Analytic calculation of power spectrum covariance:

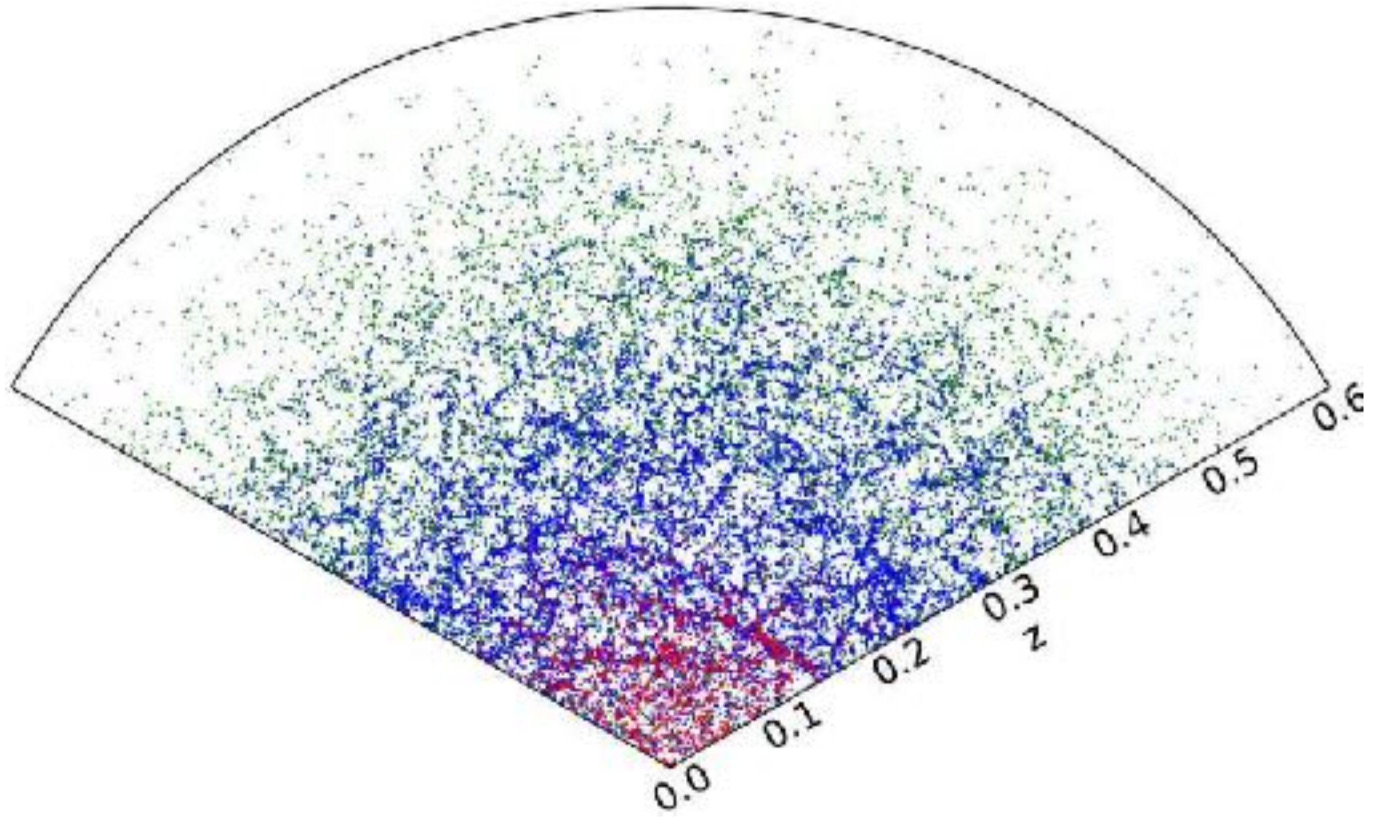
speedup by four orders of magnitude

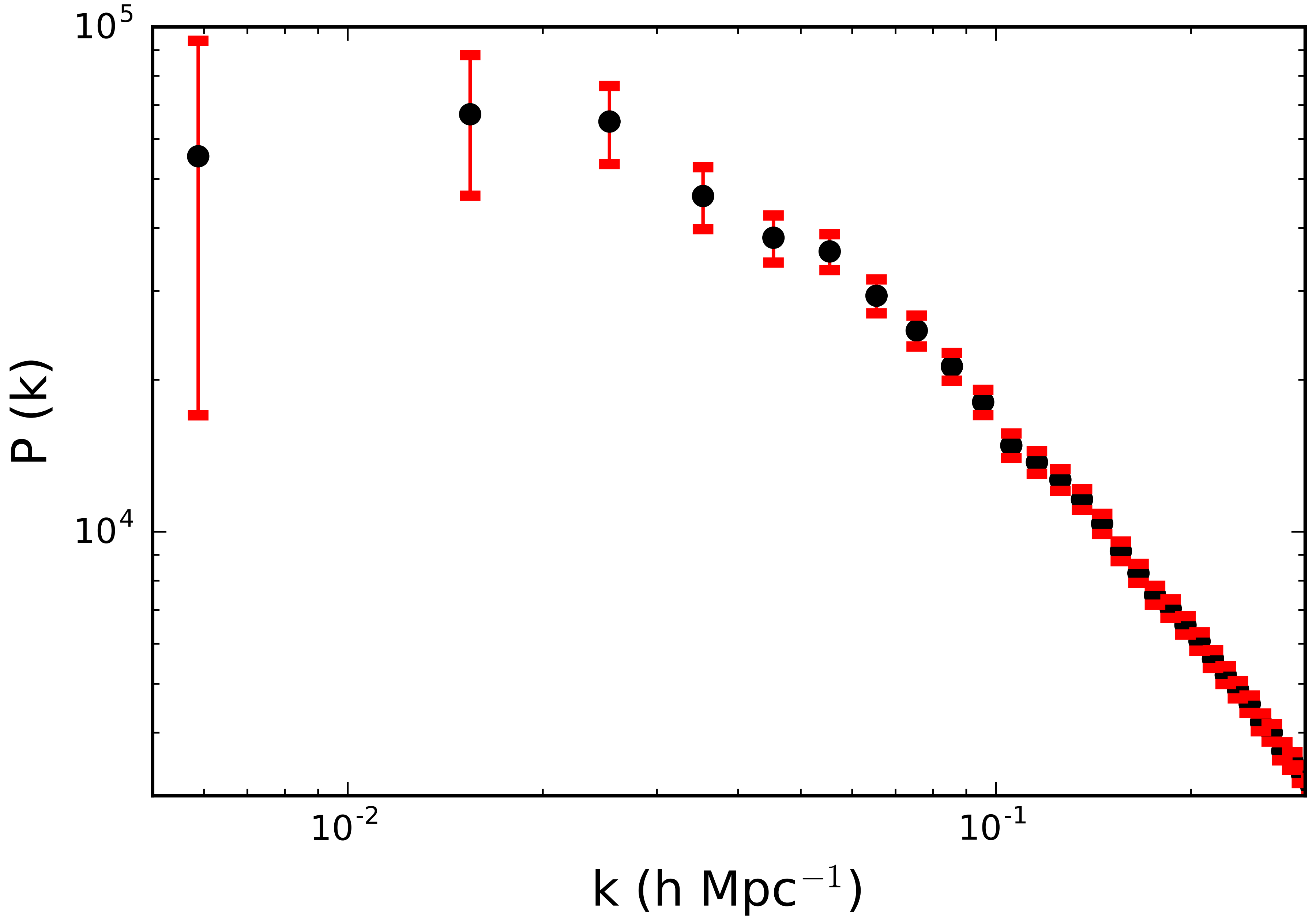
(Jay) Digvijay Wadekar

New York University

DW, Roman Scoccimarro (arXiv 1910.02914)
DW, Misha Ivanov, Roman Scoccimarro (aXiv:2008.xxxx)

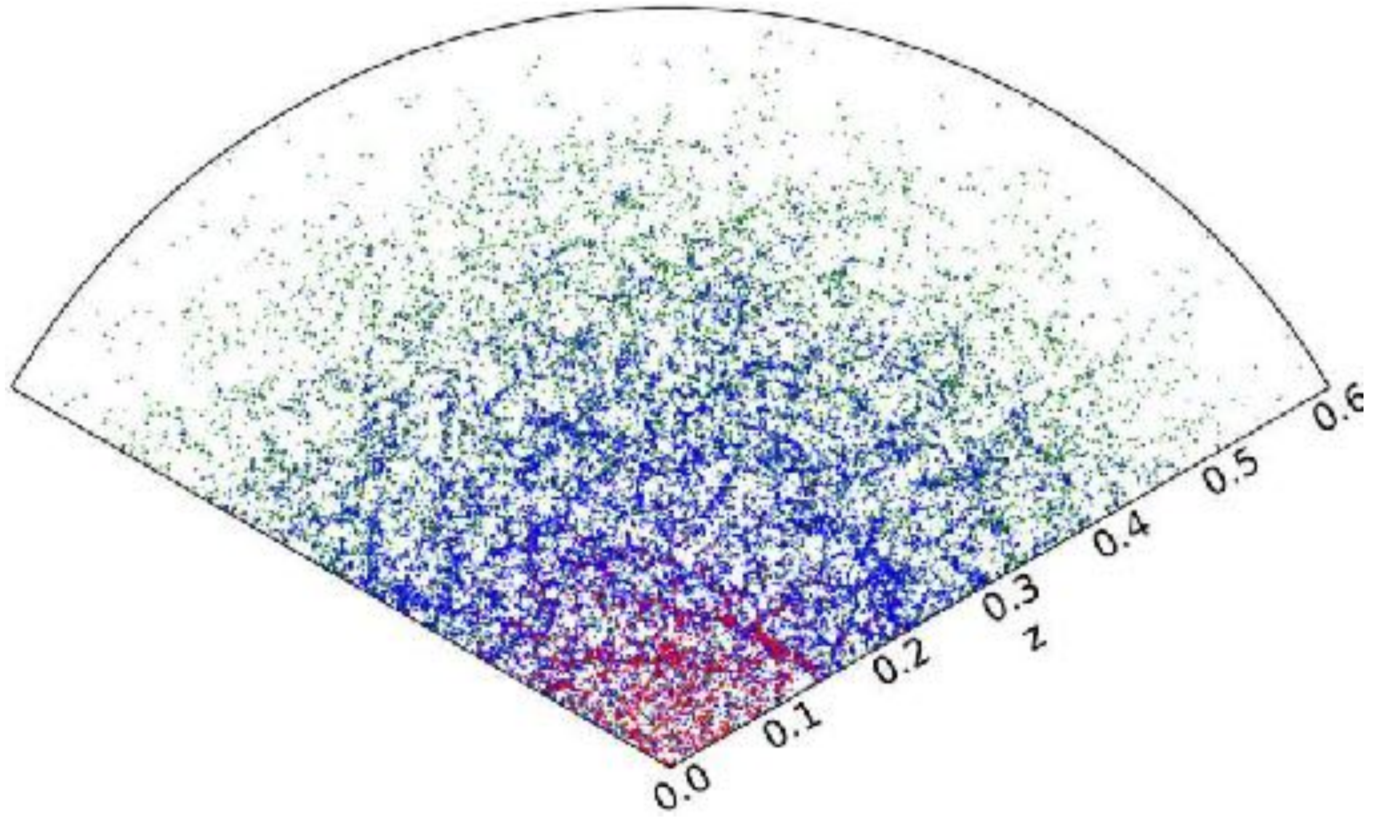
Galaxy power spectrum covariance

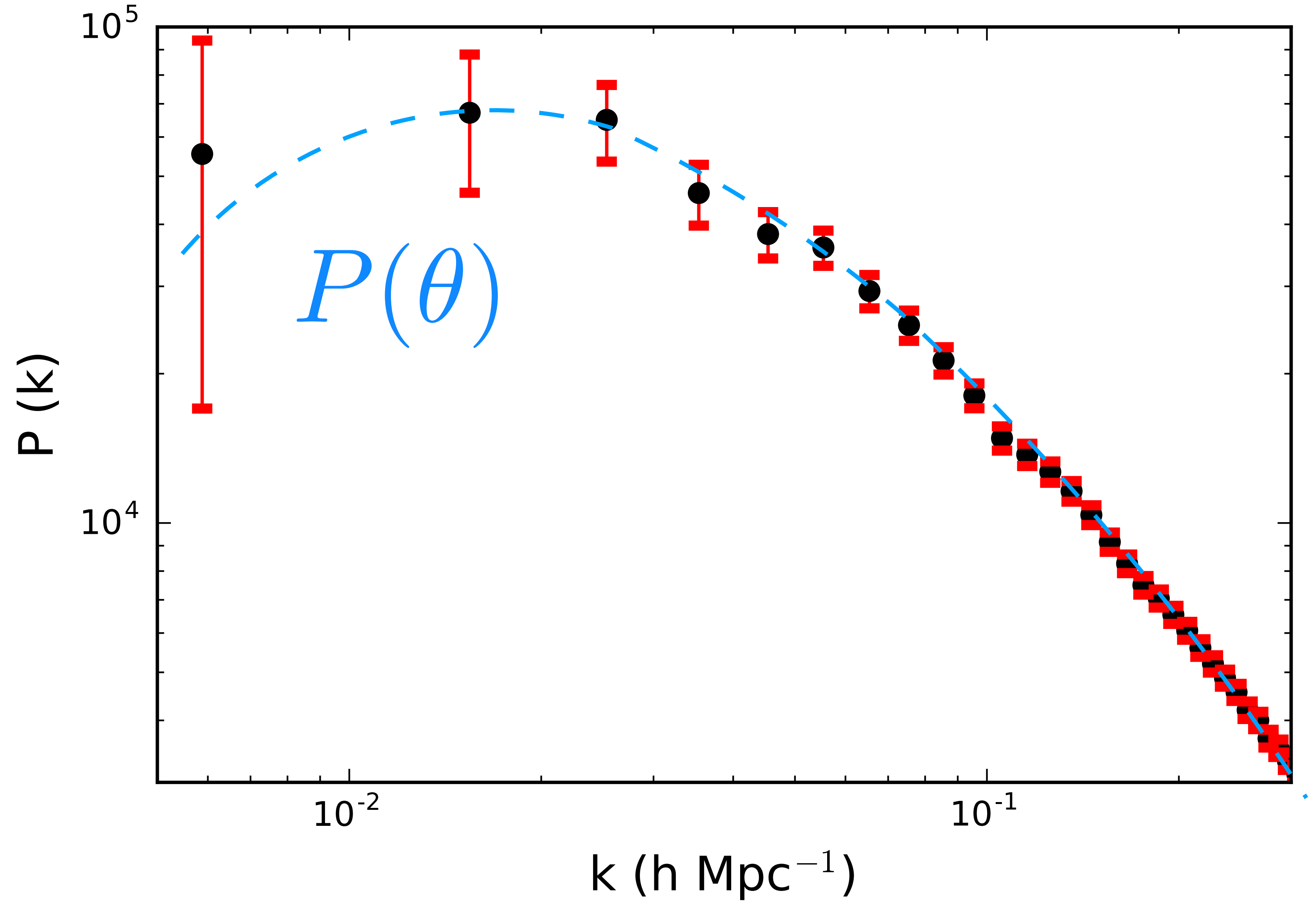




$$\left\{ \begin{matrix} H_0 \\ \Omega_m \\ \sigma_8 \\ b_1 \\ \dots \end{matrix} \right\}$$

Galaxy power spectrum covariance





$$\left\{ \begin{array}{c} H_0 \\ \Omega_m \\ \sigma_8 \\ b_1 \\ \ldots \end{array} \right\}$$

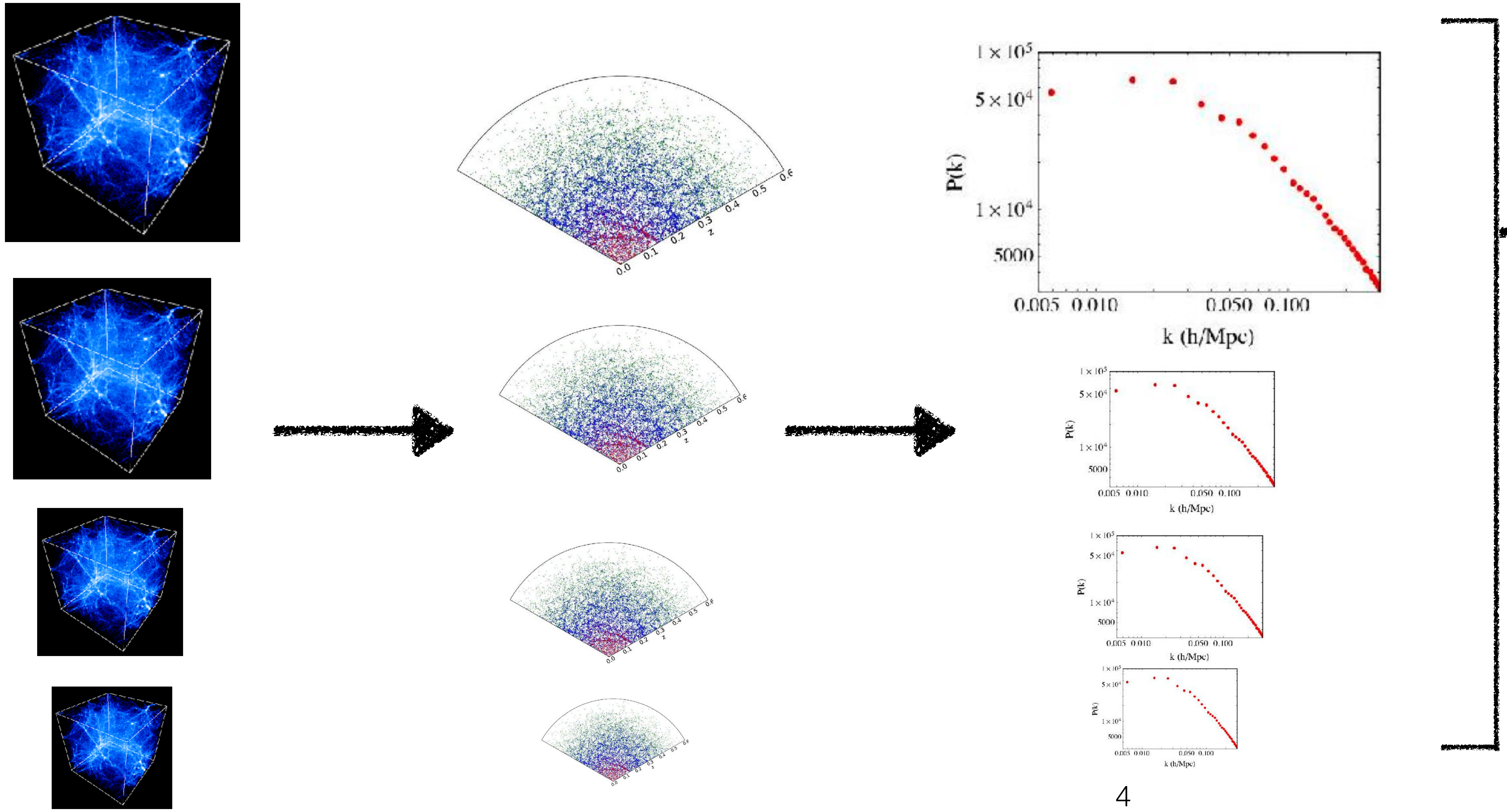
$$p(P_d|\theta) = \frac{1}{(2\pi)^{n/2}\sqrt{\det C(\theta)}} \exp\left[-\frac{1}{2}(P_d - P(\theta))^T C(\theta)^{-1}(P_d - P(\theta)) \right]$$

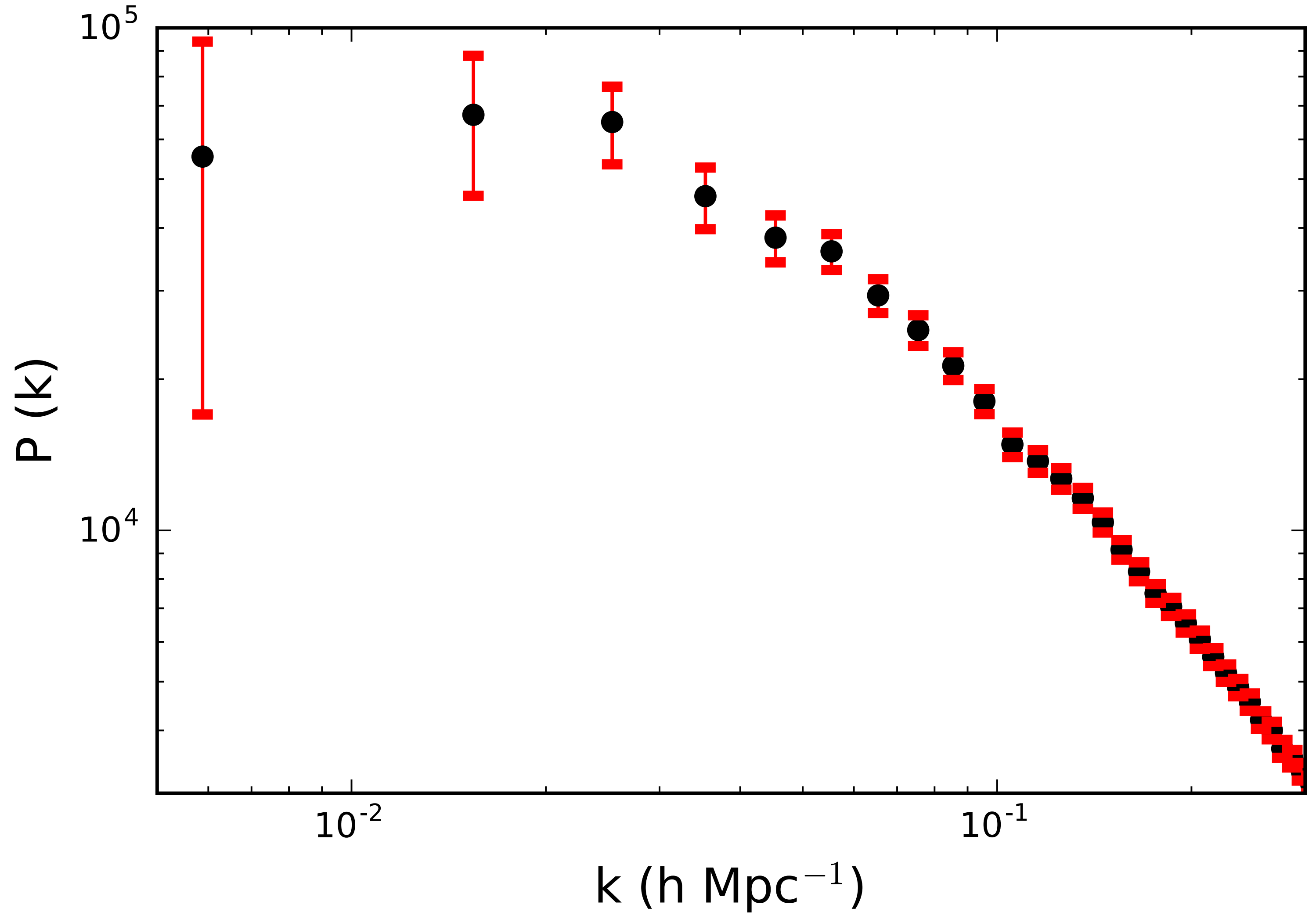
Covariance from mock catalogs

- Need to simulate mock surveys (~ thousands)

$$C_{1,2} = \frac{1}{N_{\mathrm{mock}} - 1} \sum_{i=1}^{N_{\mathrm{mock}}} (\hat{P}_i(k_1) - \bar{P}(k_1))(\hat{P}_i(k_2) - \bar{P}(k_2))$$

~$\mathcal{O}$ (Gpc)^3





$$\{H_0, \Omega_m, \sigma_8, b_1, \ldots\}$$

Covariance from mock catalogs

- As survey volume increases, mock catalogs become tougher to simulate (LSST, DESI, Euclid and others)

- Dependence of covariance on cosmology and bias parameters is computationally prohibitive

$$p(P_d|\theta) = \frac{1}{(2\pi)^{n/2}\sqrt{\det C(\theta)}} \exp\left[-\frac{1}{2}(P_d - P(\theta))^T C(\theta)^{-1}(P_d - P(\theta))\right]$$

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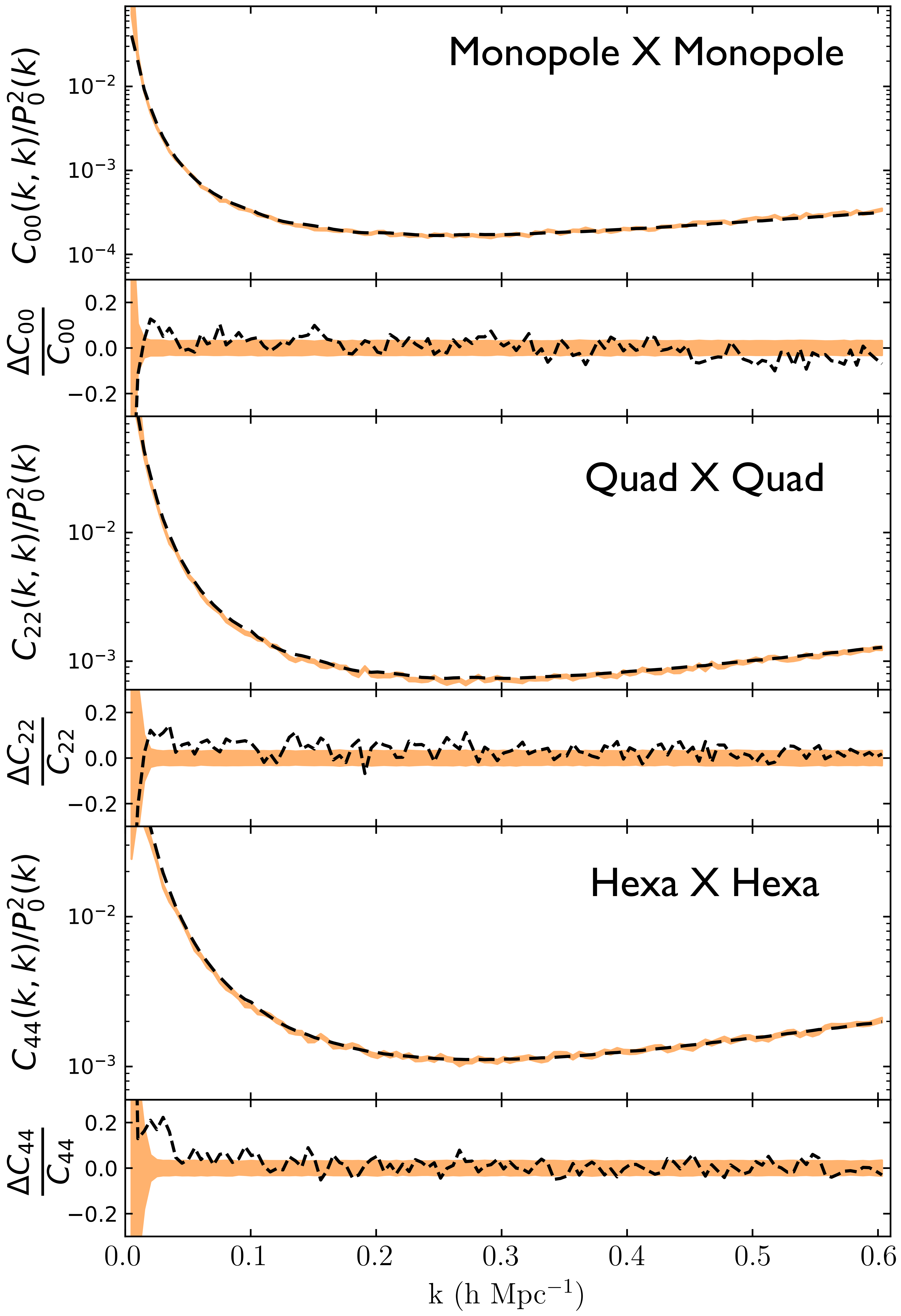
- Mocks suffer from sampling noise
  - Need to artificially inflate constraints

Results

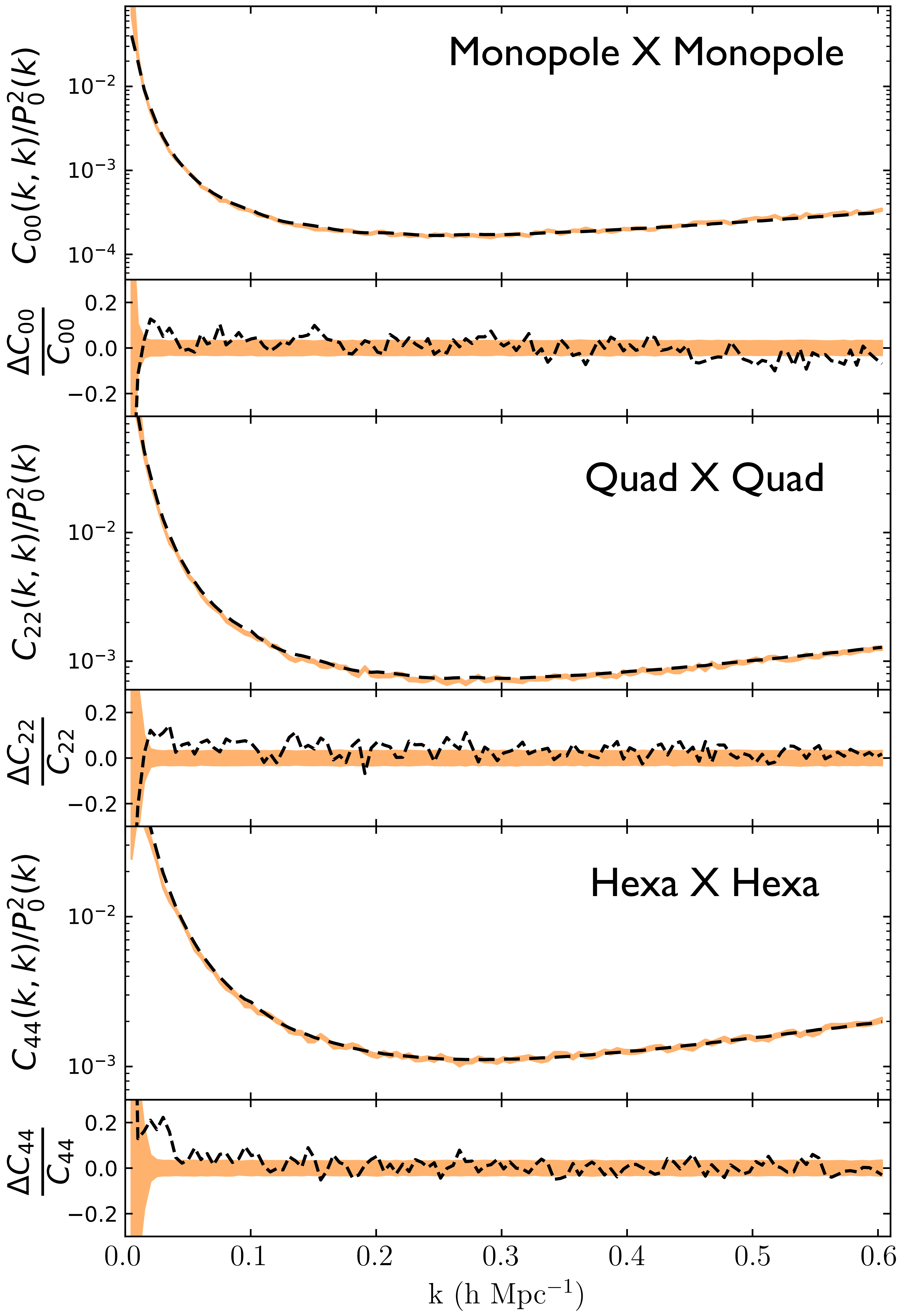


----- ***Our analytic method***

Patchy Mocks
(state-of-the-art mocks used for SDSS BOSS parameter estimation)

DW & Scoccimarro 19

Results



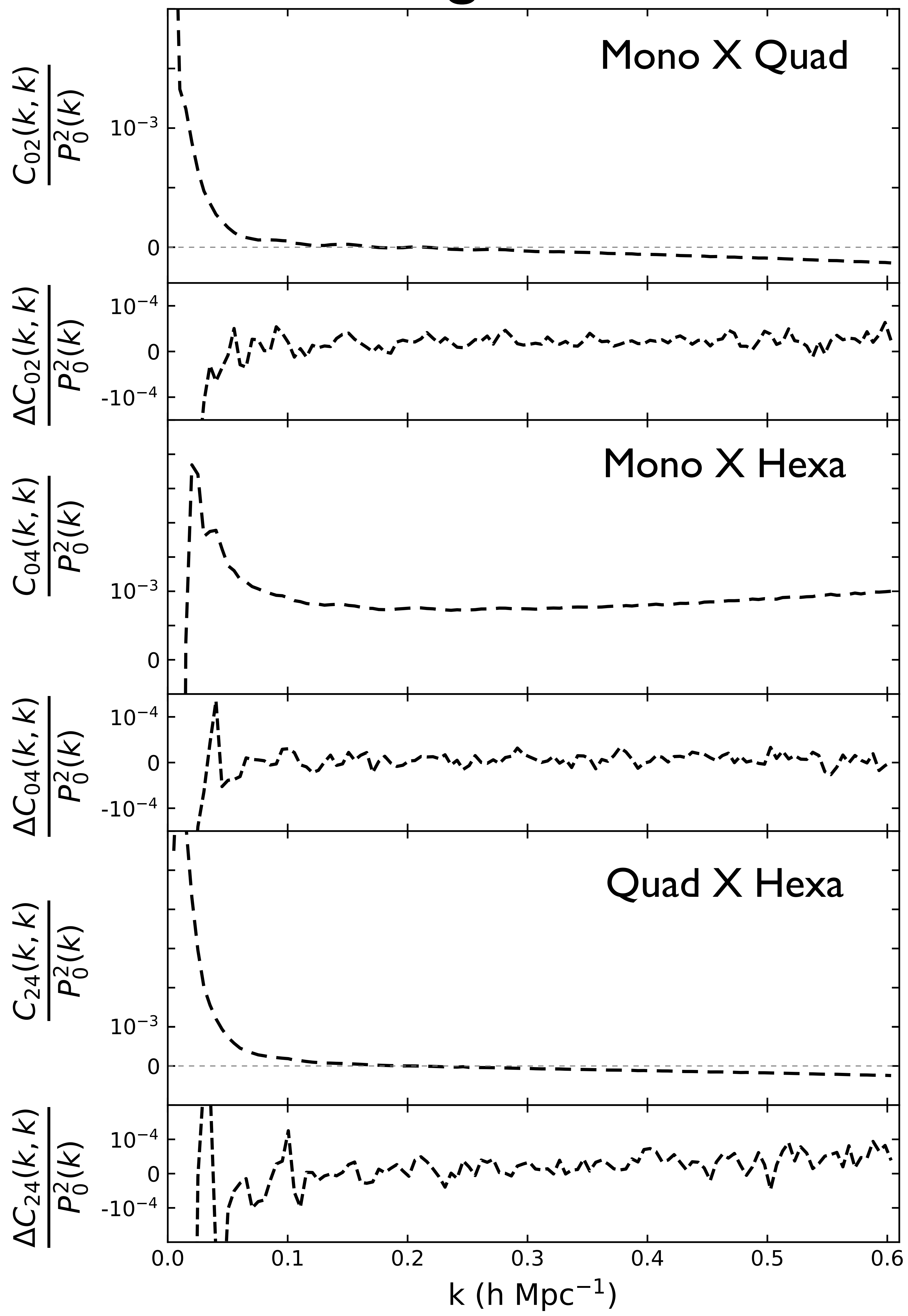
- - - - - ***Our analytic method (≤ MINUTE)***

—— Patchy Mocks ***(MONTHS)*** (state-of-the-art mocks used for SDSS BOSS parameter estimation)

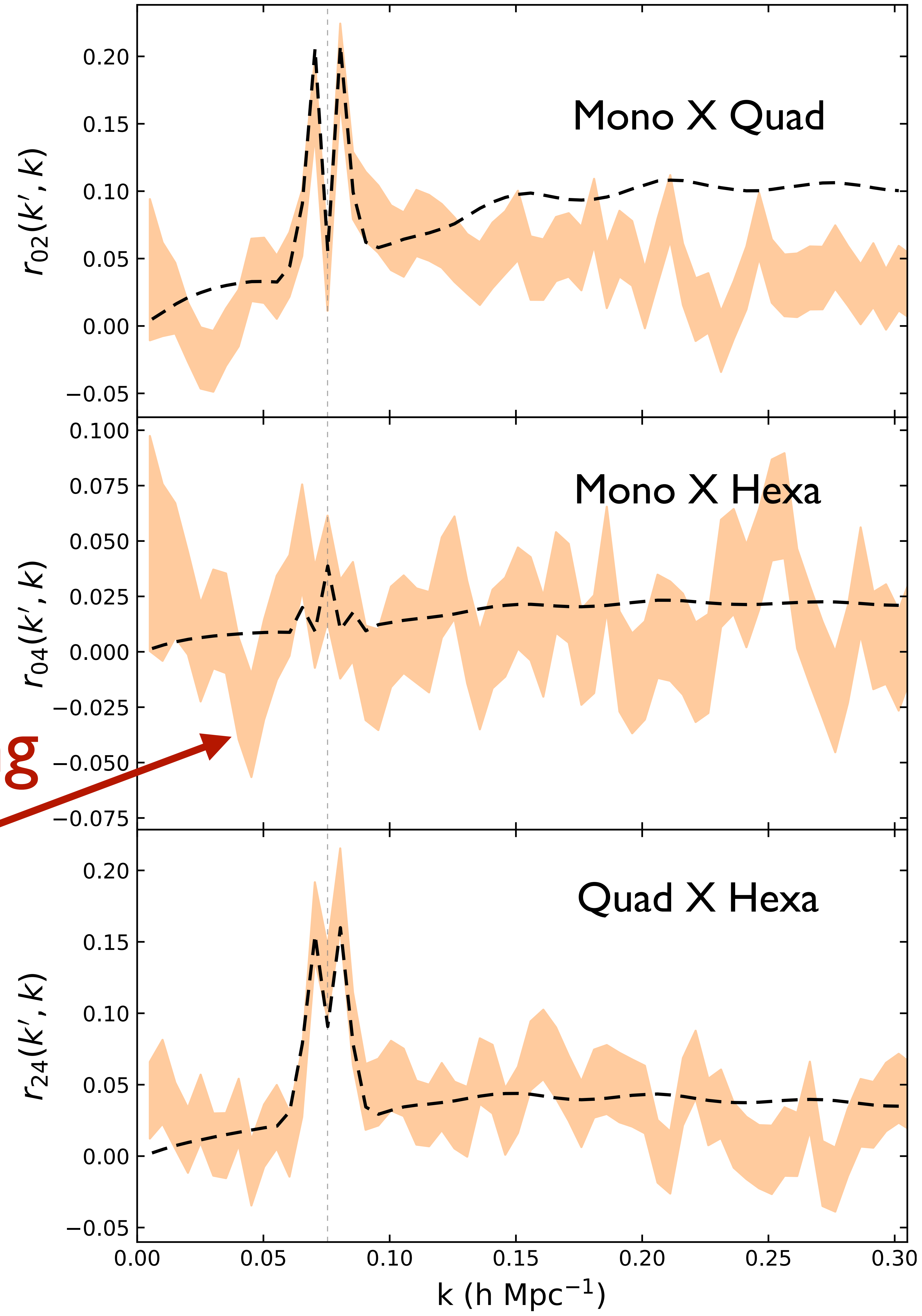
DW & Scoccimarro 19

Cross-covariance

Diagonals

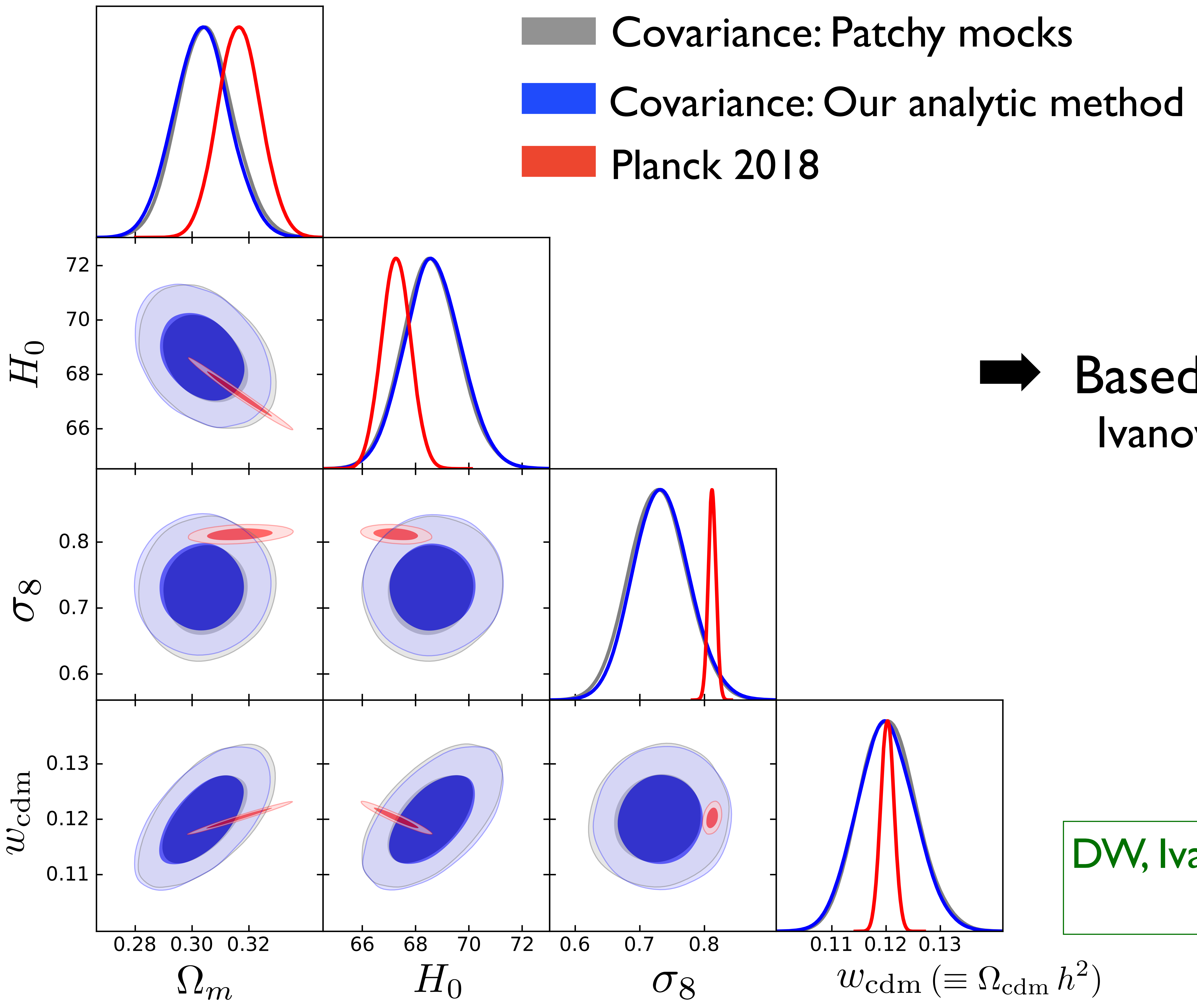


Row of matrix



Large sampling noise in mocks

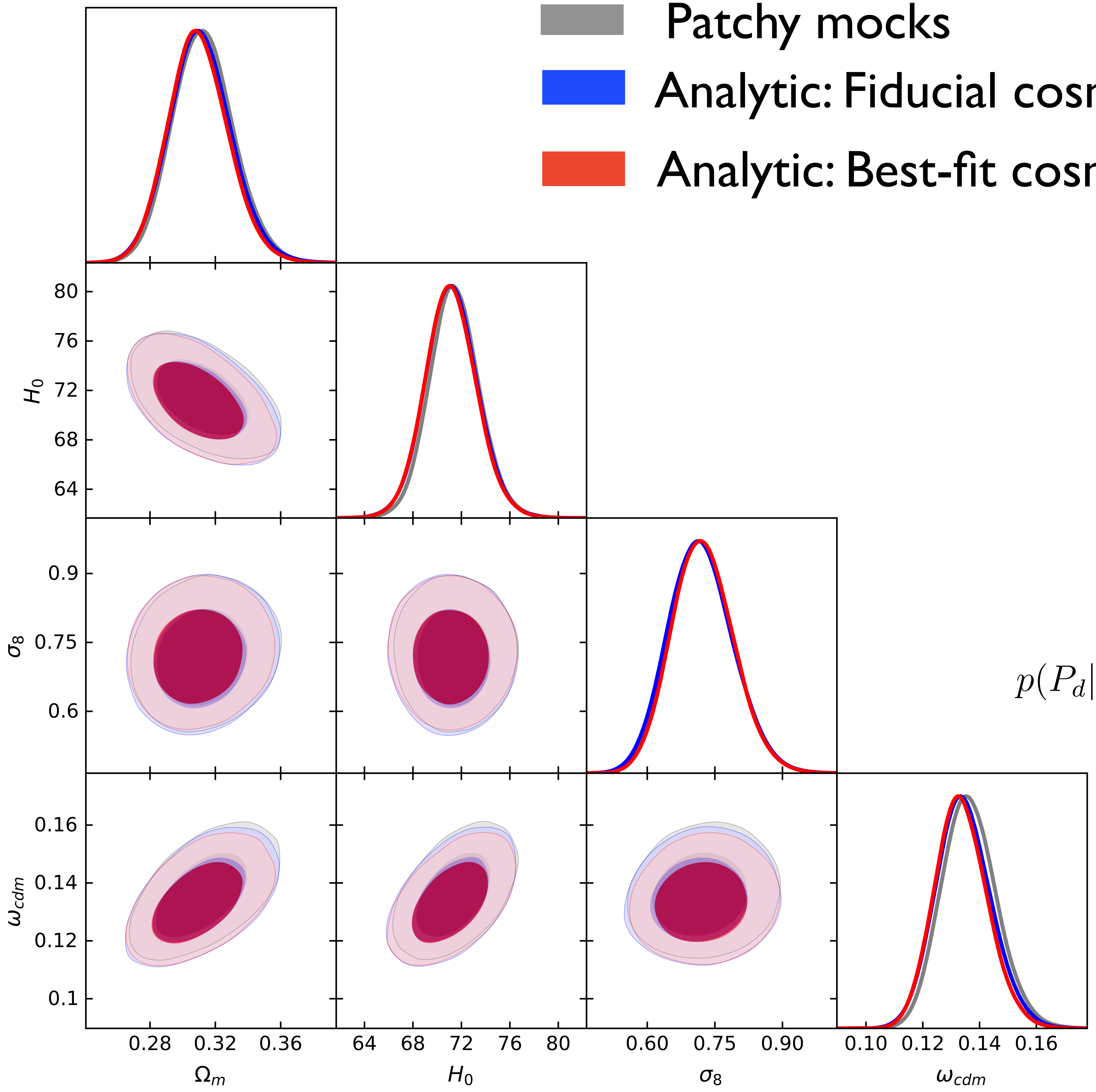
Results: BOSS DR12 data analysis



➡ Based on BOSS analysis pipeline of
Ivanov, Simonovic, Zaldarriaga, JCAP 20

DW, Ivanov & Scoccimarro, 2020
(to appear)

Case study: BOSS sample of NGC high-z



Patchy mocks

Analytic: Fiducial cosmology

Analytic: Best-fit cosmology

✓ BOSS results are robust w.r.t change in cosmology of covariance matrix

$$p(P_d|\theta) = \frac{1}{(2\pi)^{n/2}\sqrt{\det C(\theta)}} \exp\left[-\frac{1}{2}(P_d - P(\theta))^T C(\theta)^{-1}(P_d - P(\theta))\right]$$

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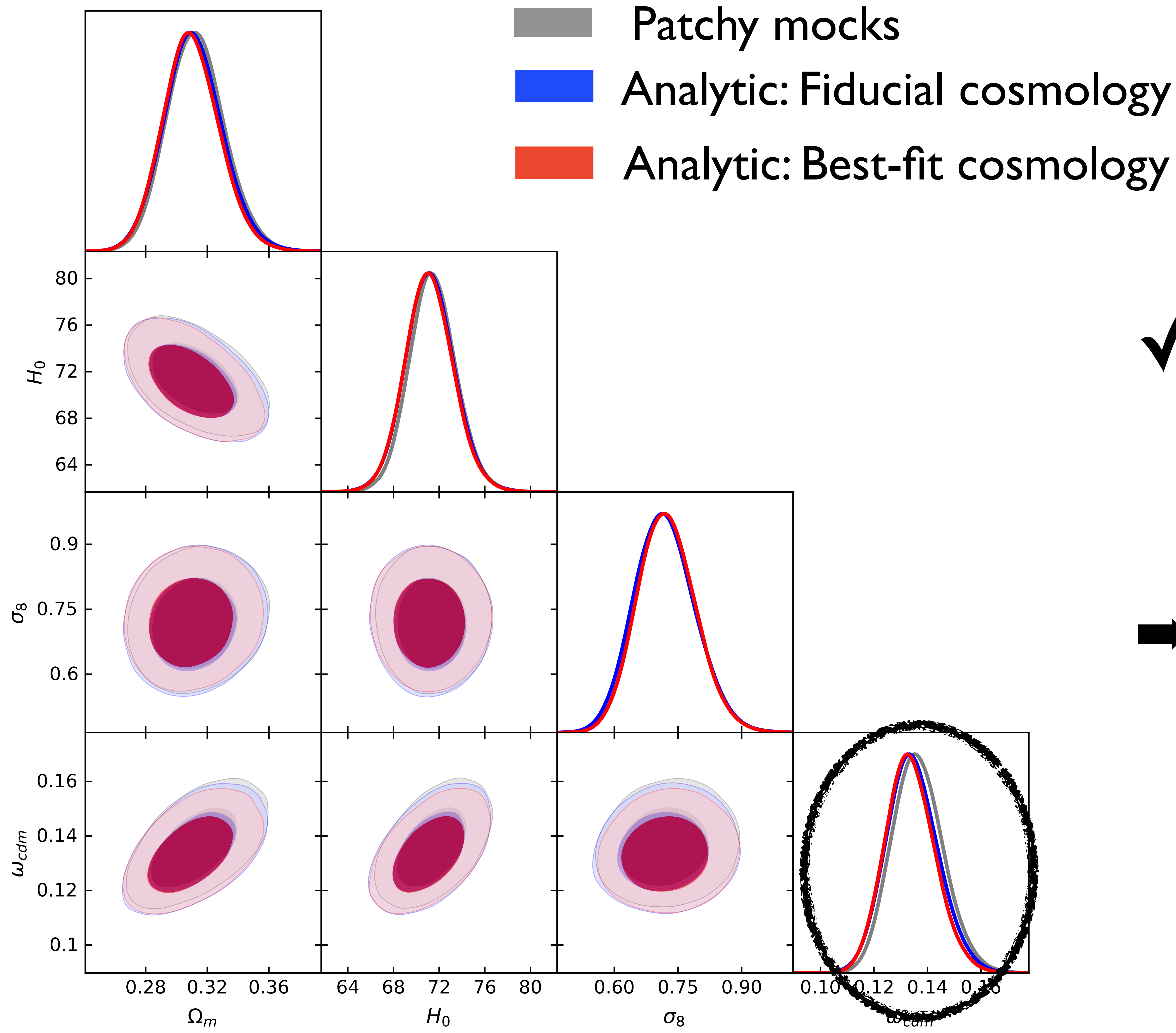
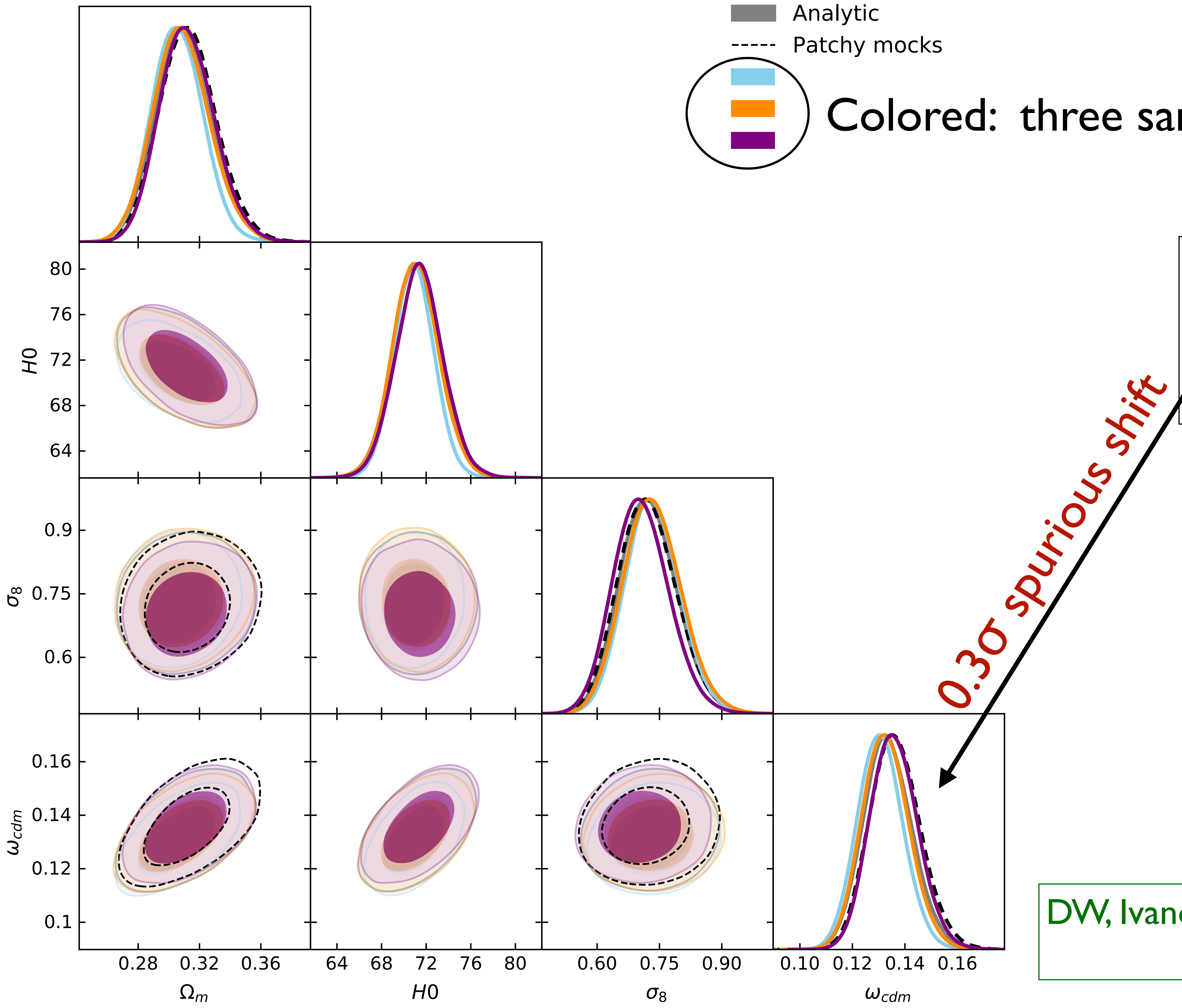


✓ BOSS results are robust w.r.t change in cosmology of covariance matrix

➡ Small shifts (0.2σ) because of sampling noise

Sampling noise in covariance from 2048 mocks

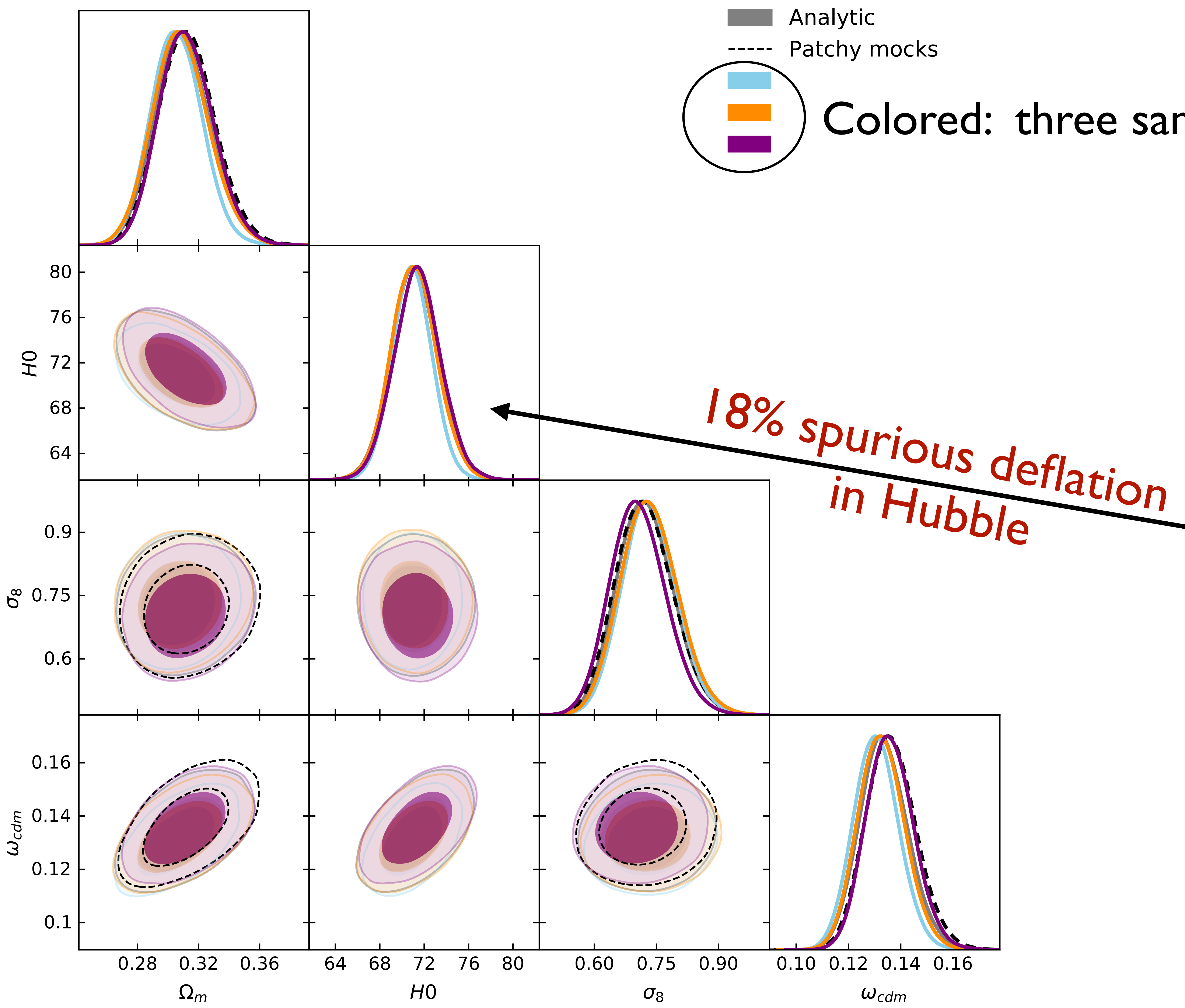


Colored: three sampled versions of Analytic covariance

1. Shifts in parameter constraints

DW, Ivanov & Scoccimarro, 2020 (to appear)

Sampling noise in covariance from 2048 mocks



1. Shifts in parameter constraints

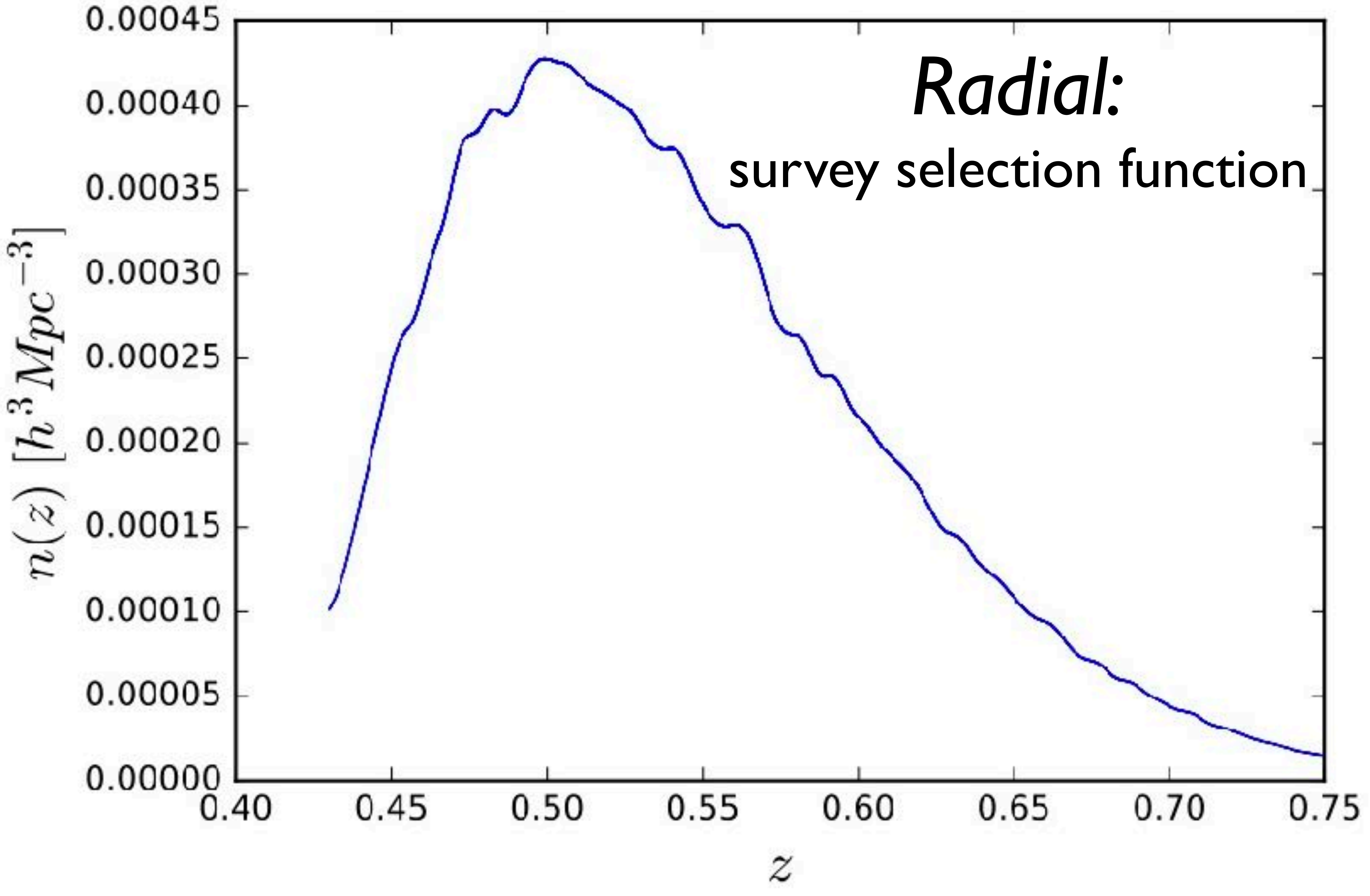
2. Inflation/deflation of parameter error bars

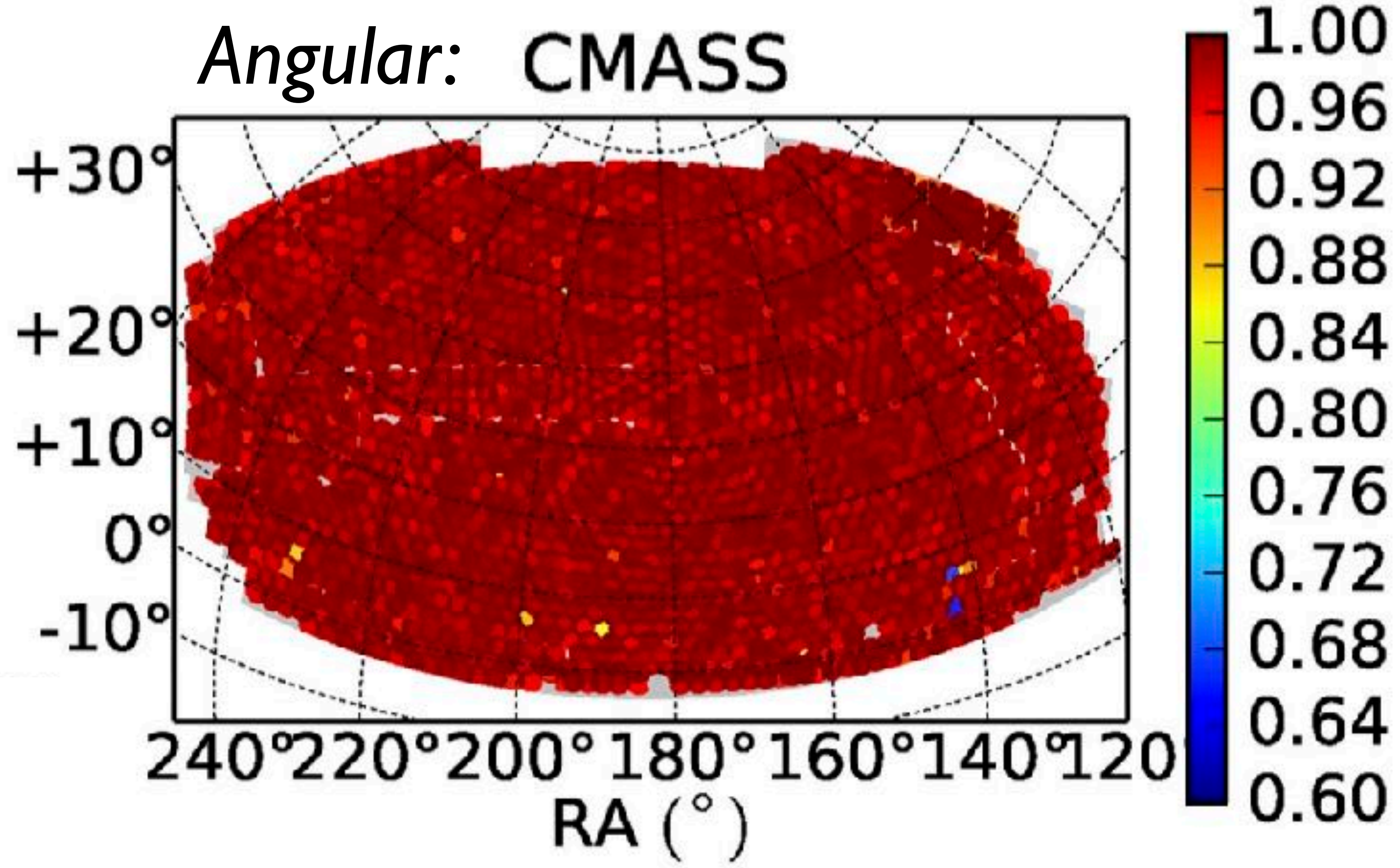
Dodelson & Schneider 2013
Percival et al 2014

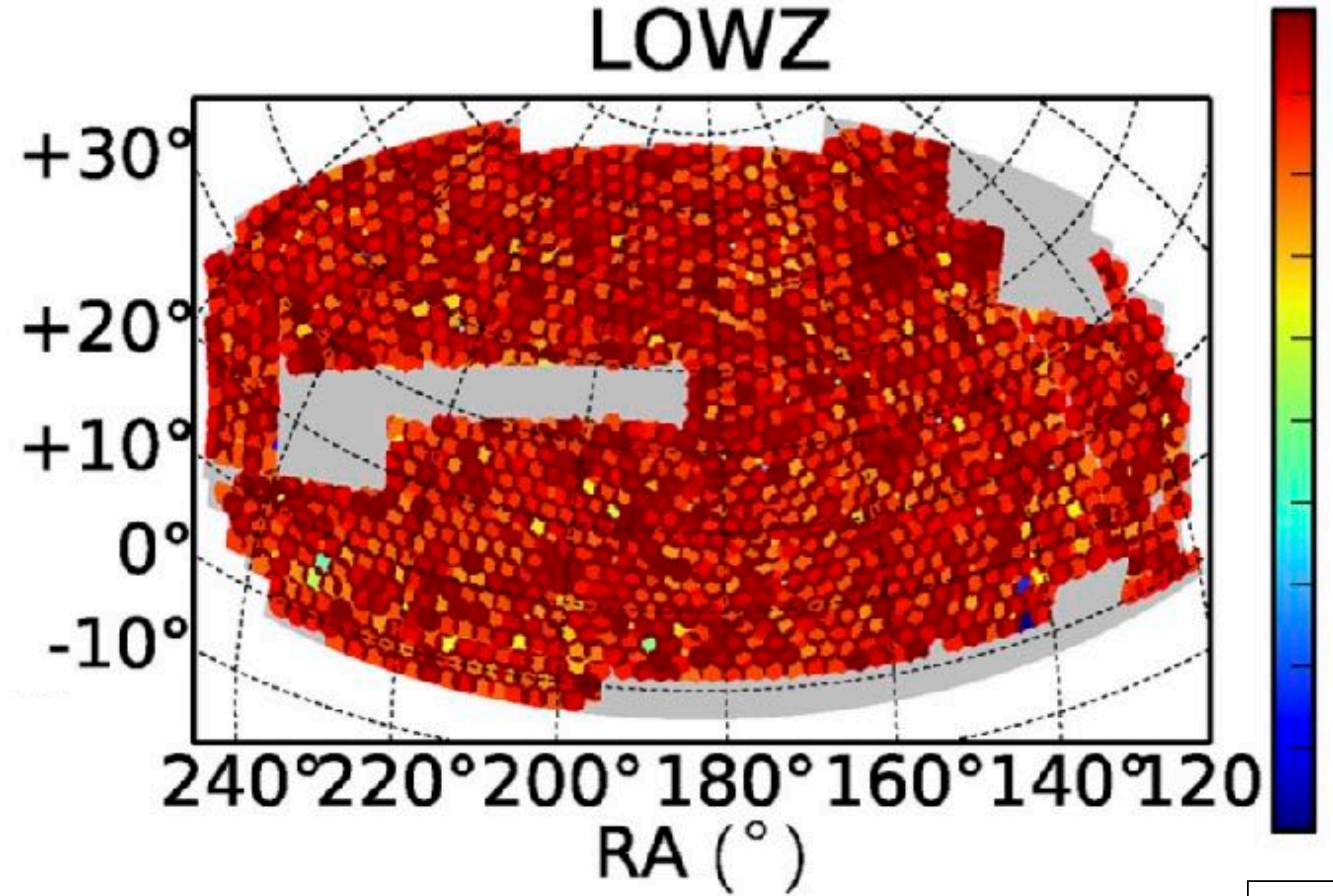
What are the challenges to analytically calculate the covariance?

Challenge I: Highly non-trivial survey window

$$\delta_W(\mathbf{x}) \equiv W(\mathbf{x})\delta(\mathbf{x})$$

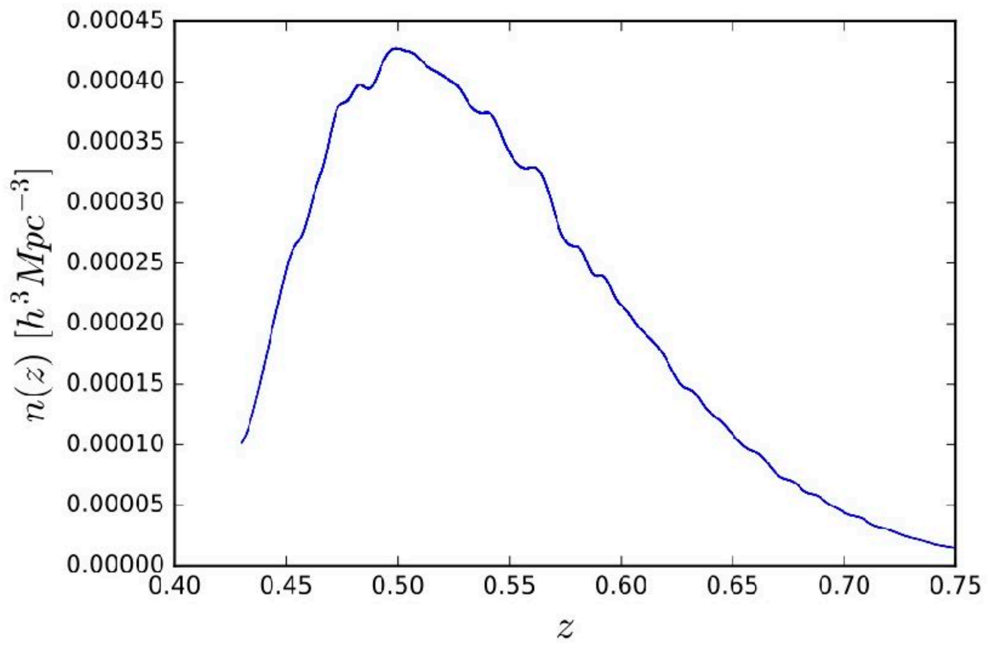


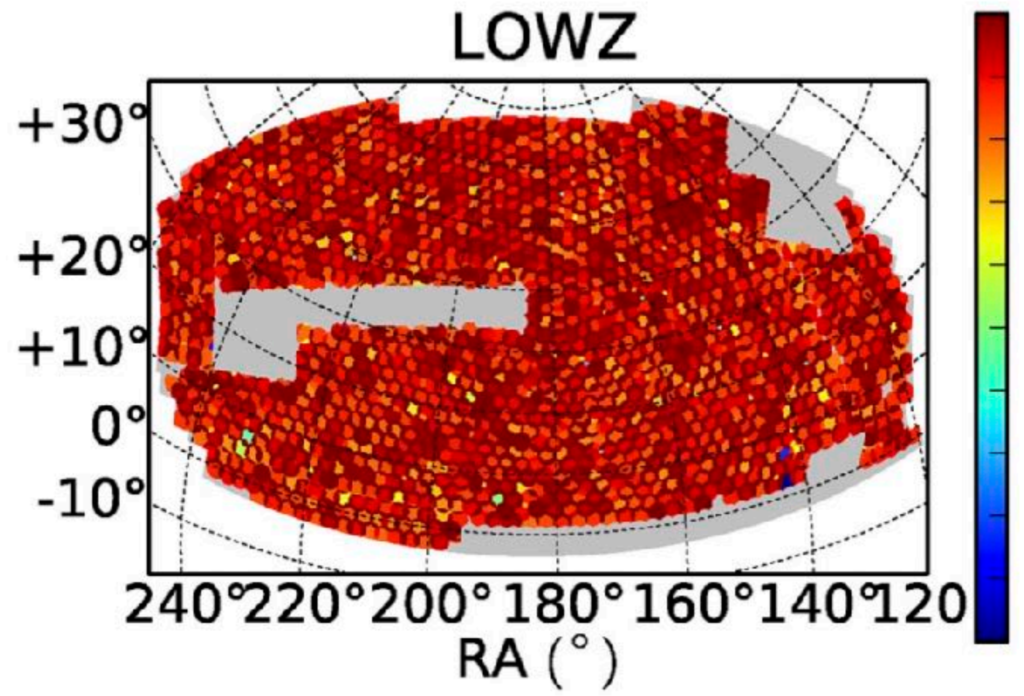




Reid et al. 2015

Survey window enters covariance





$$C(k_1, k_2) = \langle \hat{P}(k_1)\hat{P}(k_2)\rangle - \langle \hat{P}(k_1)\rangle\langle \hat{P}(k_2)\rangle$$

$$\begin{aligned}\langle \hat{P}(k_1)\hat{P}(k_2)\rangle =& \frac{1}{V_2^2}\int_{\hat{\mathbf{k}}_1,\hat{\mathbf{k}}_2} \langle \delta_W(\mathbf{k}_1)\delta_W(-\mathbf{k}_1)\delta_W(\mathbf{k}_2)\delta_W(-\mathbf{k}_2)\rangle \\ =& \frac{1}{V^2{}_2}\int_{\hat{\mathbf{k}}_1,\hat{\mathbf{k}}_2,\mathbf{p}_1,\mathbf{p}'_1,\mathbf{p}_2,\mathbf{p}'_2} W(\mathbf{k}_1-\mathbf{p}_1)W(-\mathbf{k}_1-\mathbf{p}'_1)W(\mathbf{k}_2-\mathbf{p}_2)W(-\mathbf{k}_2-\mathbf{p}'_2) \\ & \times \langle \delta(\mathbf{p}_1)\delta(\mathbf{p}'_1)\delta(\mathbf{p}_2)\delta(\mathbf{p}'_2)\rangle\end{aligned}$$

18 dimensional integral

Solution: separate clustering and window terms

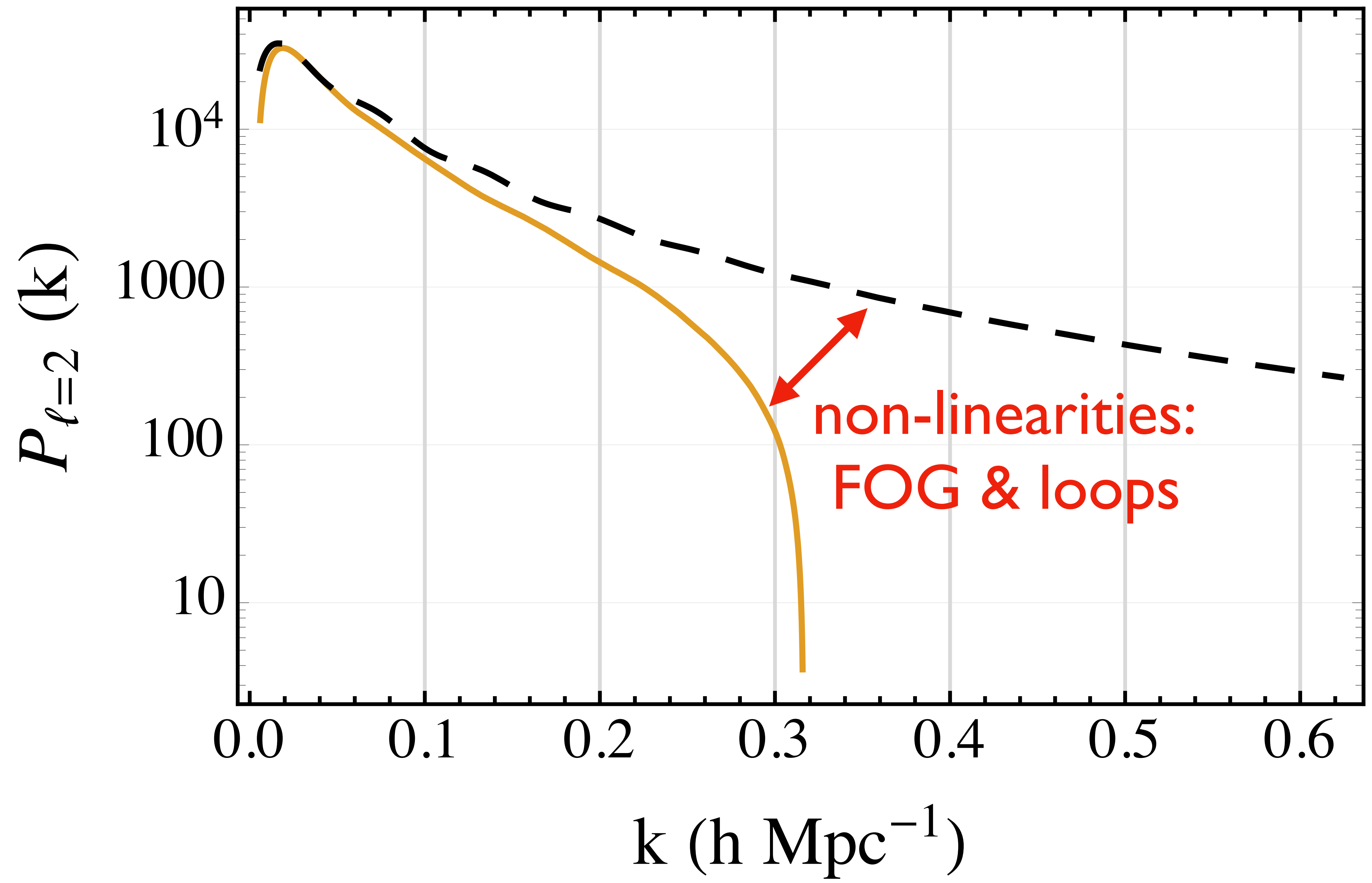
Contains all dependence on cosmology and bias parameters

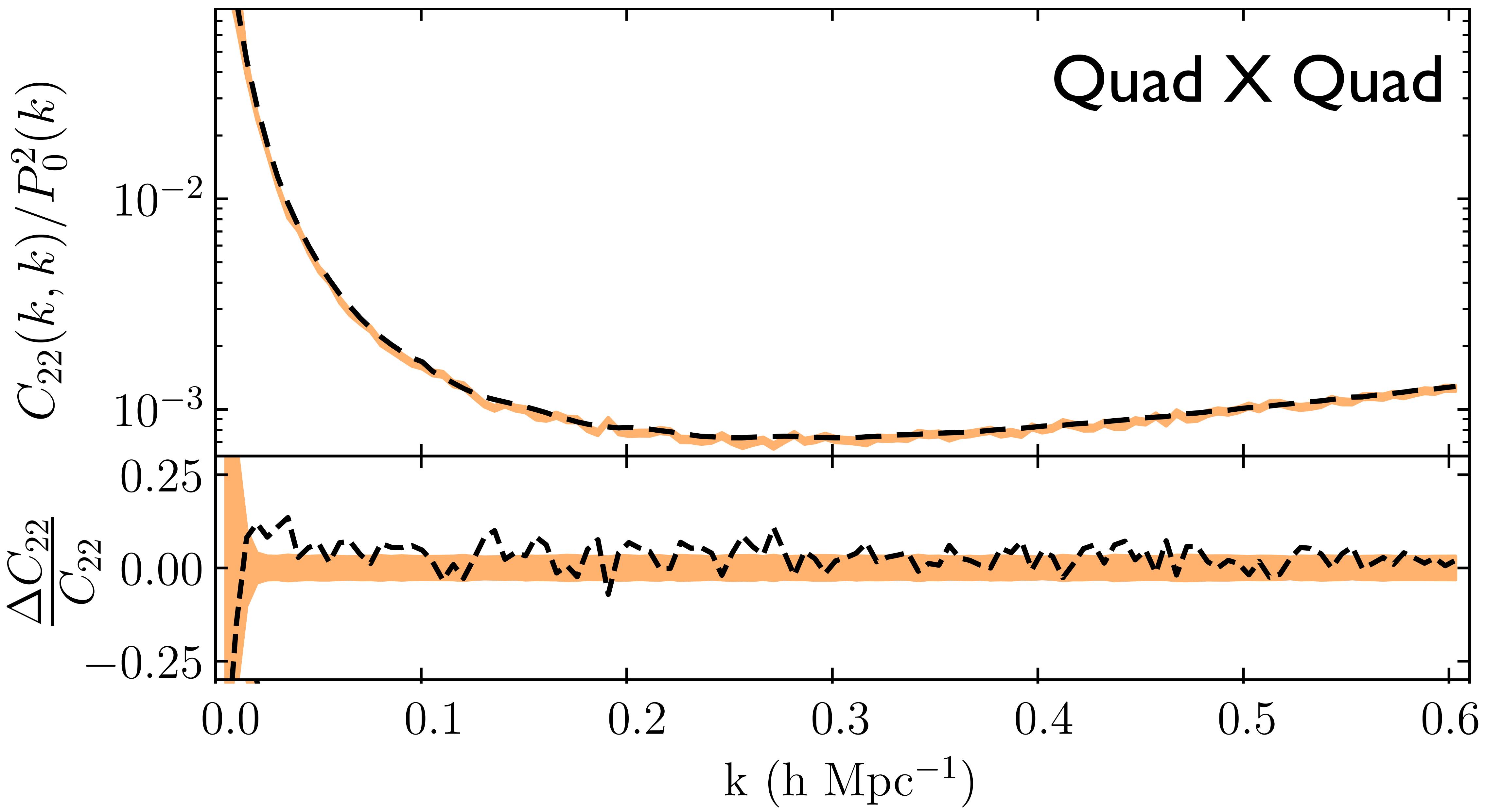
$$\mathbf{C}^{\mathrm{G}}_{\ell_1\ell_2}(k_1,k_2) \simeq \sum_{\ell_1',\ell_2'} P_{\ell_1'}(k_2)P_{\ell_2'}(k_1)\left\{\frac{(2\ell_1+1)(2\ell_2+1)}{\mathrm{I}_{22}^2}\int_{\hat{\mathbf{k}}_1,\hat{\mathbf{k}}_2,\mathbf{x}_1,\mathbf{x}_2} W_{22}(\mathbf{x}_1)W_{22}(\mathbf{x}_2)\, e^{-i(\mathbf{x}_1-\mathbf{x}_2)\cdot(\mathbf{k}_1-\mathbf{k}_2)}\right.$$
$$\left.\times\mathcal{L}_{\ell_1}(\hat{\mathbf{x}}_1\cdot\hat{\mathbf{k}}_1)\mathcal{L}_{\ell_1'}(\hat{\mathbf{x}}_1\cdot\hat{\mathbf{k}}_1)\mathcal{L}_{\ell_2'}(\hat{\mathbf{x}}_2\cdot\hat{\mathbf{k}}_2)\left[\mathcal{L}_{\ell_2}(\hat{\mathbf{x}}_2\cdot\hat{\mathbf{k}}_2)+\mathcal{L}_{\ell_2}(\hat{\mathbf{x}}_1\cdot\hat{\mathbf{k}}_1)\right]\right\}$$

{ Computed from survey random catalog }

See also:
O'Connell et al. 2016
Li et al. 2019

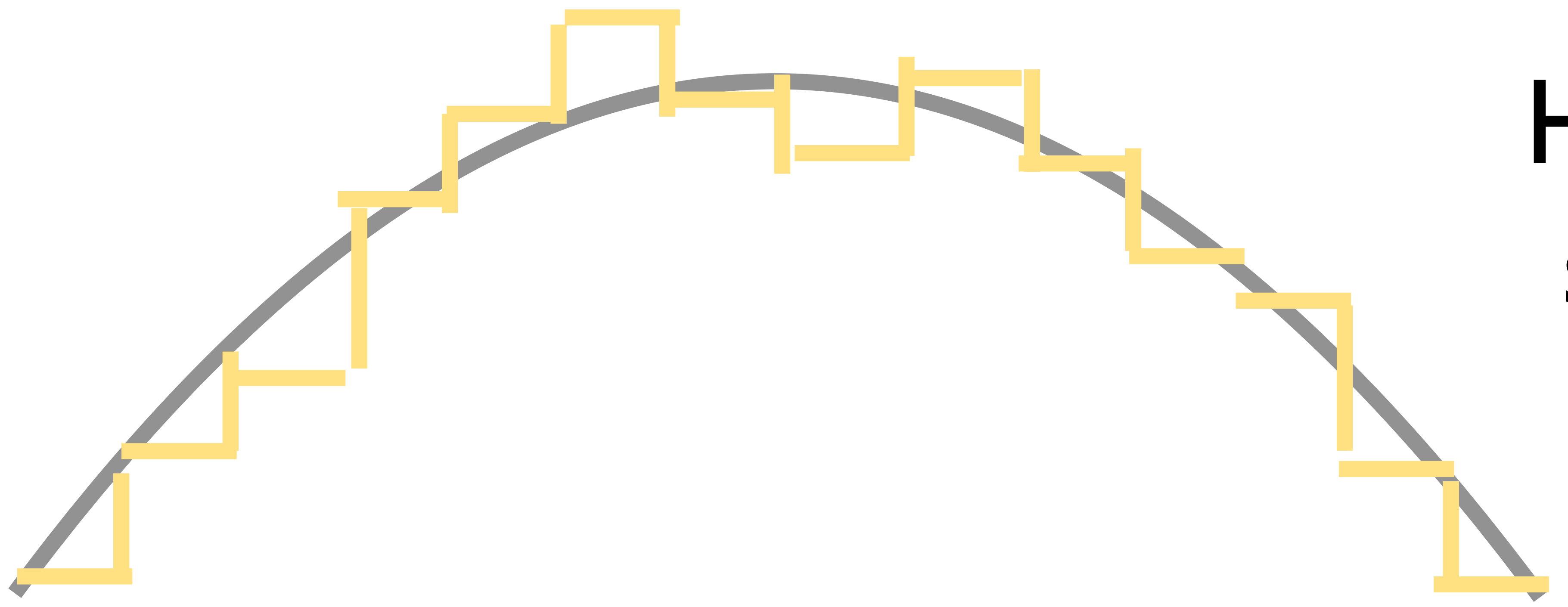
Challenge II:
Analytic modeling in the non-linear regime



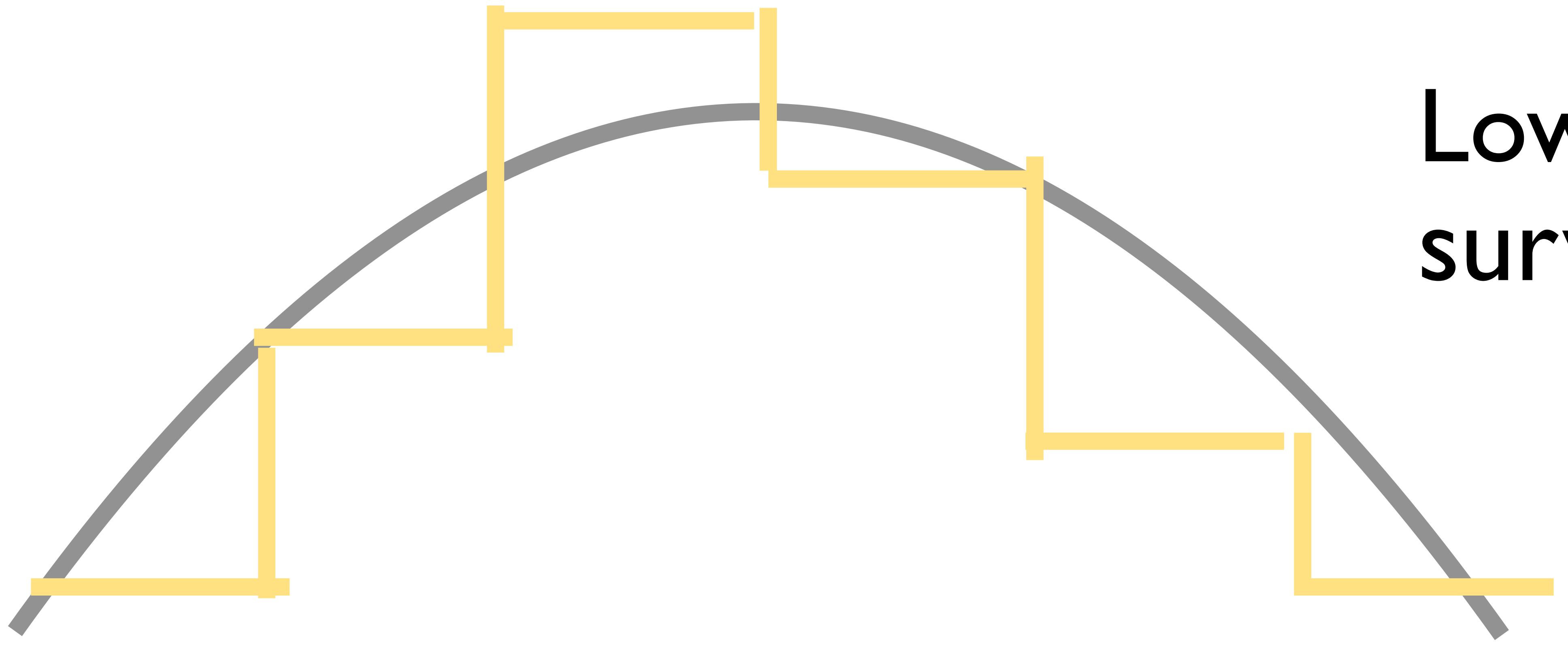


Analytic covariance works
very well at high-k.
WHY!?

Why does analytic work in the non-linear regime?



High $\bar{n}$ survey



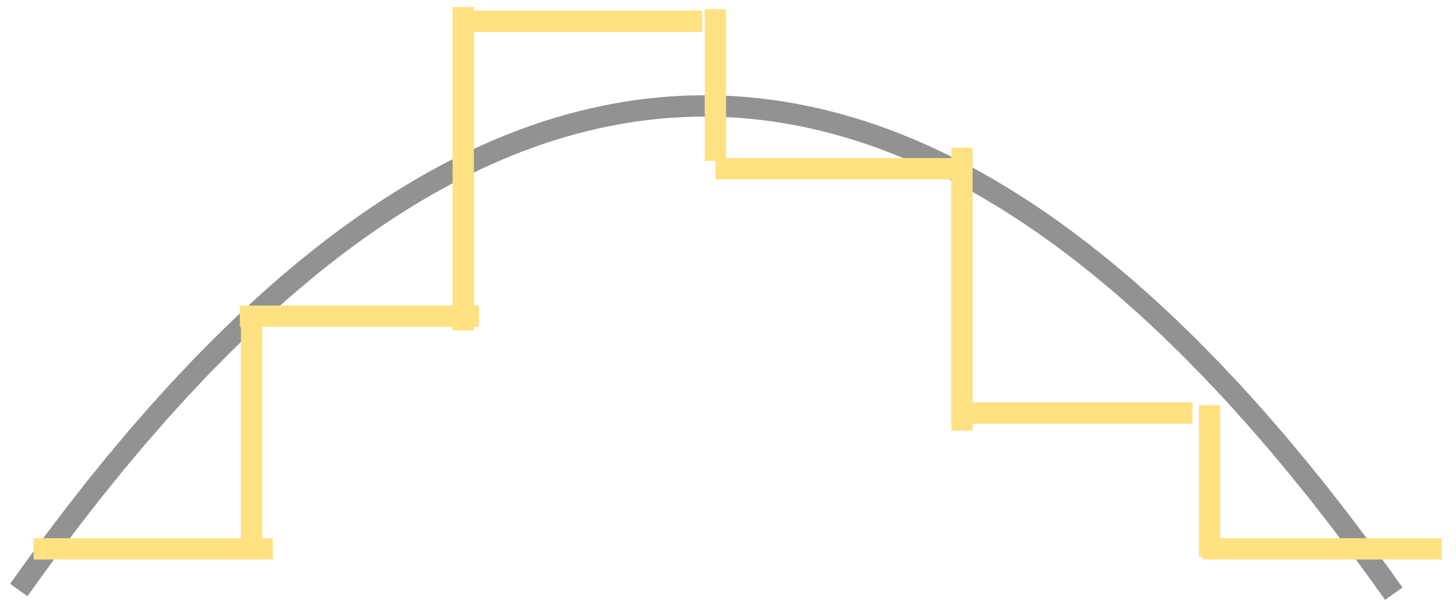
Low $\bar{n}$ survey

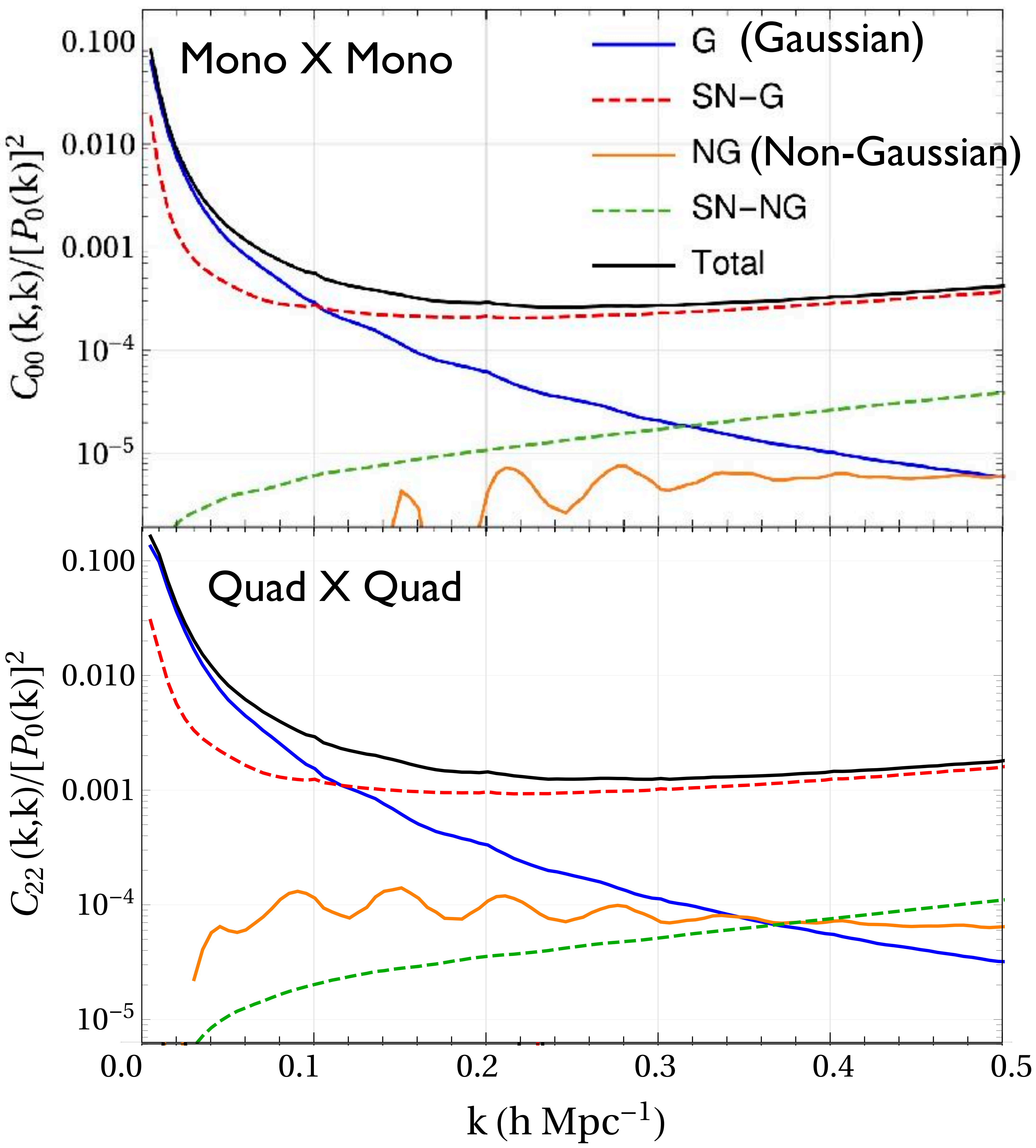
- Poisson fluctuations dominate the error bars at small scales:

✓ Can be well modeled analytically

Why does analytic work in the non-linear regime?

Shot noise (dashed) dominates at high-k





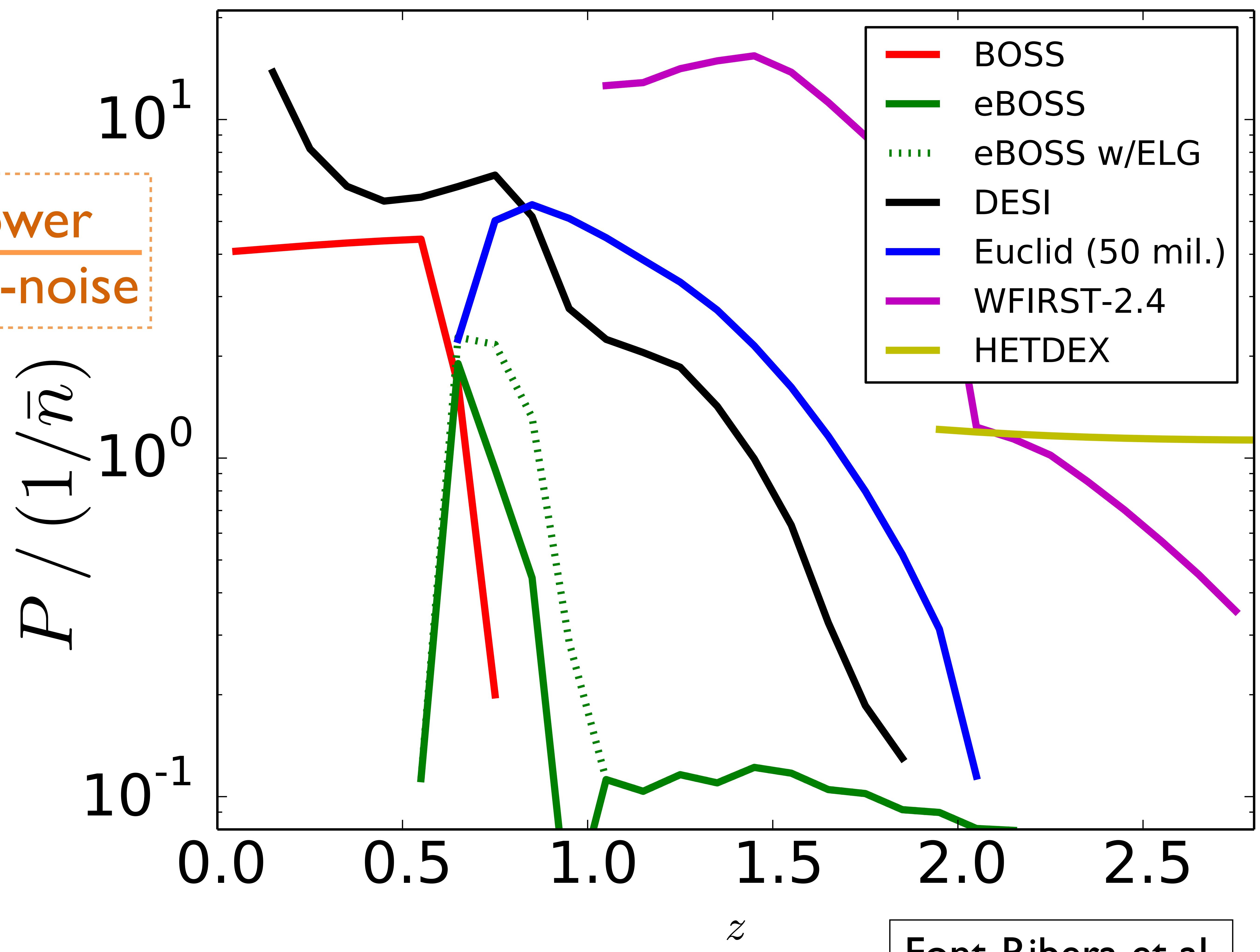
Analytic cov. should work at small scales for upcoming surveys

- Shot noise level of upcoming surveys is comparable to BOSS

↓

✓ Shot noise will dominate covariance for upcoming surveys

Power / shot-noise



Font-Ribera et al. 2014

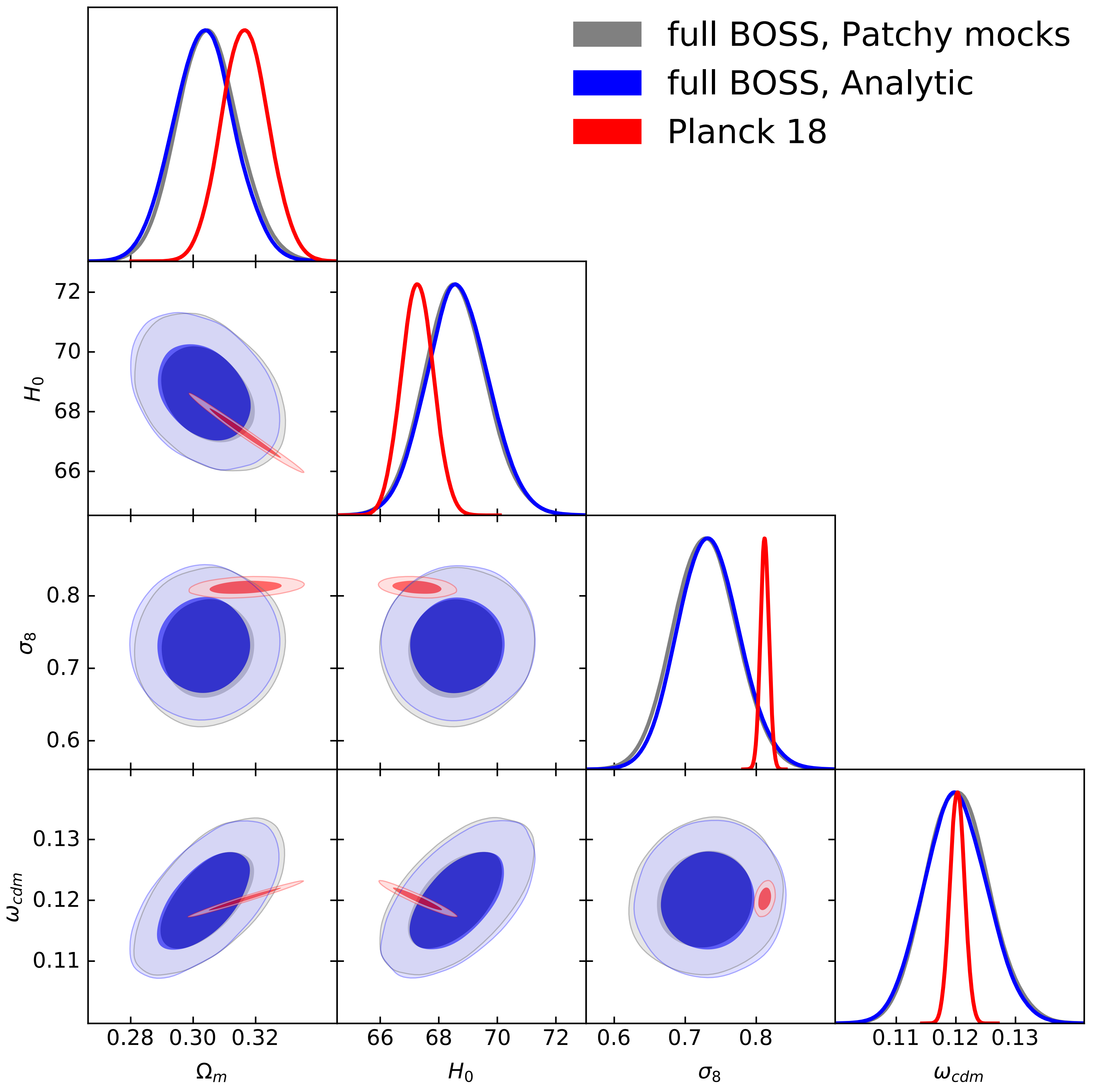
Summary

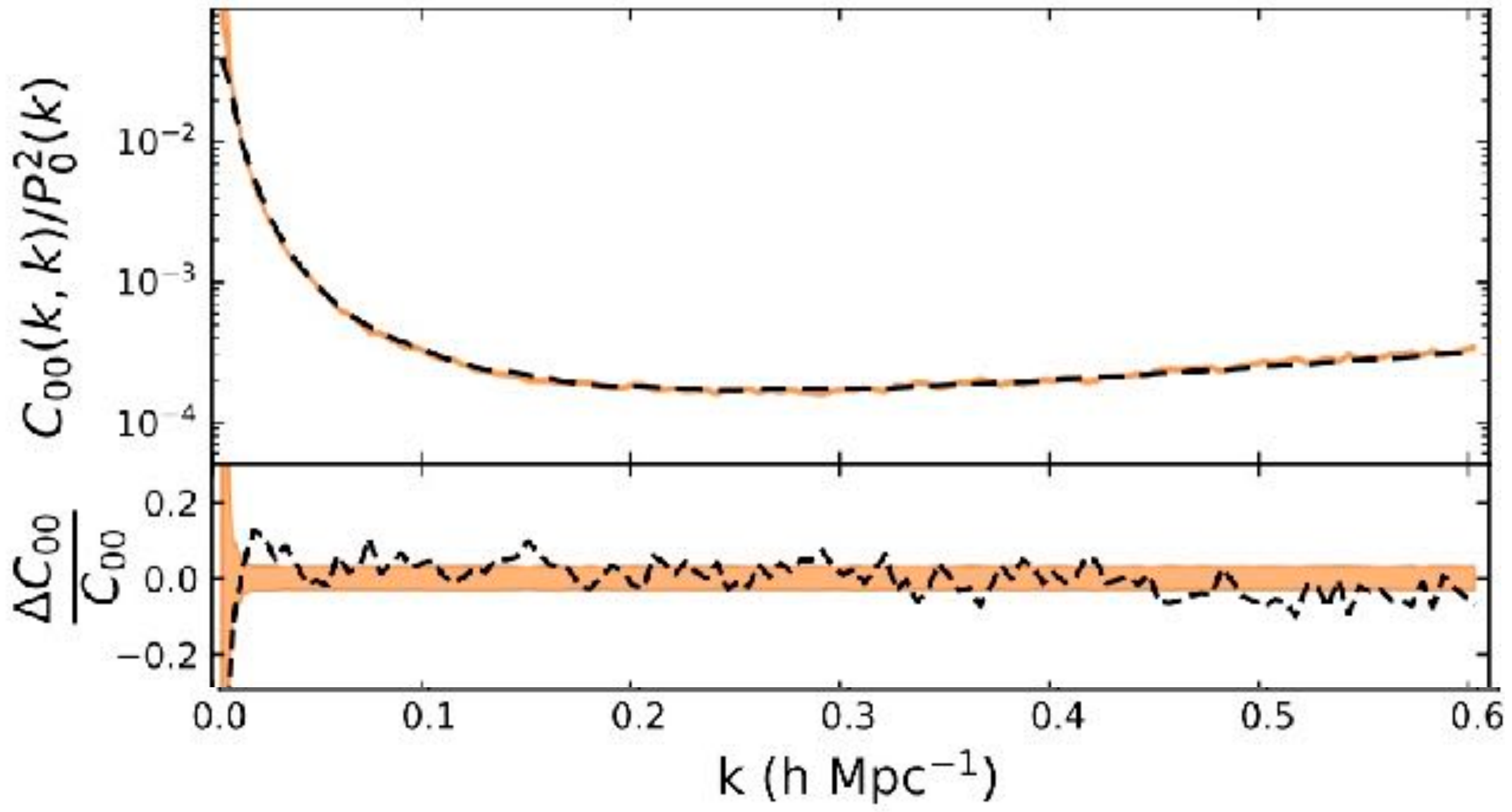
★ Analytic covariance is an excellent alternative to mock simulations for upcoming spectroscopic surveys

1. Very good agreement with the state-of-the-art mocks up to non-linear scales

2. Immense computational speedup (~ 10^4) - allows calculation at best-fit cosmology

3. No sampling noise effects
   - no parameter shifts
   - no inflation/deflation of error bars





D.Wadekar, NYU

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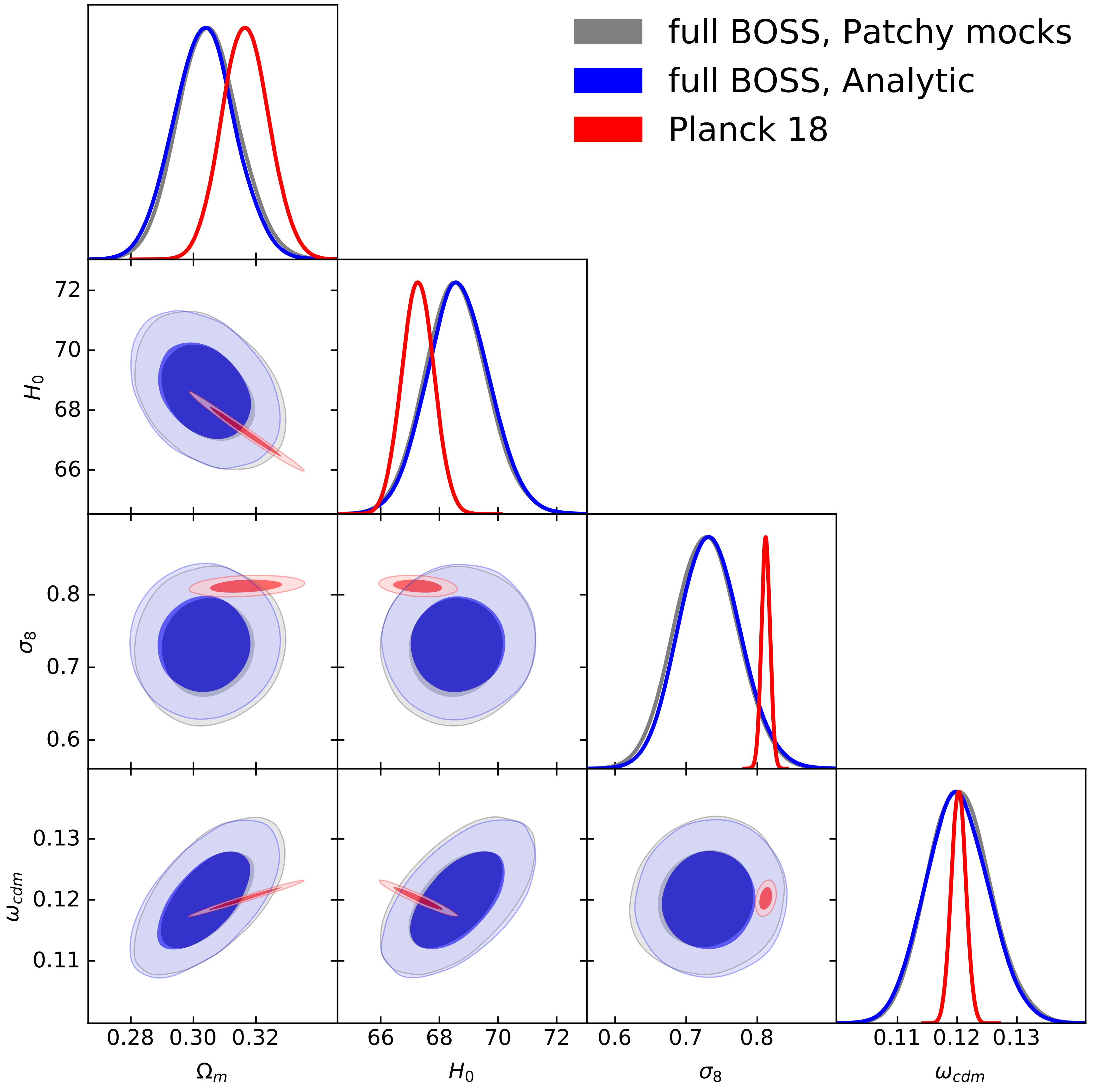

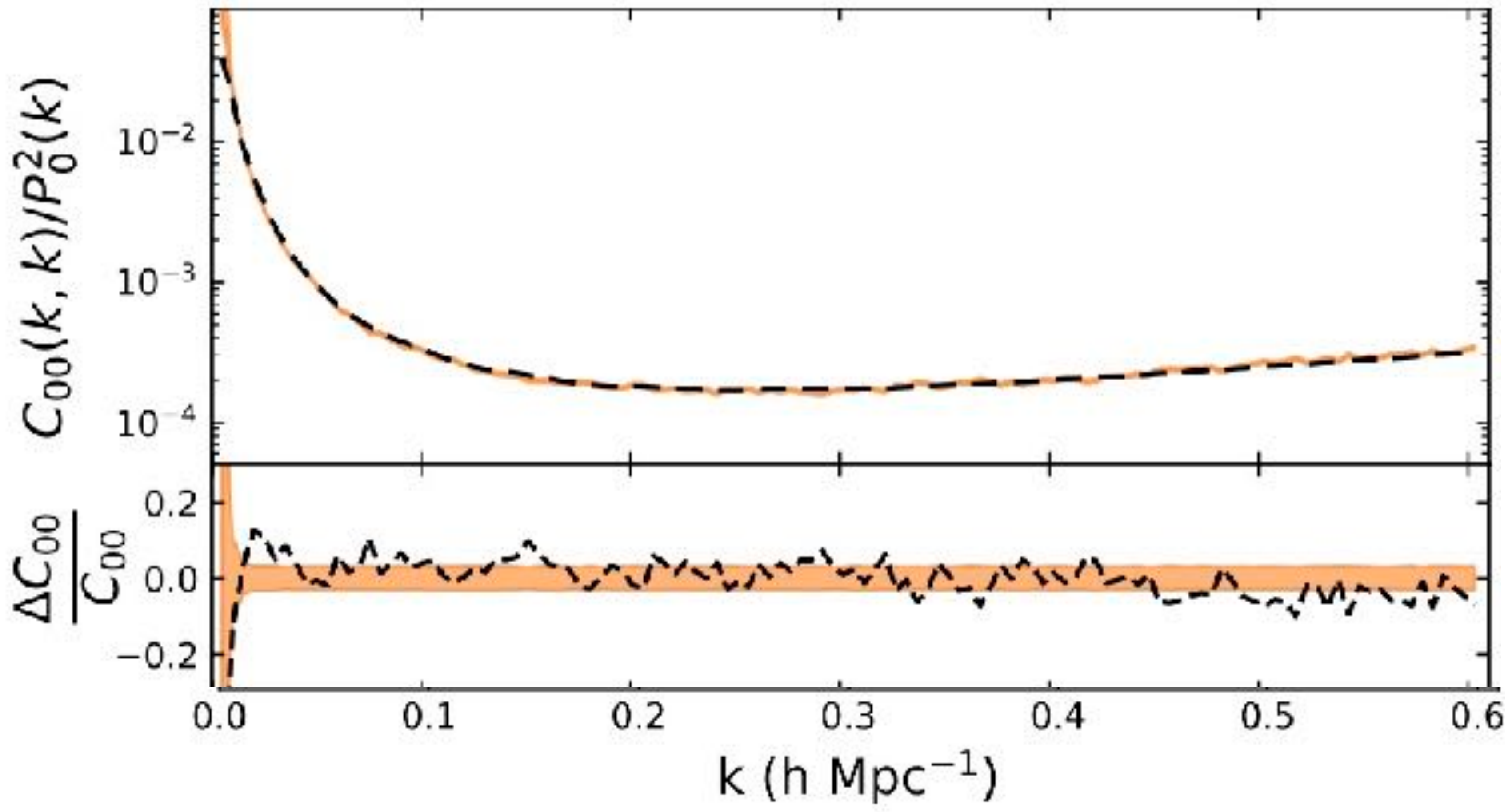

- Next: Bispectrum covariance
  - Simulations are computationally prohibitive

D.Wadekar, NYU

Literature

1. Estimator motivated by simulations

$$\hat{\delta} \equiv \frac{\Delta\rho}{(\bar{\rho})_{\rm survey}}$$

Our Approach

1. FKP estimator (motivated from surveys)

$$\hat{\delta}^{\rm FKP}(\mathbf{x}) \equiv \frac{w(\mathbf{x})[n_g(\mathbf{x}) - \alpha n_r(\mathbf{x})]}{\left[\mathrm{N_g}/\mathrm{N_r} \times \int_{\mathbf{x}'} w^2(\mathbf{x}')\bar{n}(\mathbf{x}')n_r(\mathbf{x}')\right]^{1/2}}$$

which simplifies to

$$\hat{\delta}^{\rm FKP} \simeq \frac{1}{\sqrt{\mathrm{I}_{22}}} \frac{W(\mathbf{x})\delta(\mathbf{x})}{(1+\delta_{\mathrm{N_g}})^{1/2}}$$

Effect of super sample covariance (SSC) is more than 5 x stronger *than prev. considered*