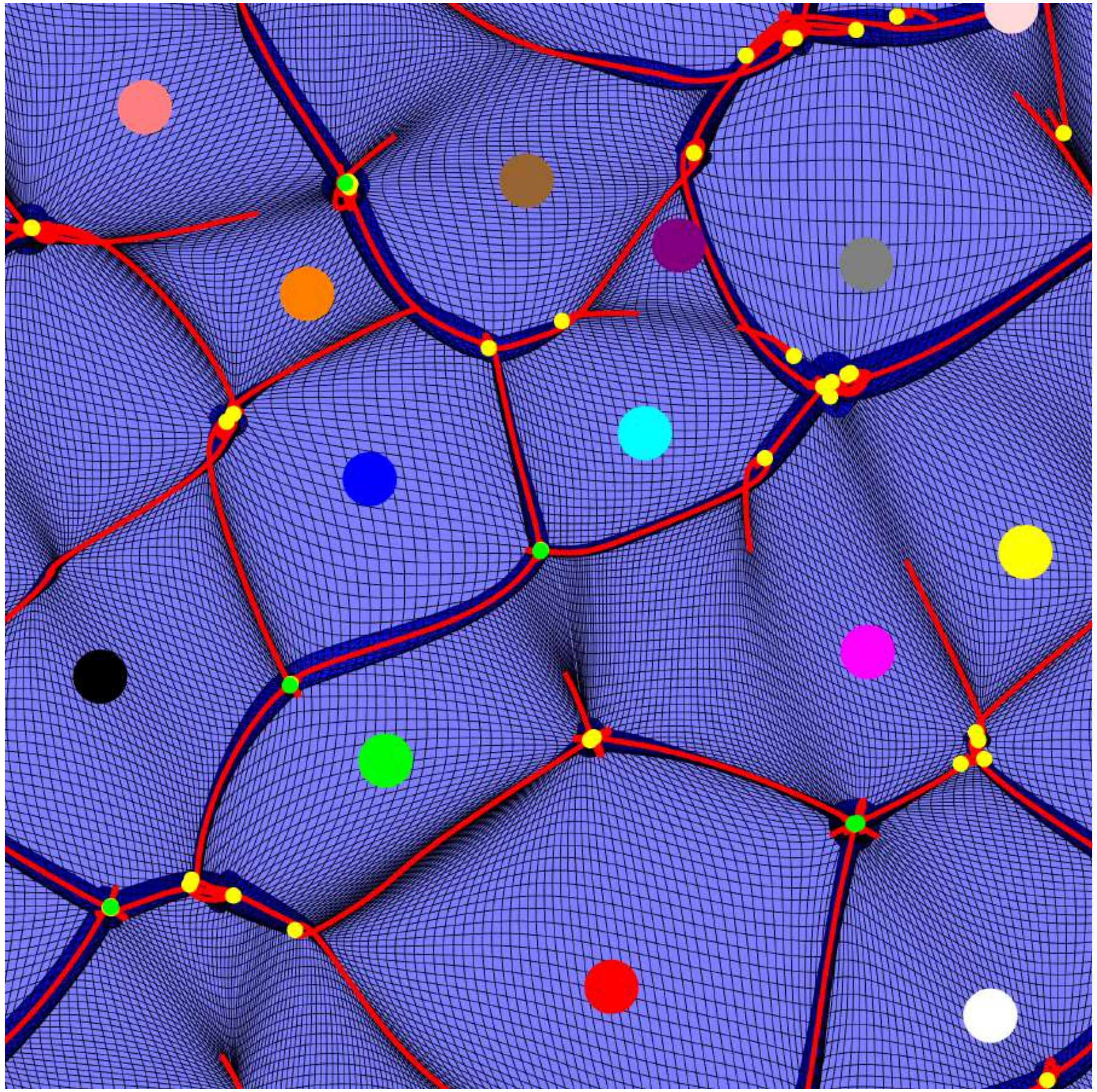
Eulerian space



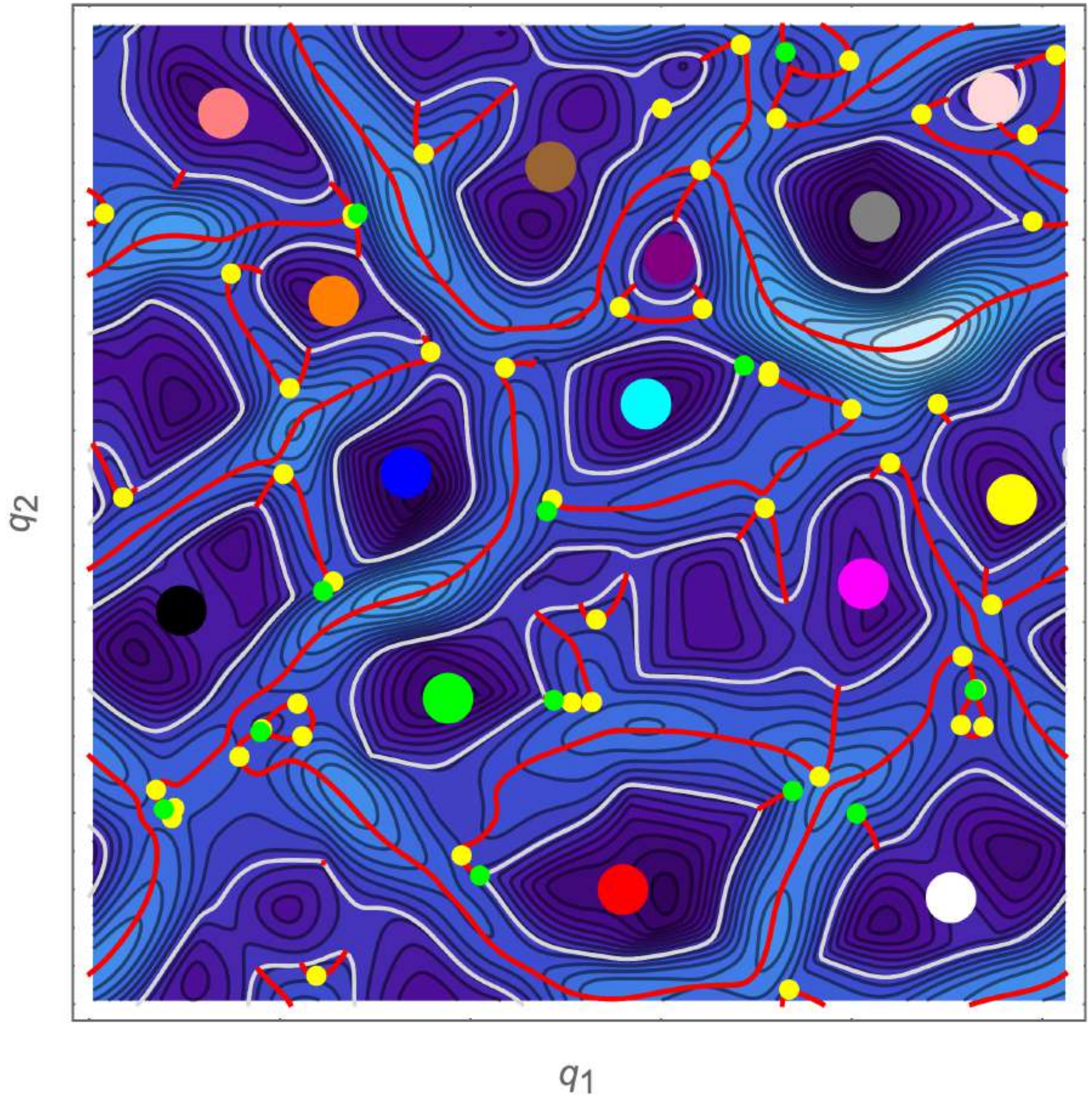
Lagrangian space



Eulerian space



Lagrangian space



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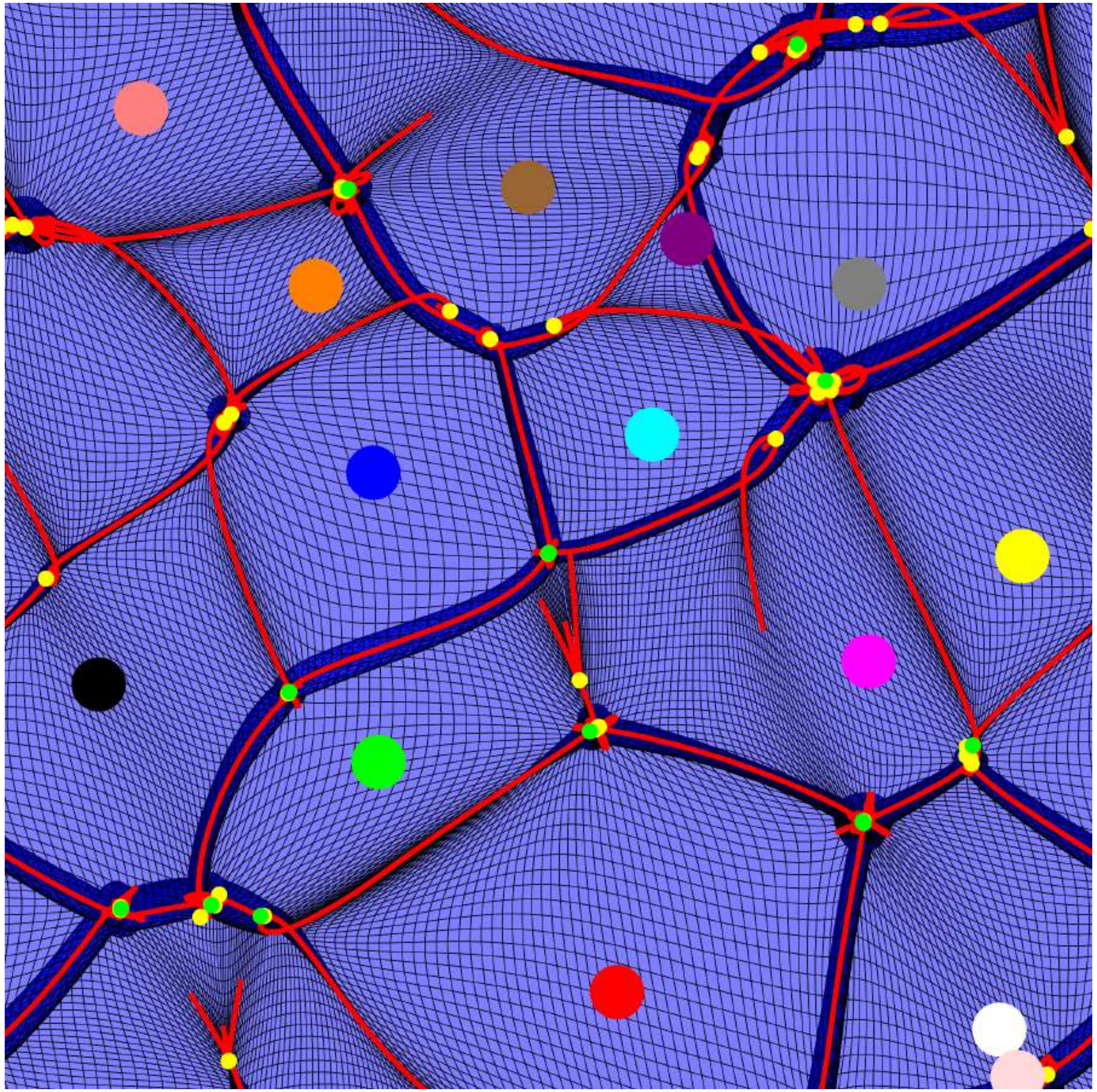
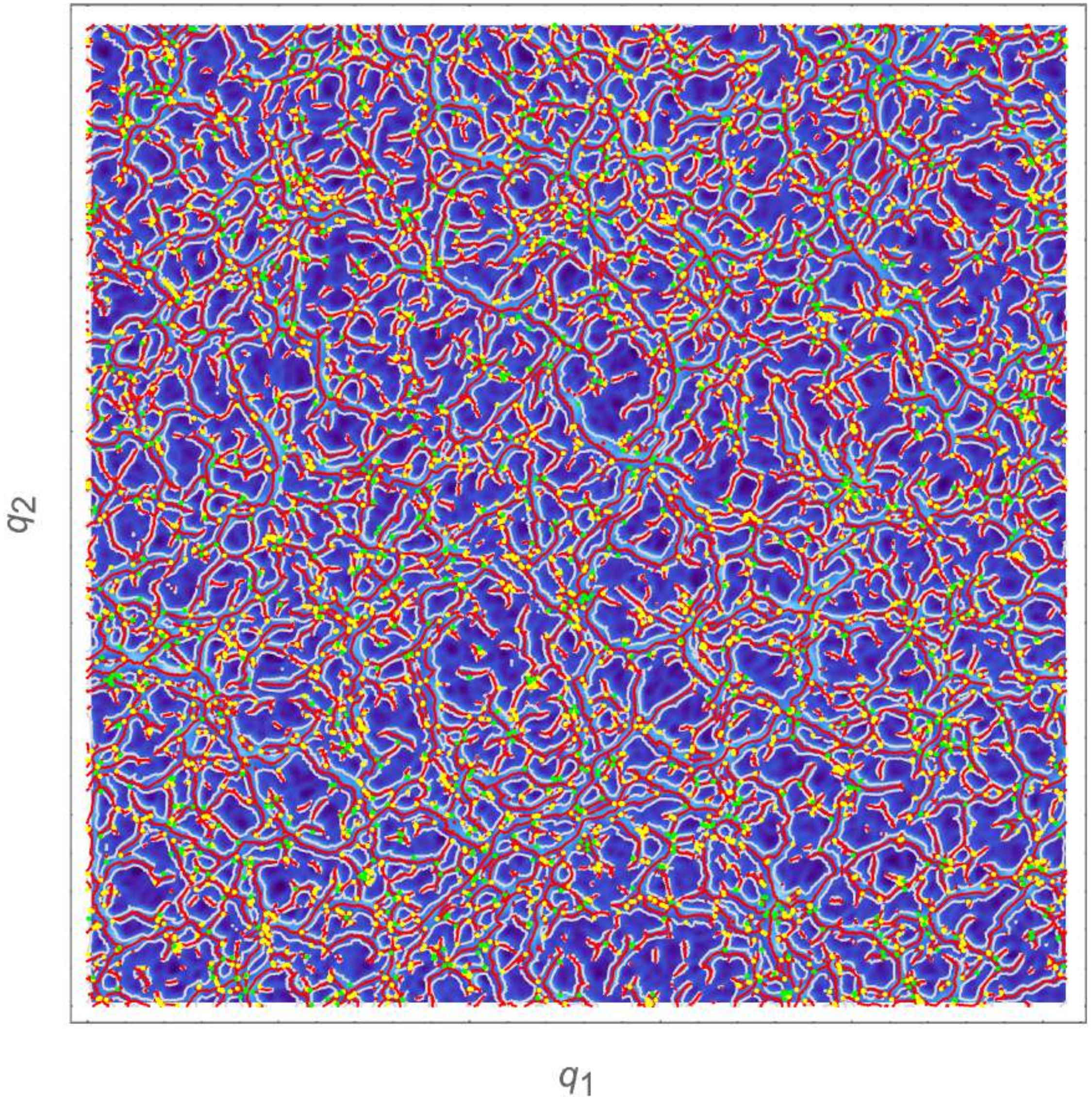


Lagrangian space



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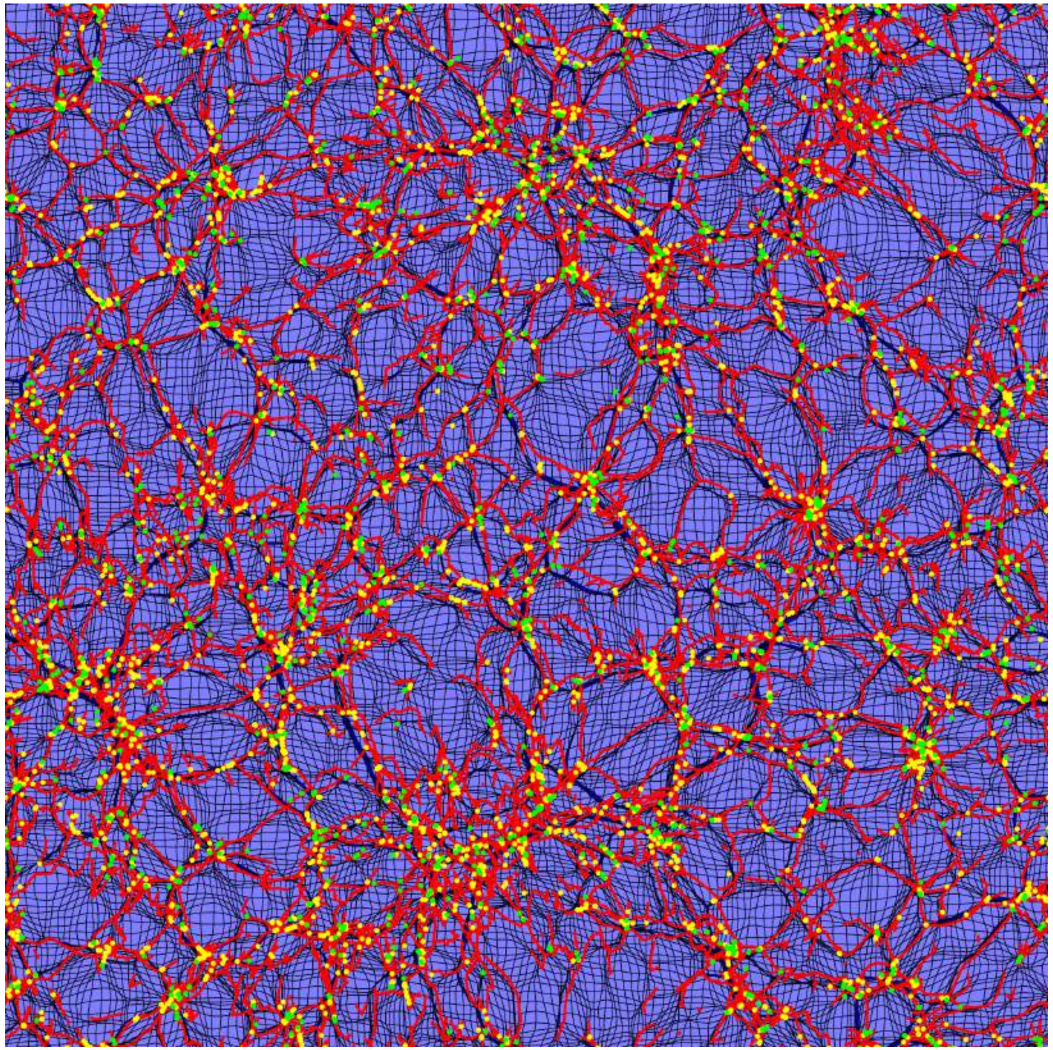
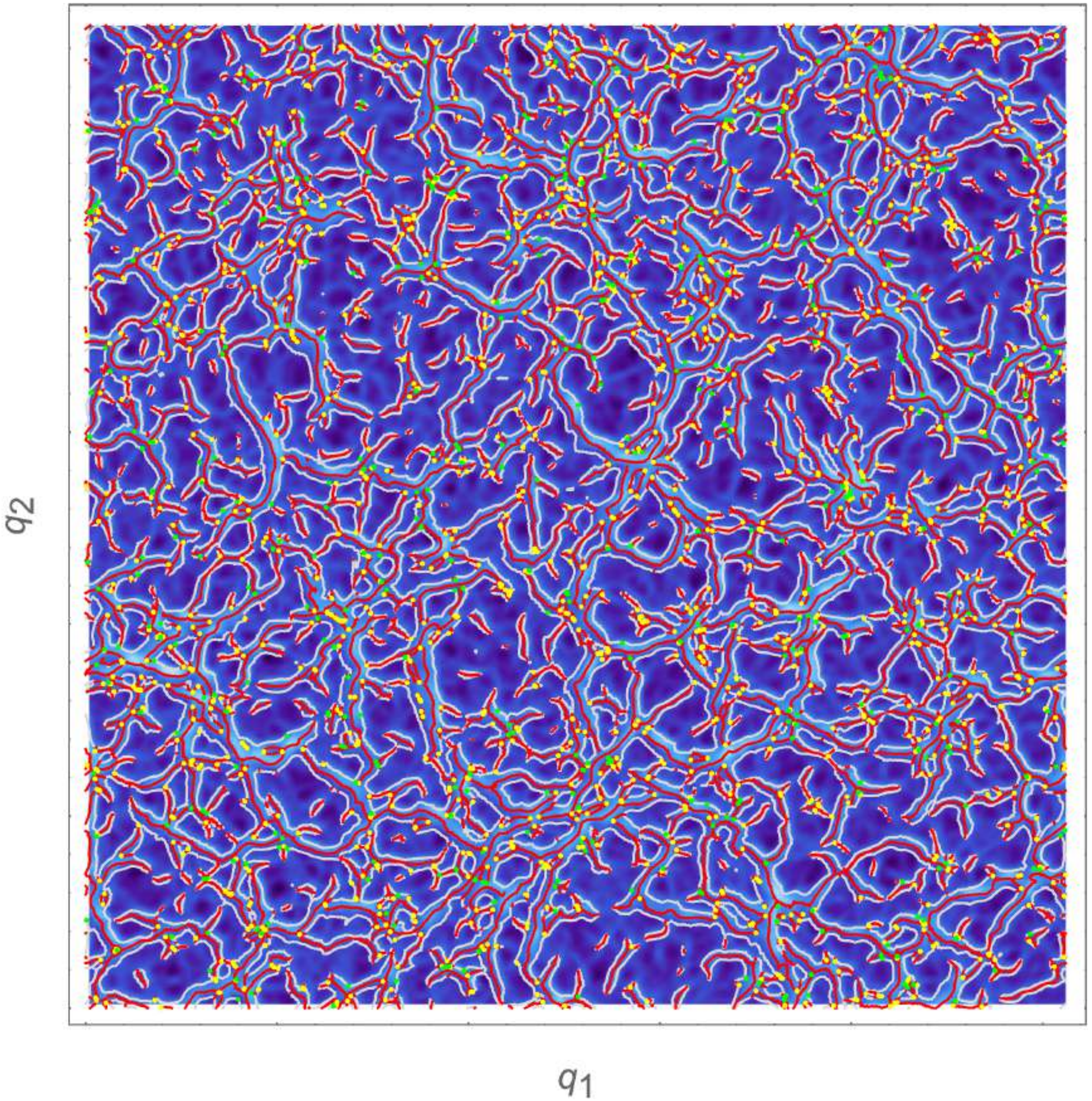


Lagrangian space



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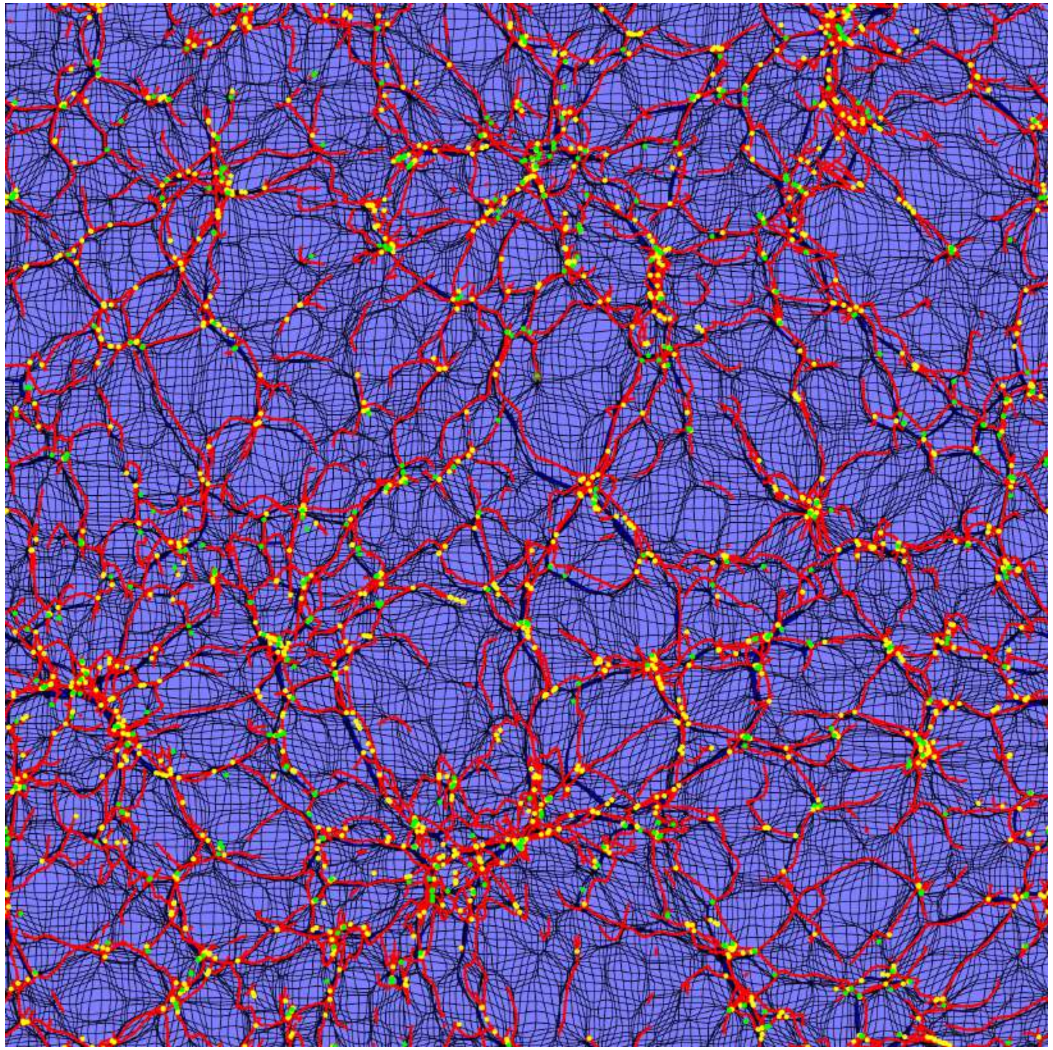
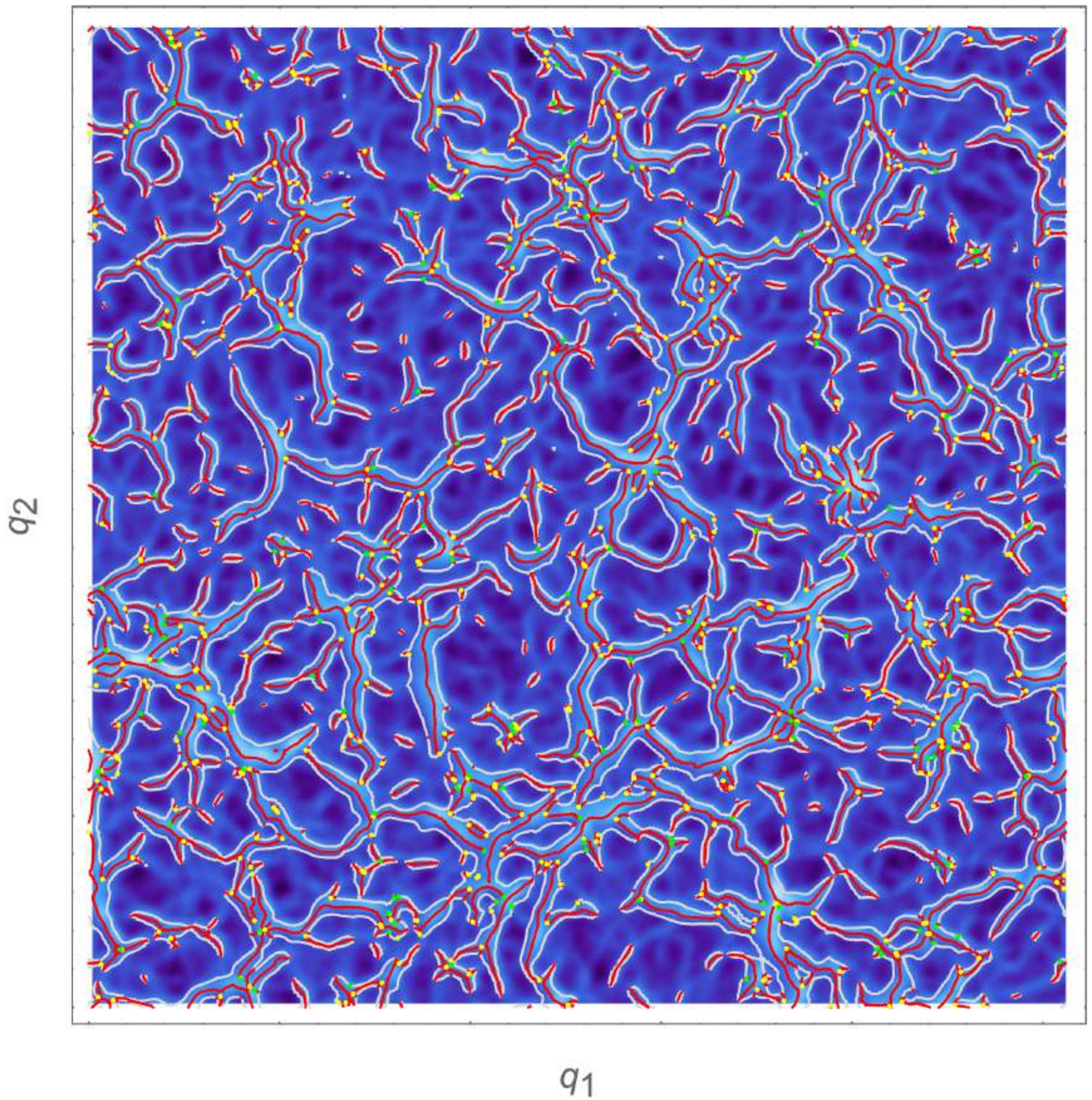


Lagrangian space



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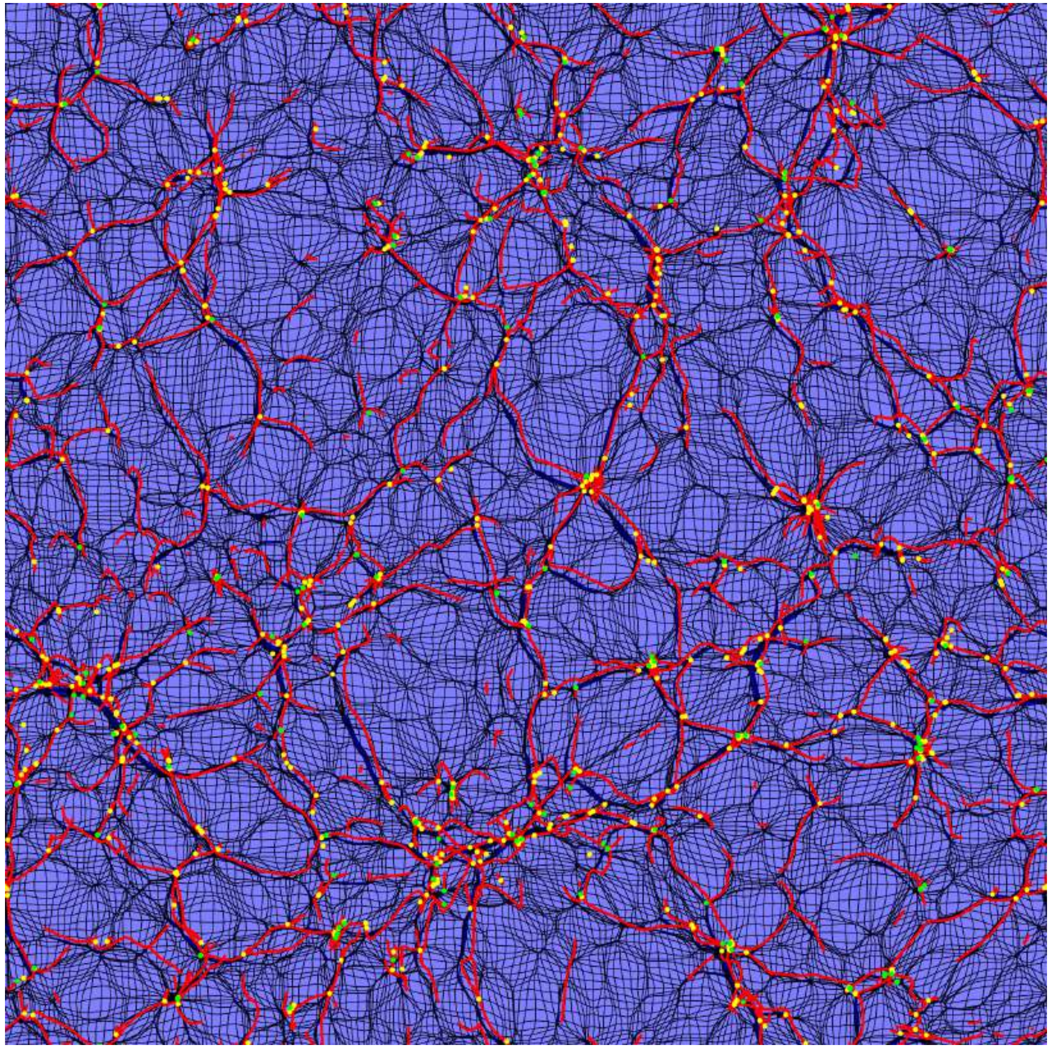
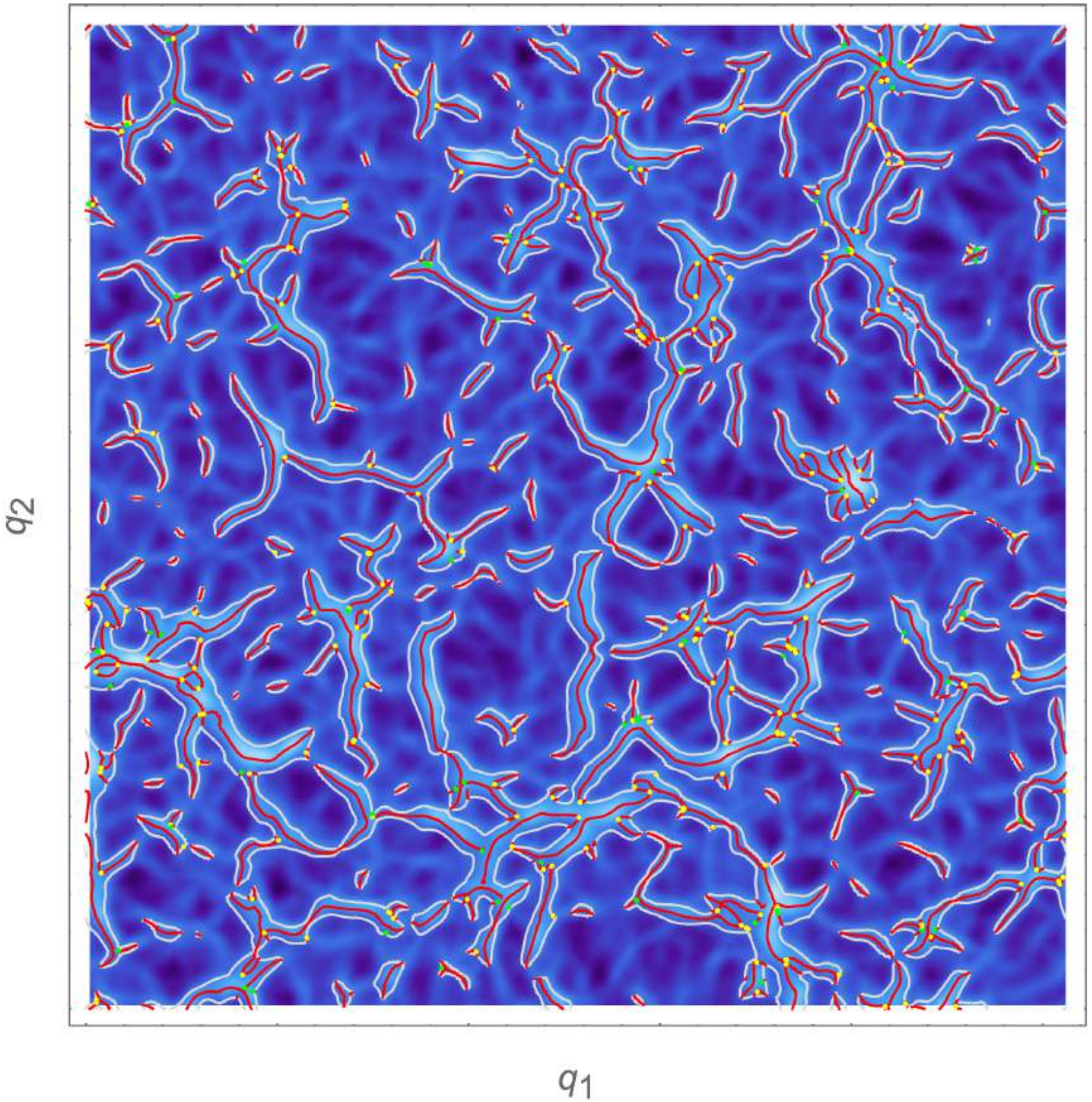
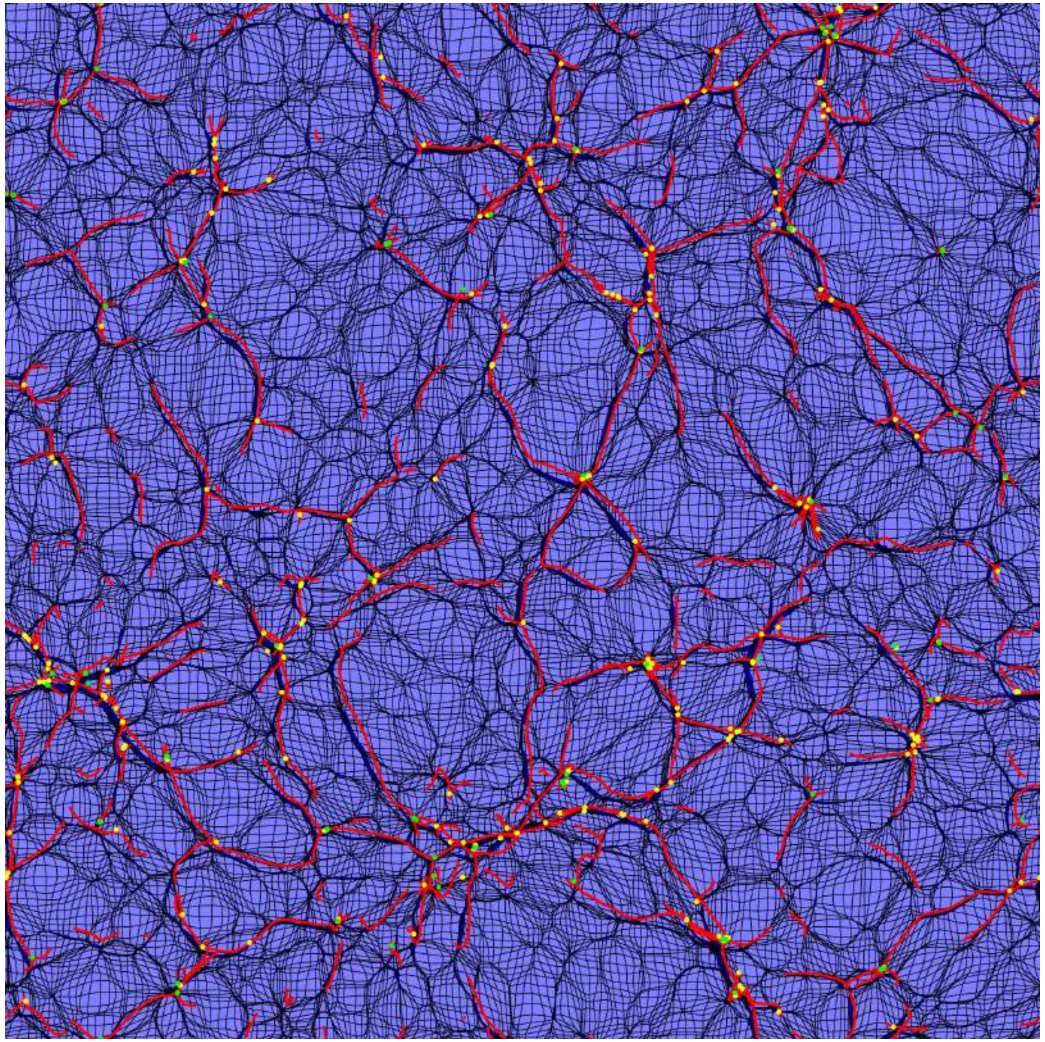


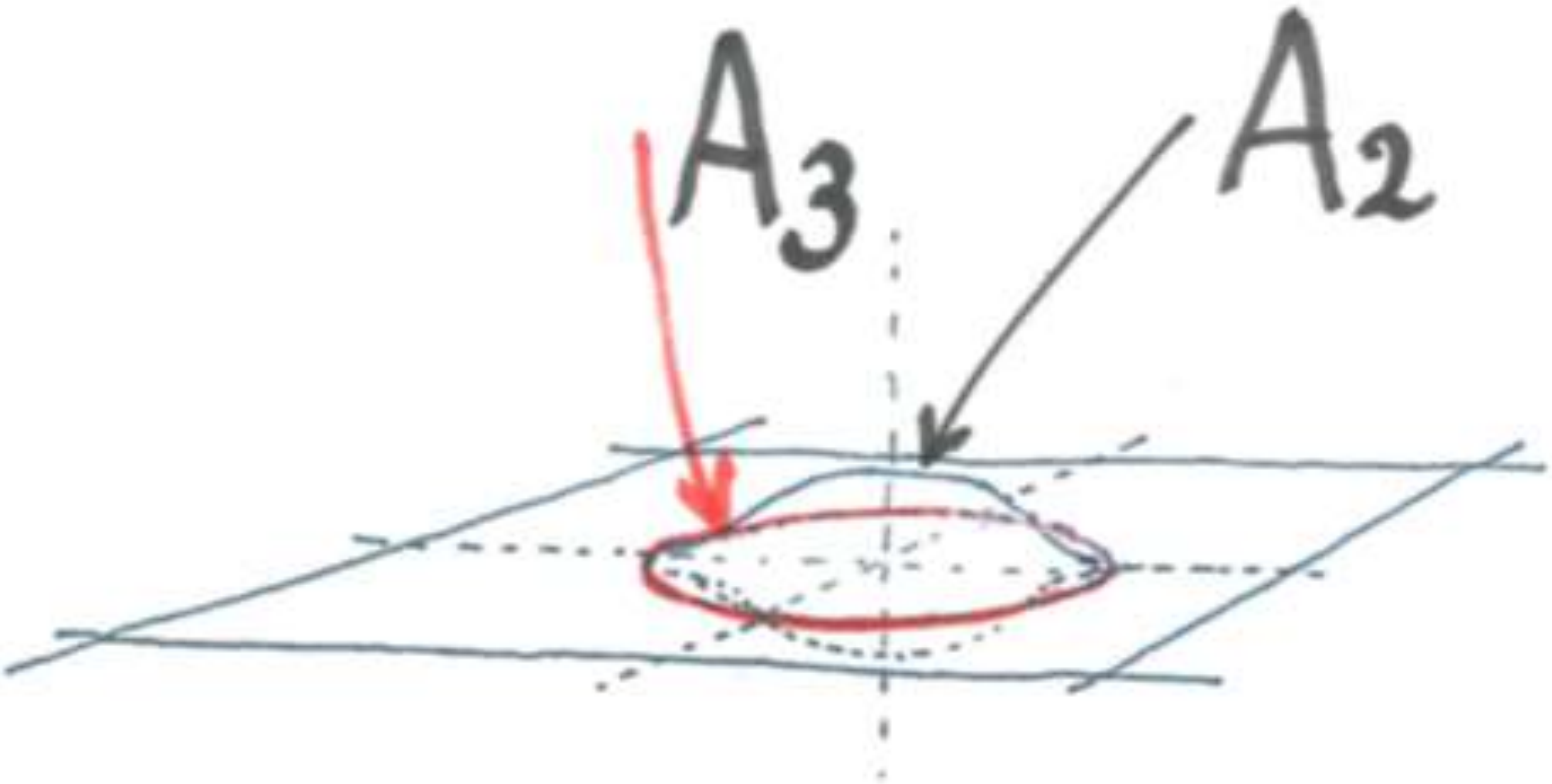
Lagrangian space



Eulerian space



Fold and cusp caustic



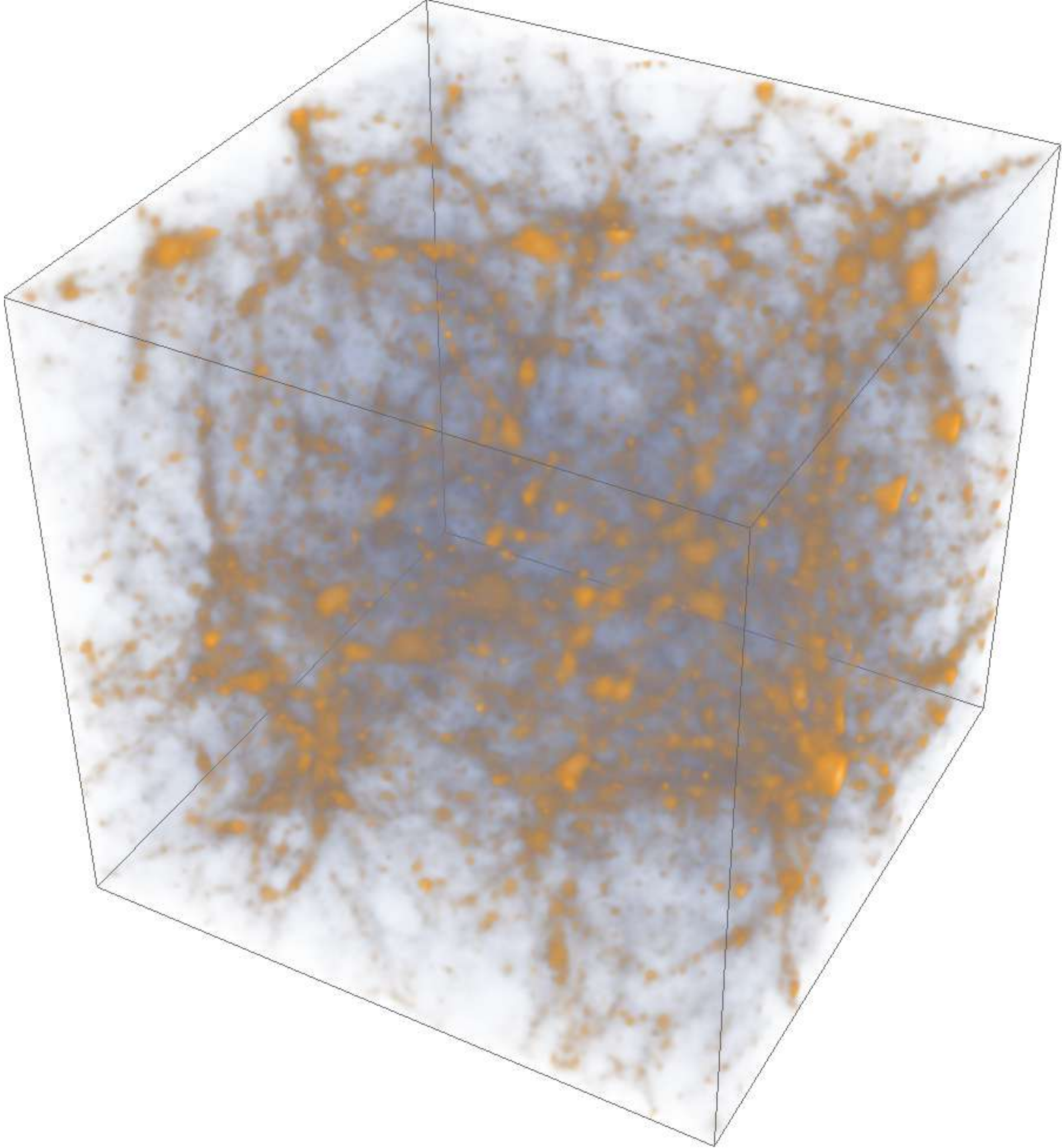
Caustic skeleton

Applied to the Zel'dovich approximation, we obtain the **caustic skeleton** in terms of the initial conditions

$$s_t(q) = x_t(q) - q$$

$$s_t(q) = -b_+(t)\nabla_q \Psi(q)$$

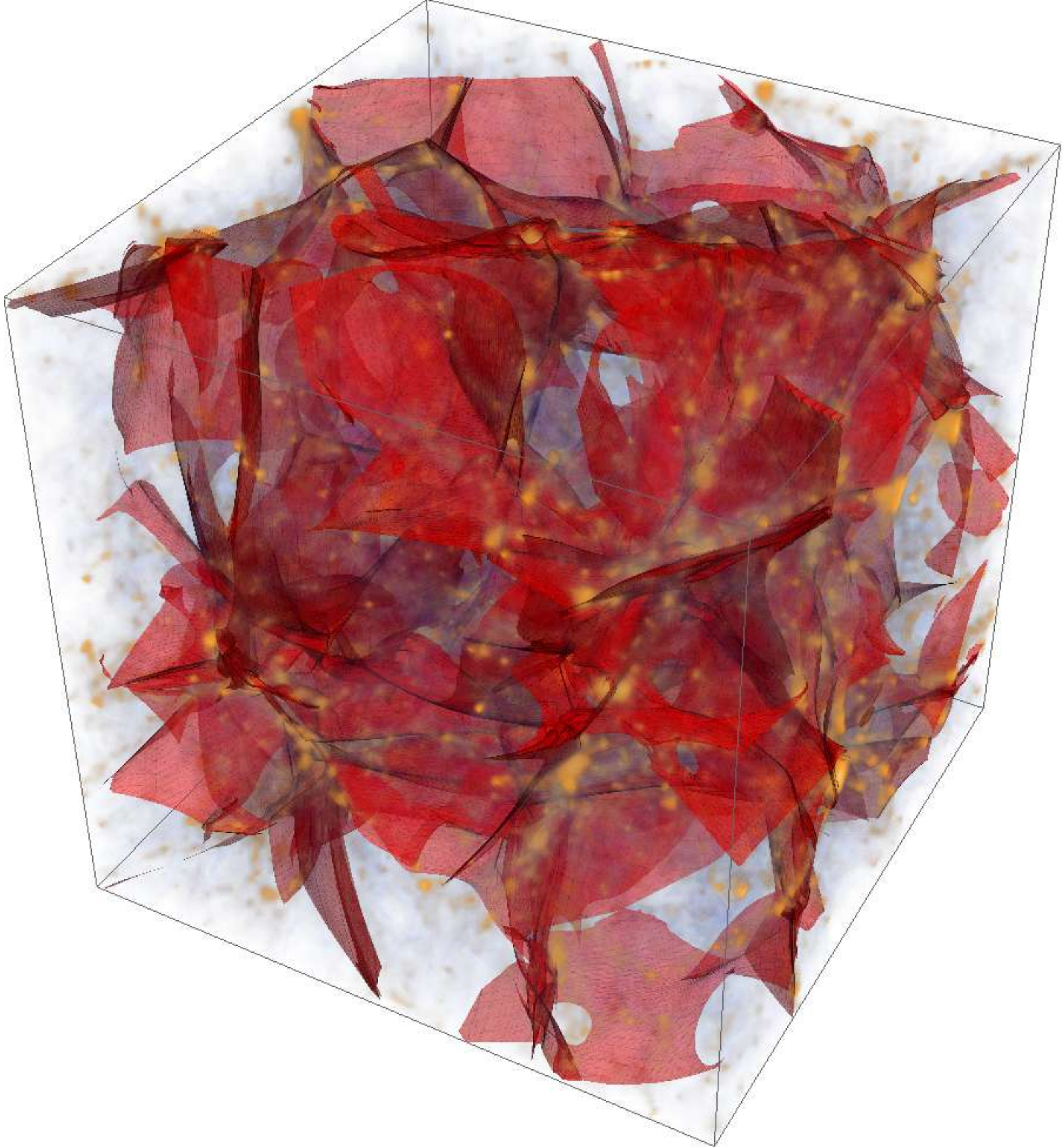
$$\Psi(q) = \frac{2}{3\Omega_0 H_0^2}\phi_0(q)$$



Caustic skeleton

Cusp sheets – walls

The cusp walls correspond to only one eigenvalue field

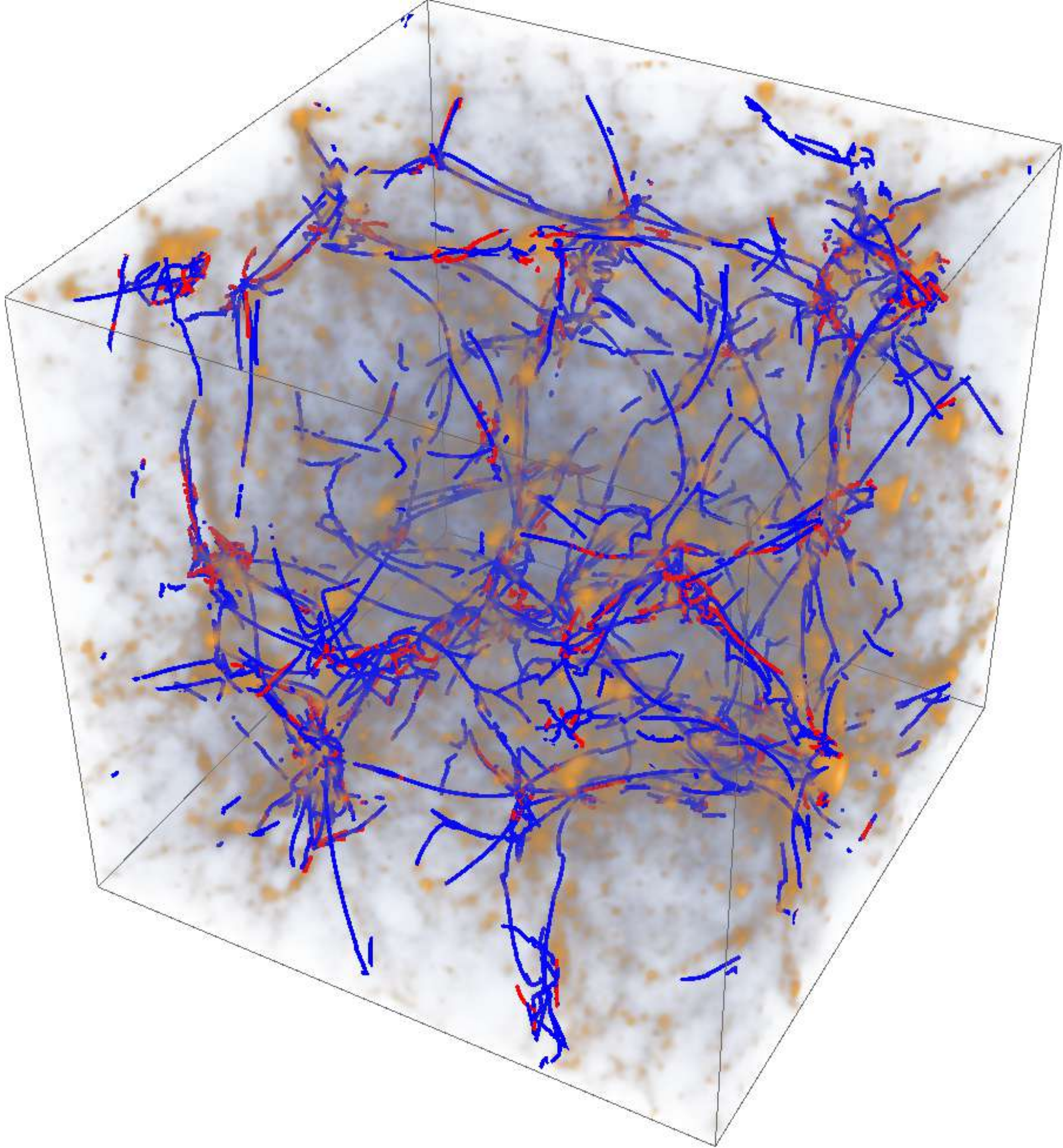


Caustic skeleton

Swallowtail line – filament

Elliptic/hyperbolic – filament

The swallowtail filaments correspond to only one eigenvalue field

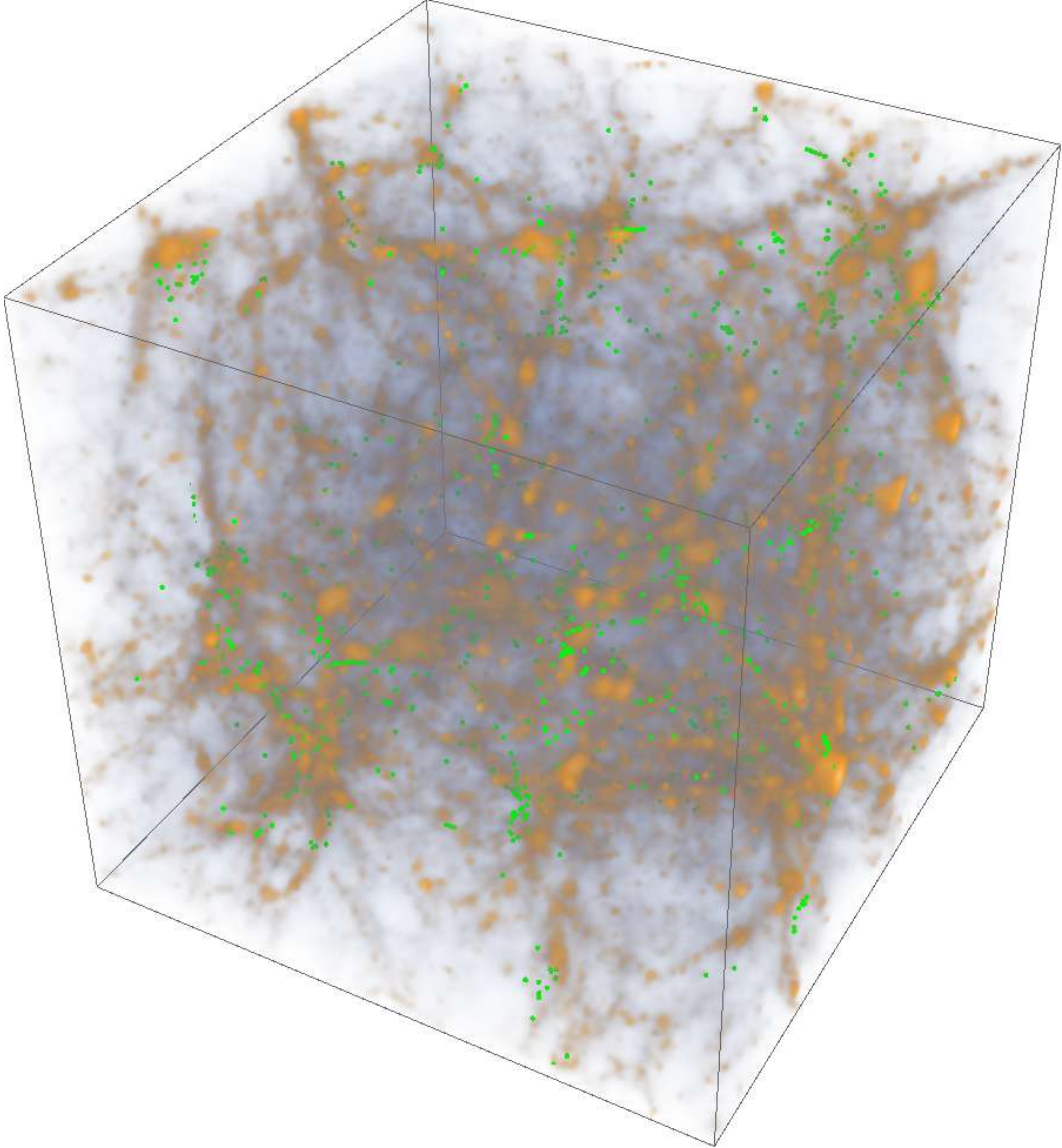


Caustic skeleton

Butterfly – cluster

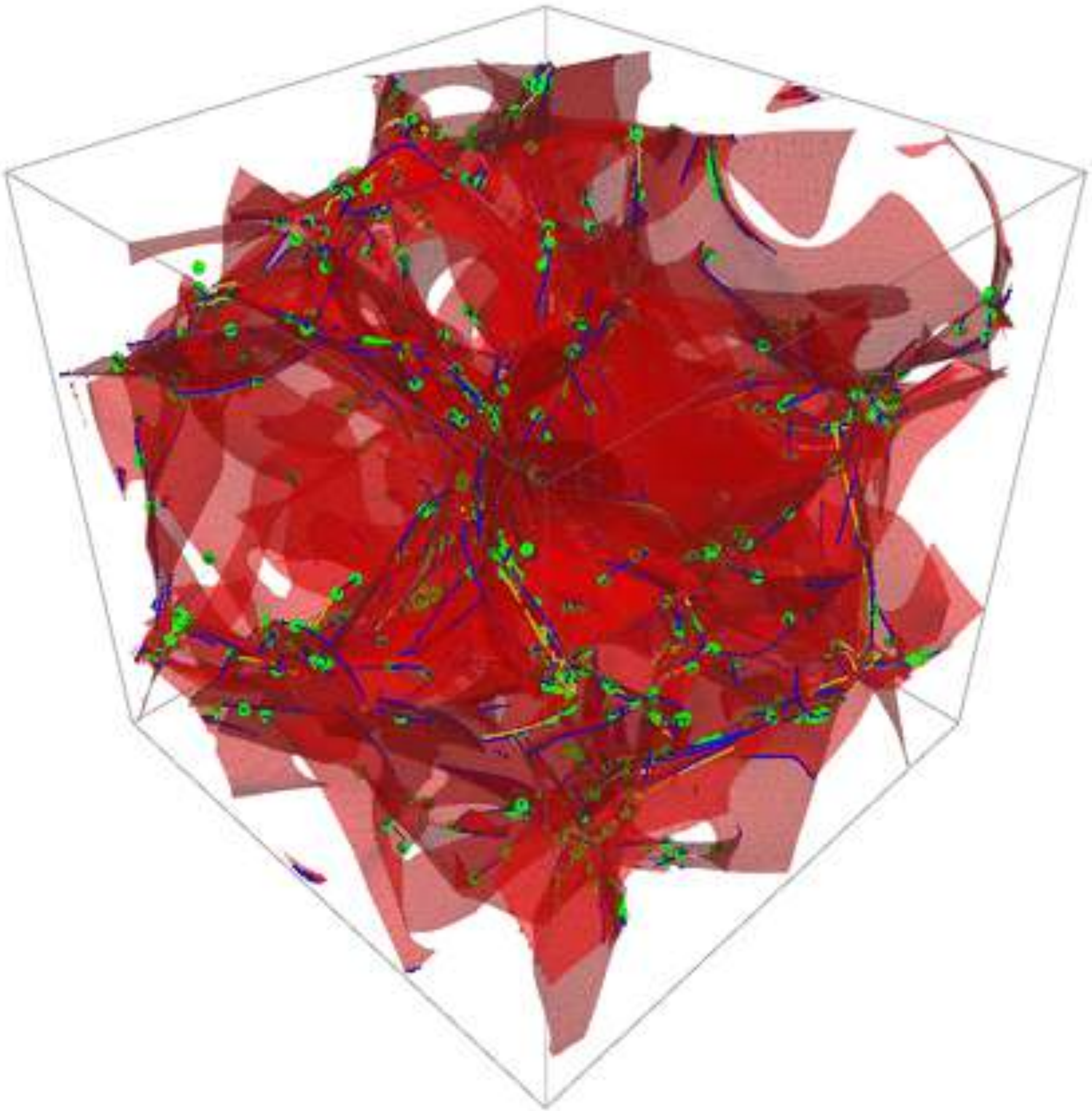
Parabolic – cluster

The butterfly clusters correspond to only one eigenvalue field

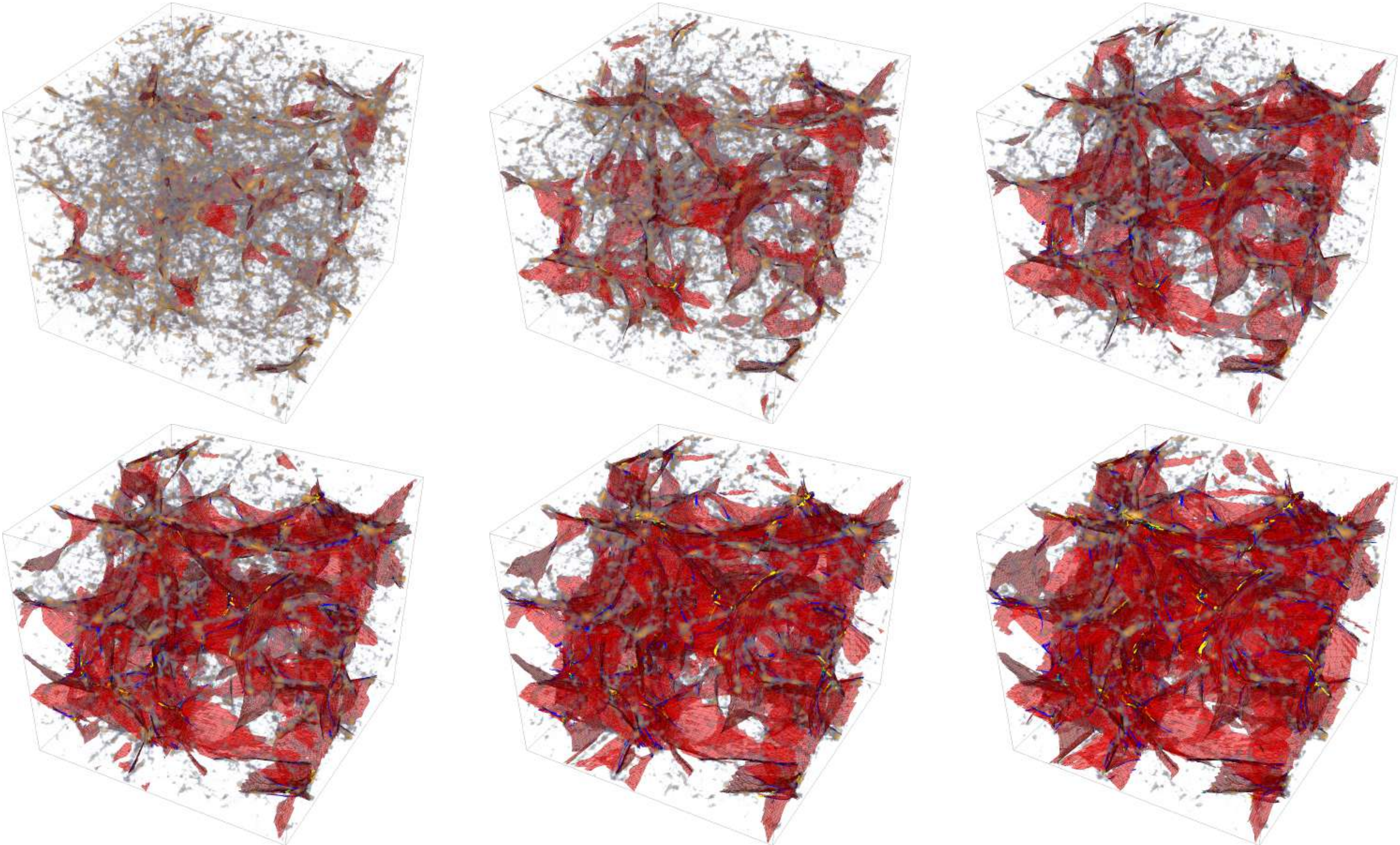


Caustic skeleton

The caustic skeleton accurately follows the large-scale geometry of the N-body simulation.



The evolving Cosmic skeleton

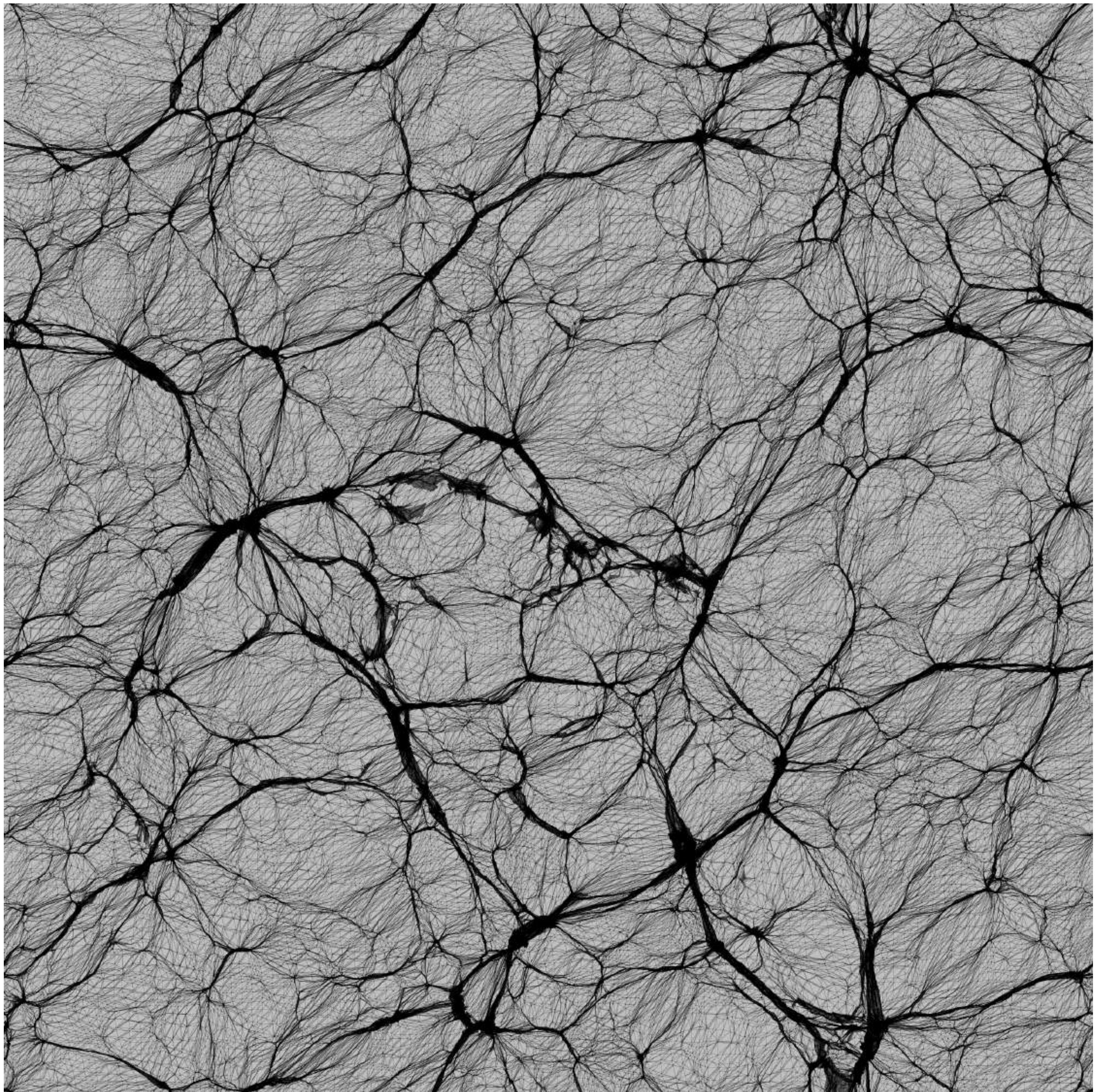


The flip-flop field

The accuracy of the caustic skeleton can be studied in more detail by comparing it with dark matter N-body simulations.

The flip-flop field of 3D N-body simulation used by the comparison project Libeskind et al. (2018)

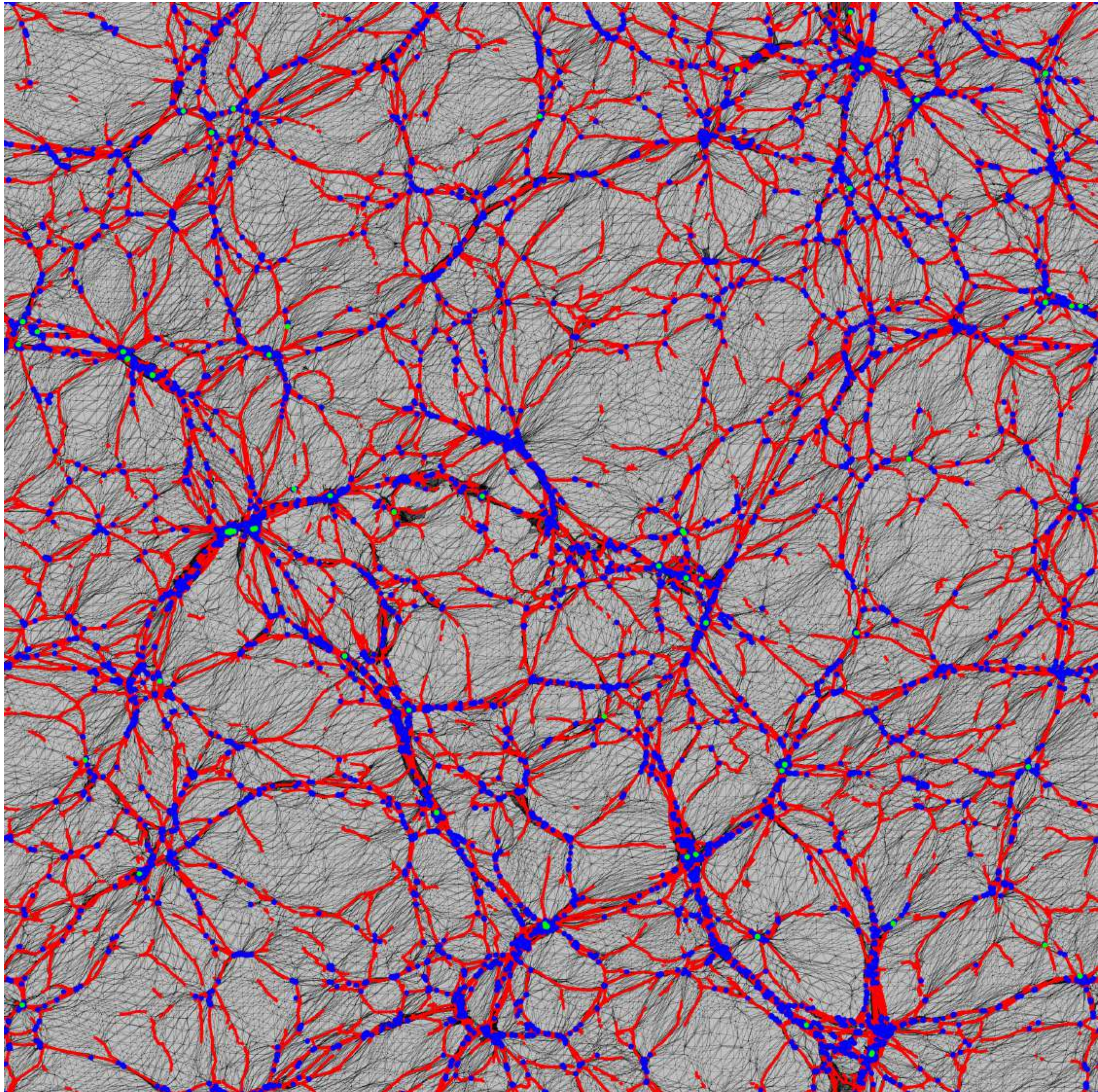
Abel, Oliver, Ralf (2012),
Shandarin, Medvedev (2016, 2017)



The flip-flop field

Gaussian smoothing of initial conditions:

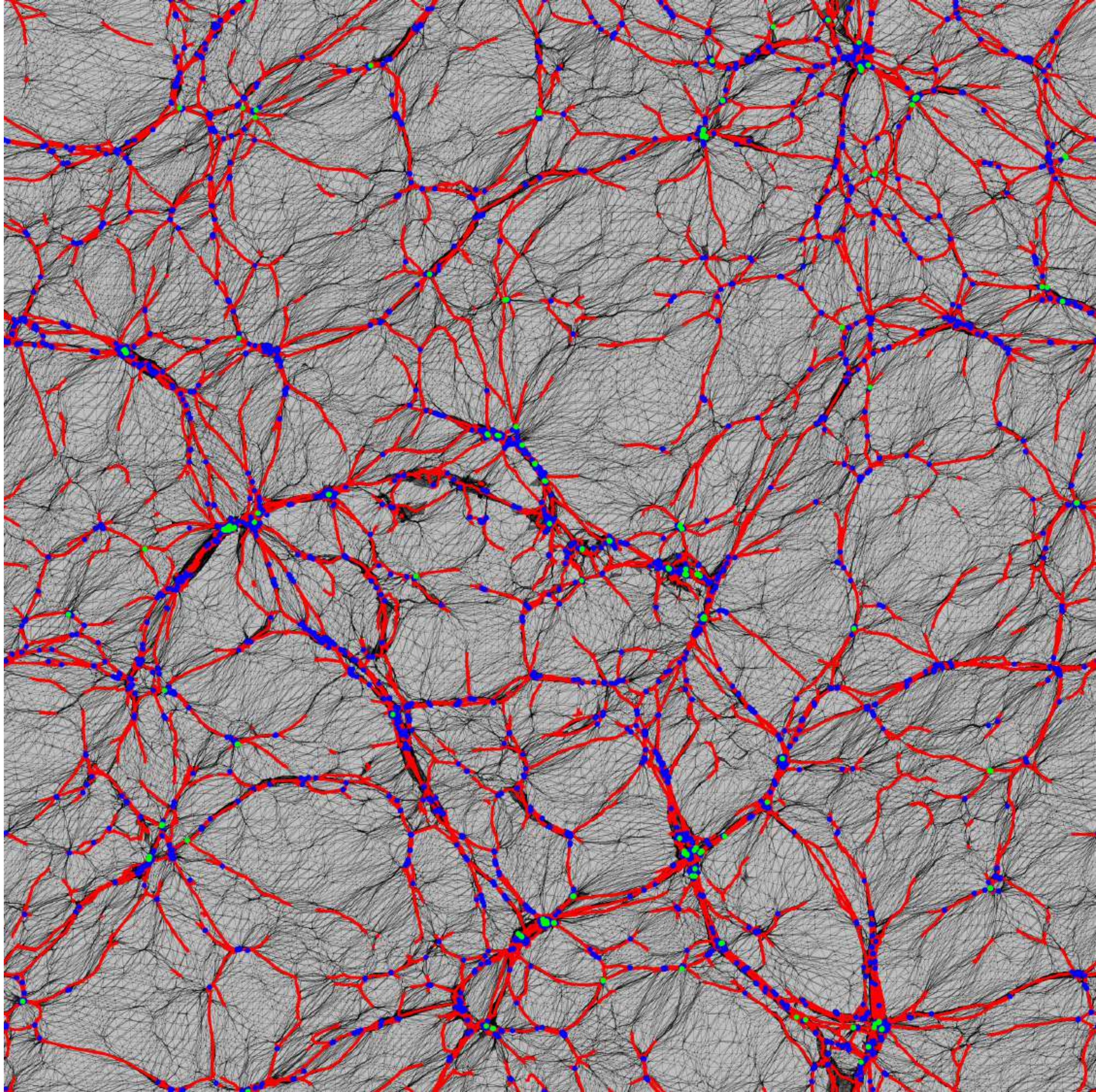
$\sigma = 0.78\, h^{-1} Mpc$



The flip-flop field

Gaussian smoothing of initial conditions:

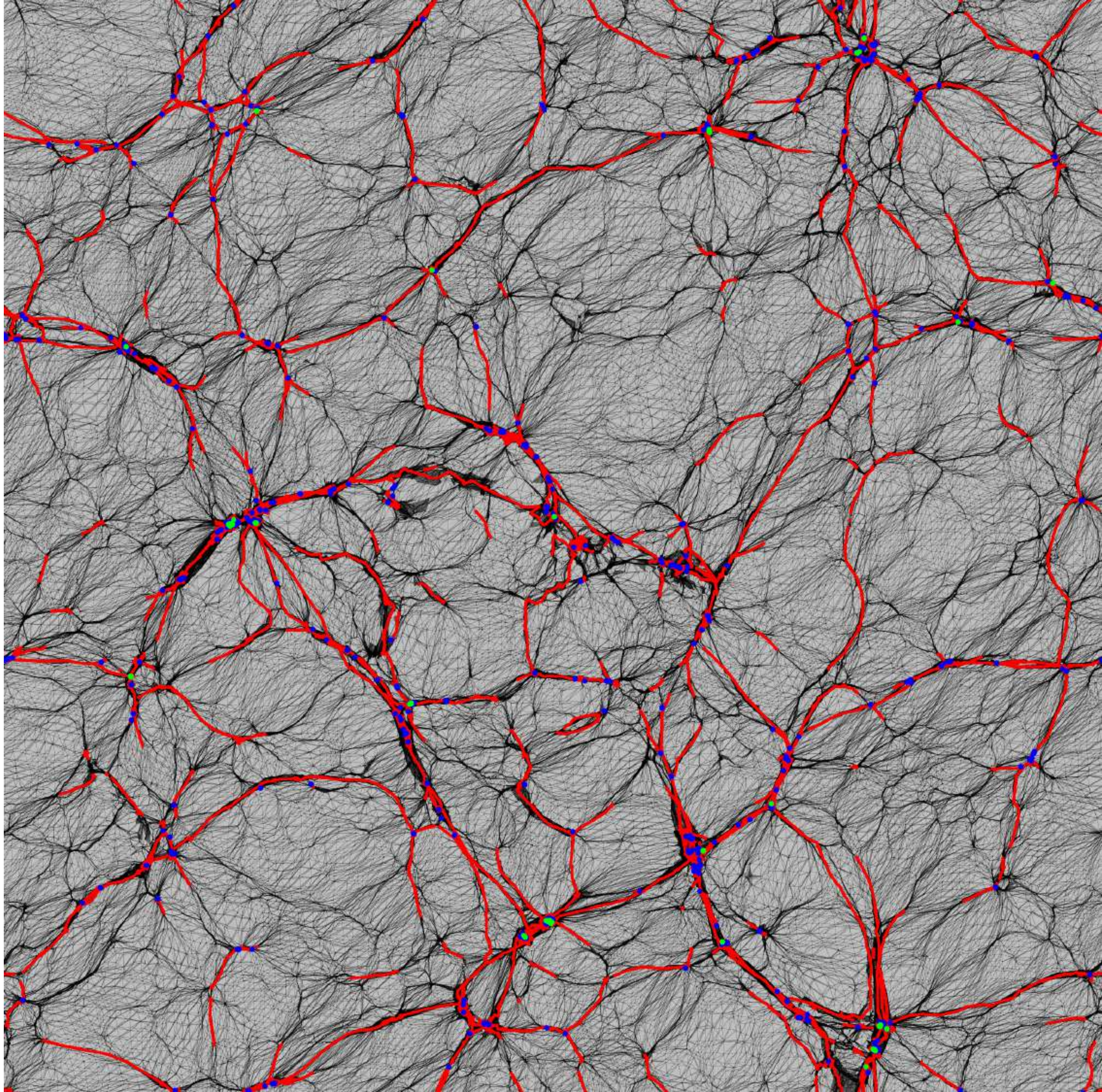
$\sigma = 1.6\, h^{-1} Mpc$



The flip-flop field

Gaussian smoothing of initial conditions:

$\sigma = 3.1\, h^{-1} Mpc$



Summary

- **Shell-crossing condition** enables us to derive **caustic conditions in 3D**
- **Caustic skeleton of cosmic web** depends on the **eigenvalue** and **eigenvector fields**
- **Filaments and walls do not require multiple shell-crossings**
- **Two types of filaments**
- **Caustic skeleton** closely resembles **N-body simulations** without **free parameters**
- What **are the statistical properties** of the skeleton in comparison to N-body?
- Caustic web of the **local universe** with **constraint simulations.**