Spherical Harmonic Tomography for Emission Line Galaxy Surveys

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Outline

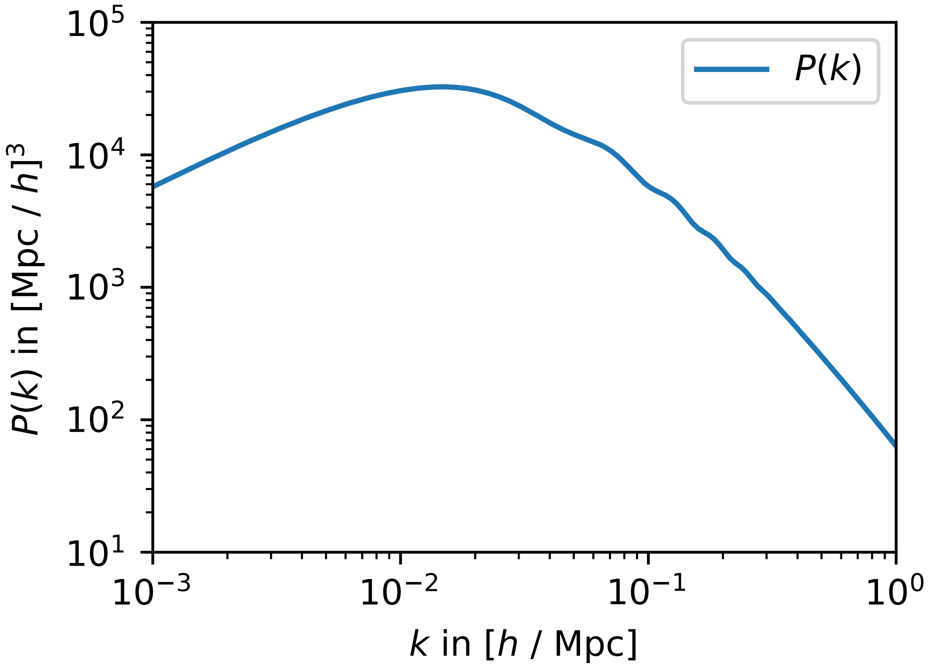
- Motivation for Spherical Harmonic Tomography (SHT)
- 2-FAST: Projecting to SHT
- Redshift-Space SHT
- Conclusion

The power spectrum contains lots of cosmological information!

- Baryon acoustic oscillations (BAO)
  → Distances!
- Alcock-Paczynski test
  → Distances!
- Growth of structure
  → Test modified gravity
- Non-Gaussianity

$$\delta(\mathbf{x}) = \frac{\rho(\mathbf{x}) - \bar{\rho}}{\bar{\rho}}$$

$$P(k) \propto \langle |\delta_k^2| \rangle$$

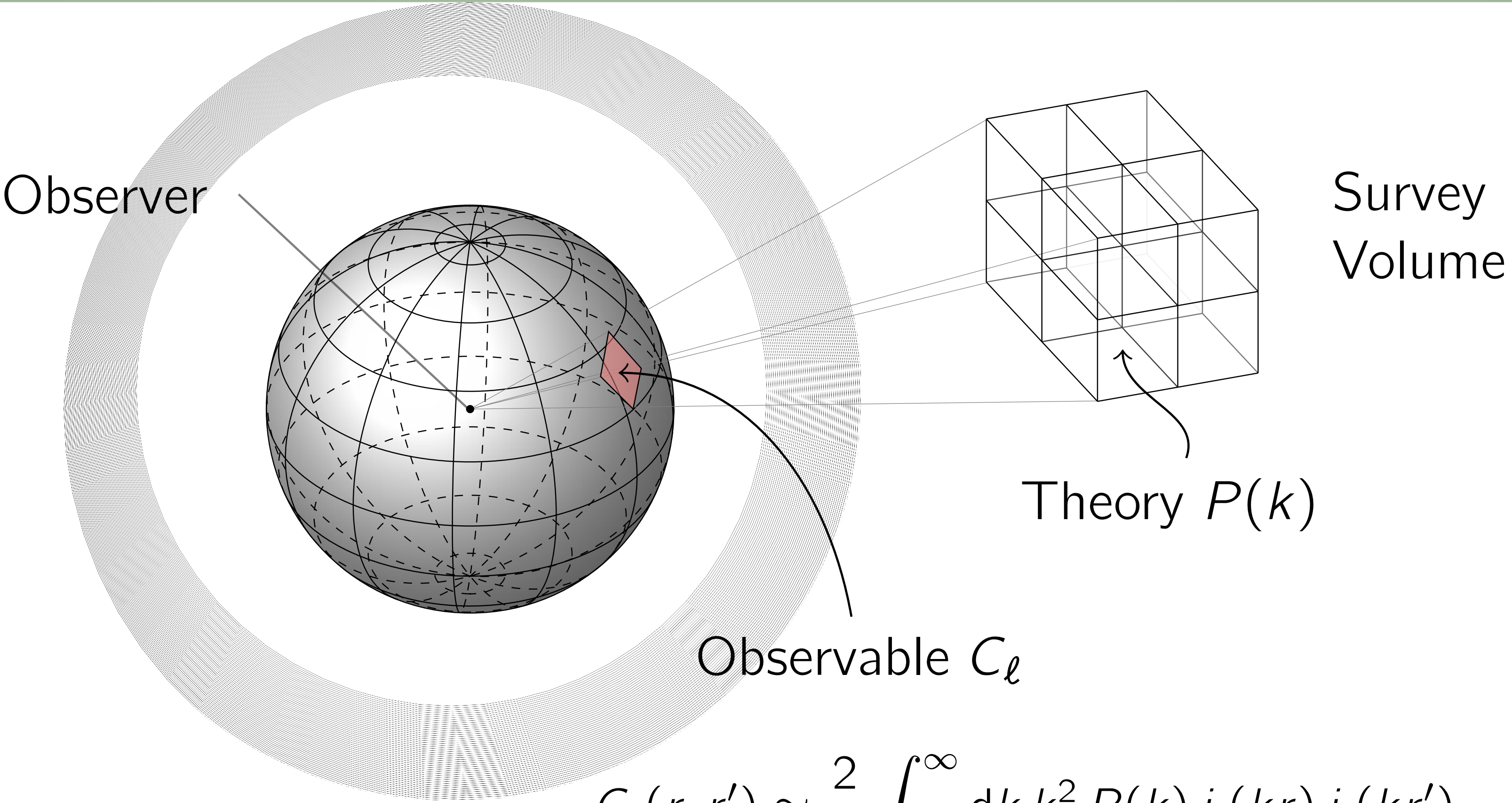


Advantages of Spherical Harmonic Space

- **Natural coordinate system** for observer position and radial/angular separation
- **Redshift evolution** (e.g. Volume effects, growth of structure, evolution of luminosity function)
- **Lensing**: Light from high-redshift galaxies must pass through matter distribution associated with low-redshift galaxies.

→SHT is straightforward for modeling this.

Spherical Harmonic Space



$$C_\ell(r, r') \simeq \frac{2}{\pi} \int_0^\infty \mathrm{d}k\, k^2\, P(k)\, j_\ell(kr)\, j_\ell(kr')$$

Challenges with Spherical Harmonic Tomography

- Data: Complex spherical transform → HEALPix

$$a_{\ell m}(z) = \int d^2\hat{s}\, Y^*_{\ell m}(\hat{s})\, \delta(\vec{s})$$

- Theory: Need to integrate highly oscillatory functions → 2-FAST algorithm

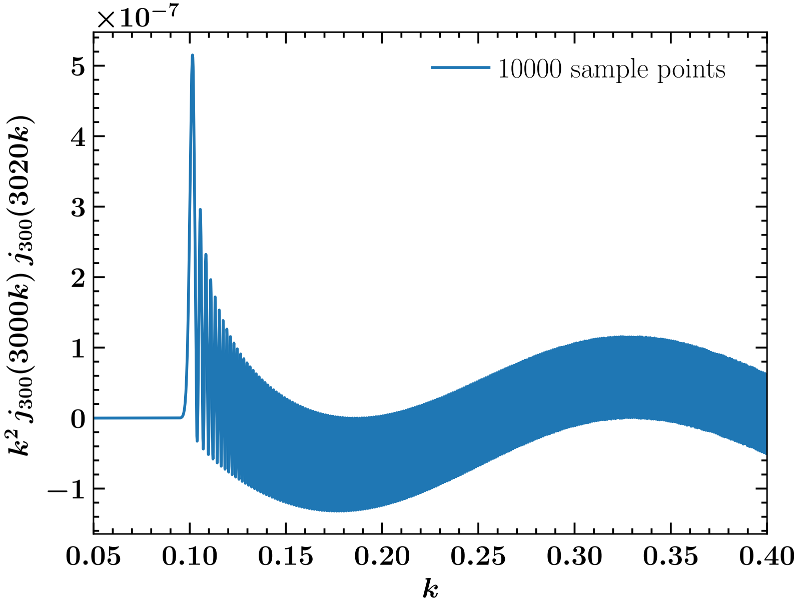
$$C_\ell(z, z') = \frac{2}{\pi} \int dk\, k^2\, P(k)\, j_\ell(kr)\, j_\ell(kr')$$

- Non-diagonal covariance matrix
- Non-linear effects can bleed into the measurement on all scales

Spherical Harmonic projection ~~is~~ *was* hard!

$$C_\ell(r, r') \simeq \frac{2}{\pi} \int_0^\infty \mathrm{d}k\, k^2\, P(k)\, j_\ell(kr)\, j_\ell(kr')$$

- Spherical Bessel functions are highly oscillatory



2-FAST: an algorithm for projecting the 2-point function into spherical harmonic space

2-point function from **F**ast and **A**ccurate **S**pherical Bessel **T**ransform

$$C_\ell(r, r') \simeq \frac{2}{\pi} \int_0^\infty \mathrm{d}k\, k^2\, P(k)\, j_\ell(kr)\, j_\ell(kr')$$

2-FAST allows us to efficiently calculate spherical harmonic projection with linear Kaiser effect.

… but may also be useful for perturbation theory!

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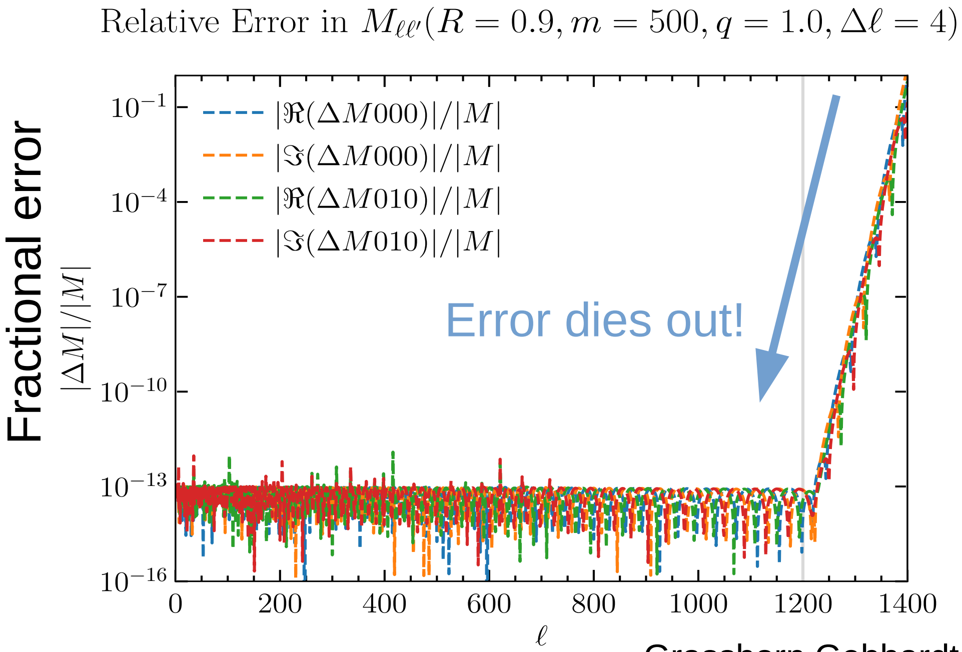
2-FAST is a spectral method

A spectral method:

- Integral becomes convolution in logarithmic space.
- Convolution in real space is multiplication in Fourier space
  1) Fast Fourier Transform (FFT) power spectrum P(k)
  2) Multiply with Fourier kernel
  3) Inverse Fast Fourier Transform → C_ℓ
- Need Fourier kernel, calculated using *Miller's algorithm*.

Miller's algorithm

- Recurrence relation: $$j_{\ell-1}(x) - \frac{2\ell+1}{x} j_\ell(x) + j_{\ell+1}(x) = 0$$
  → Unstable in the $\ell \to \ell + 1$ direction
- Error dies out as we go from $\ell \to \ell - 1$
- Match to analytical solution at $\ell = 0$.



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More Spherical Harmonic Problems to Solve: Projecting non-linear terms

- 2-FAST calculates $C_\ell(r,r')$ with linear Kaiser effect.
- Non-linear terms in P(k) → Integral diverges!

  → Fingers of God (FoG): Bang!

  → Non-linear Kaiser: Bang!

$$C_\ell(r,r') \simeq \frac{2}{\pi}\int_0^\infty \mathrm{d}k\, k^2\, P(k)\, j_\ell(kr)\, j_\ell(kr')$$

- Solution:

  → FoG result in 2D convolution of $C_\ell(r,r')$ with, e.g., Gaussian

  → Non-linear Kaiser can be calculated as derivatives on the window function.

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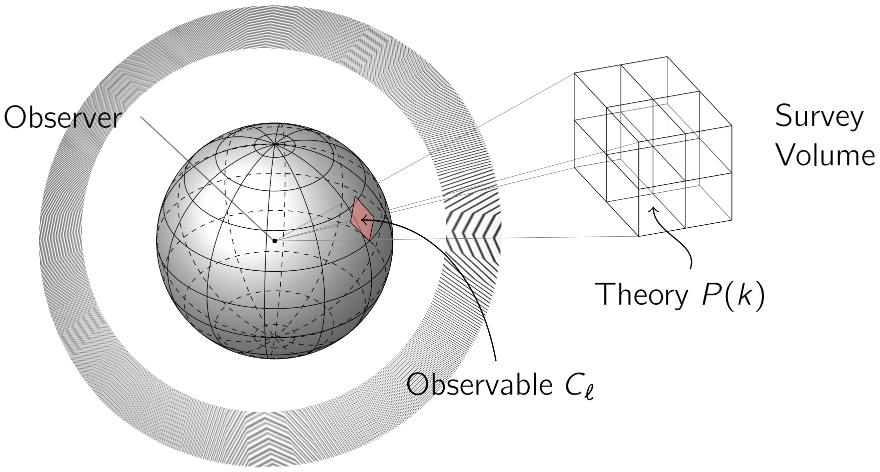
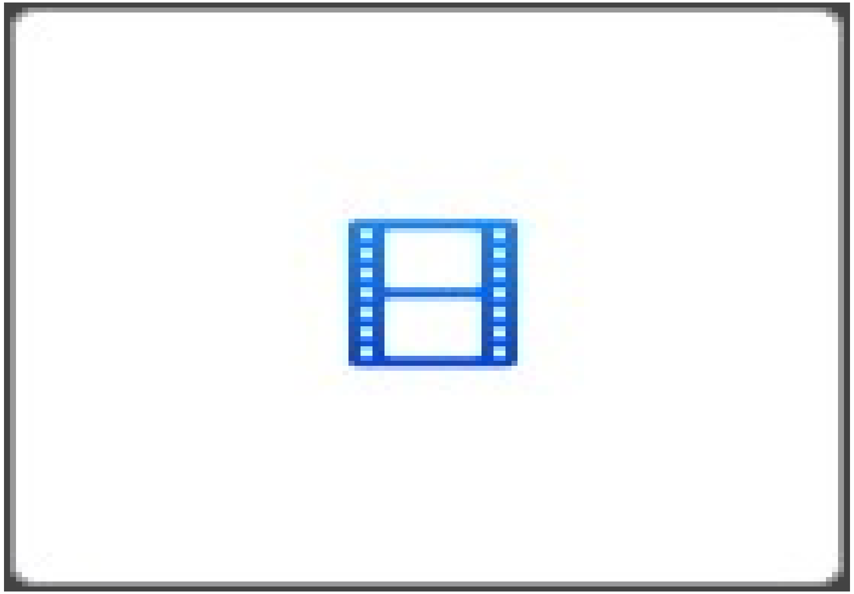
Calculating $C_\ell(z,z')$

- Projection:

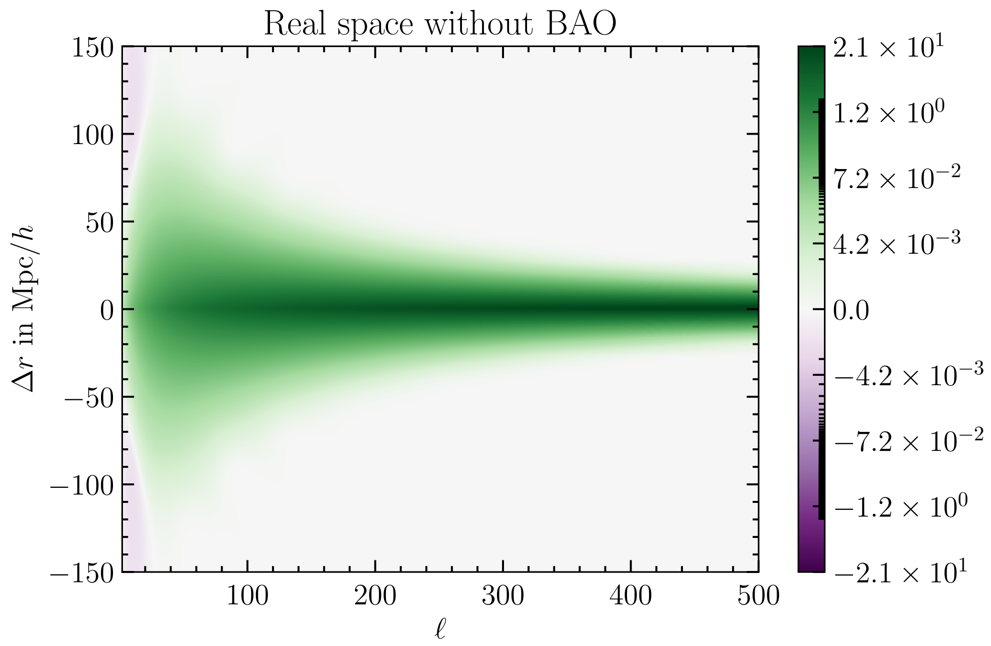
  $P(k_\perp, k_\parallel) \rightarrow C_\ell(z,z')$





Spherical harmonic Space: $C_\ell(r_{mid},\Delta r)$

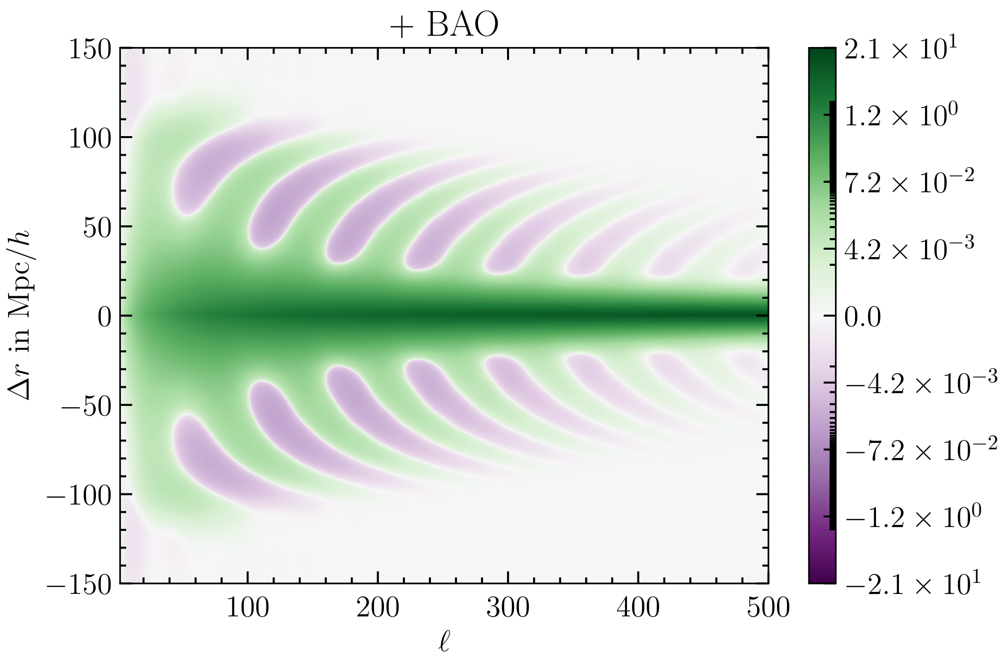
- Real space, no BAO



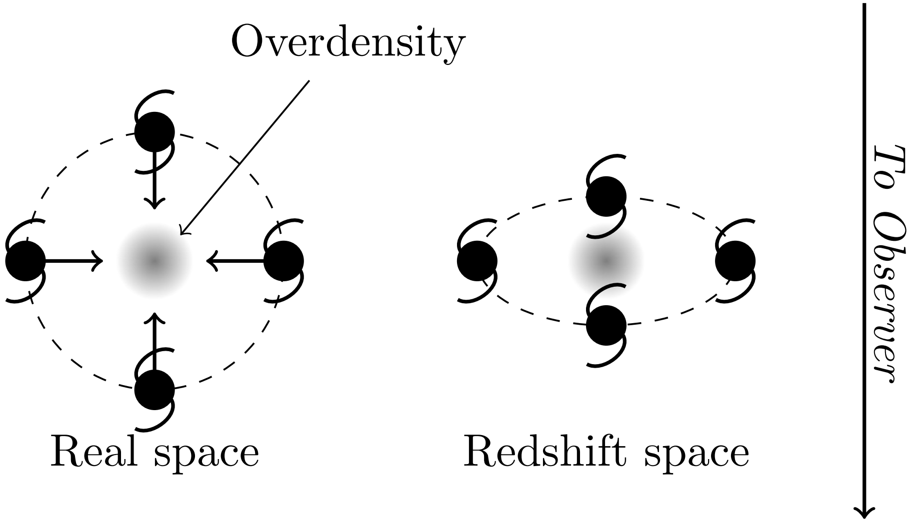
$k_\perp r \sim \ell + 0.5$

$C_\ell(r_{mid},\Delta r)$

- Real space
  + BAO



The Kaiser Effect



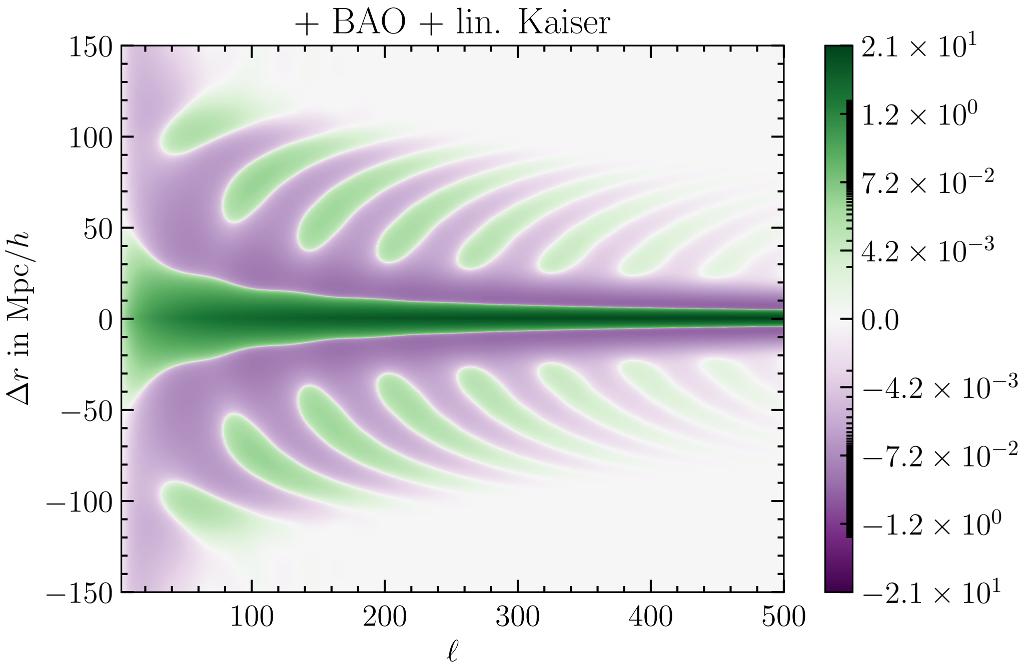
- Galaxies tend to move towards overdensities.
- ➔ Increased clustering along the line of sight direction.

$C_\ell(r_{mid},\Delta r)$

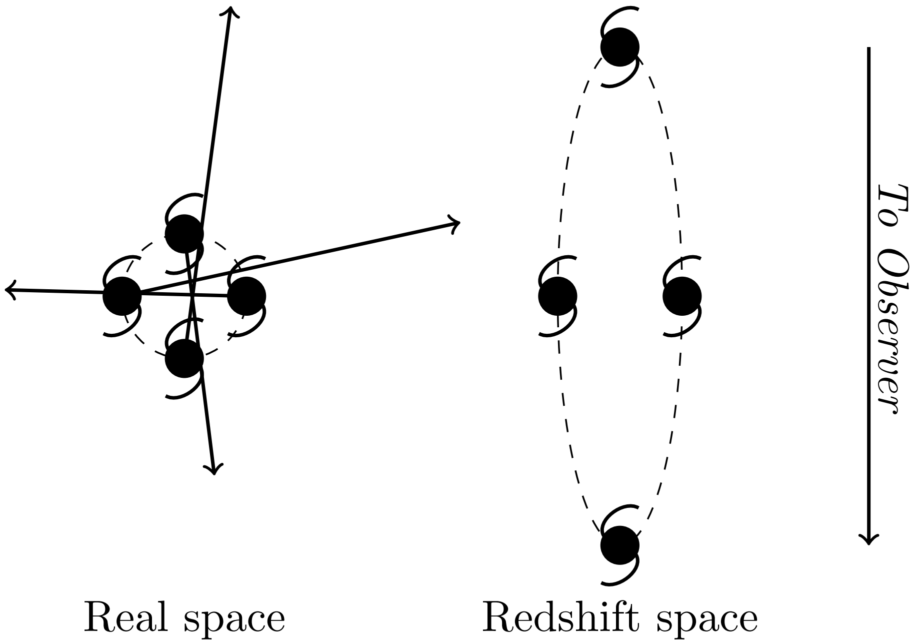
- Real space

  + BAO

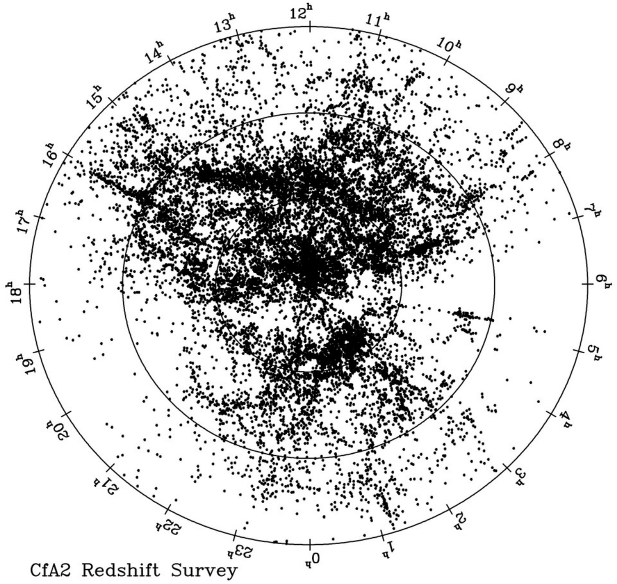
  + Linear Kaiser: coherent inflow into galaxy clusters



Fingers of God



- Stochastic motions of galaxies
- ➔ Suppression of clustering in line of sight direction.



$C_\ell(r_{mid},\Delta r)$

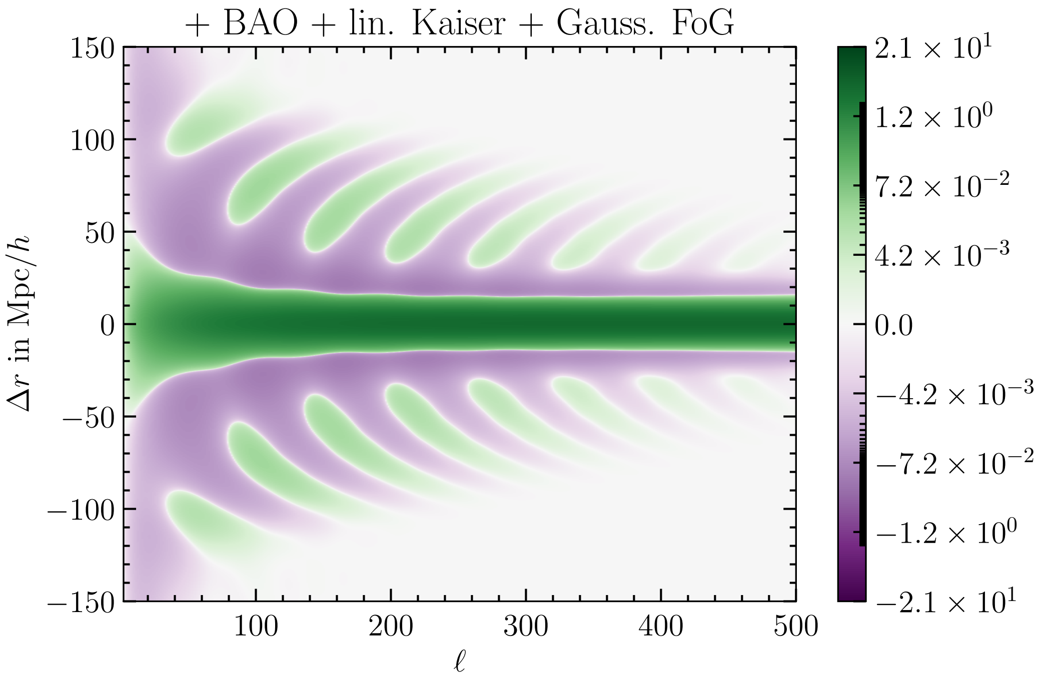
- Real space

  + BAO

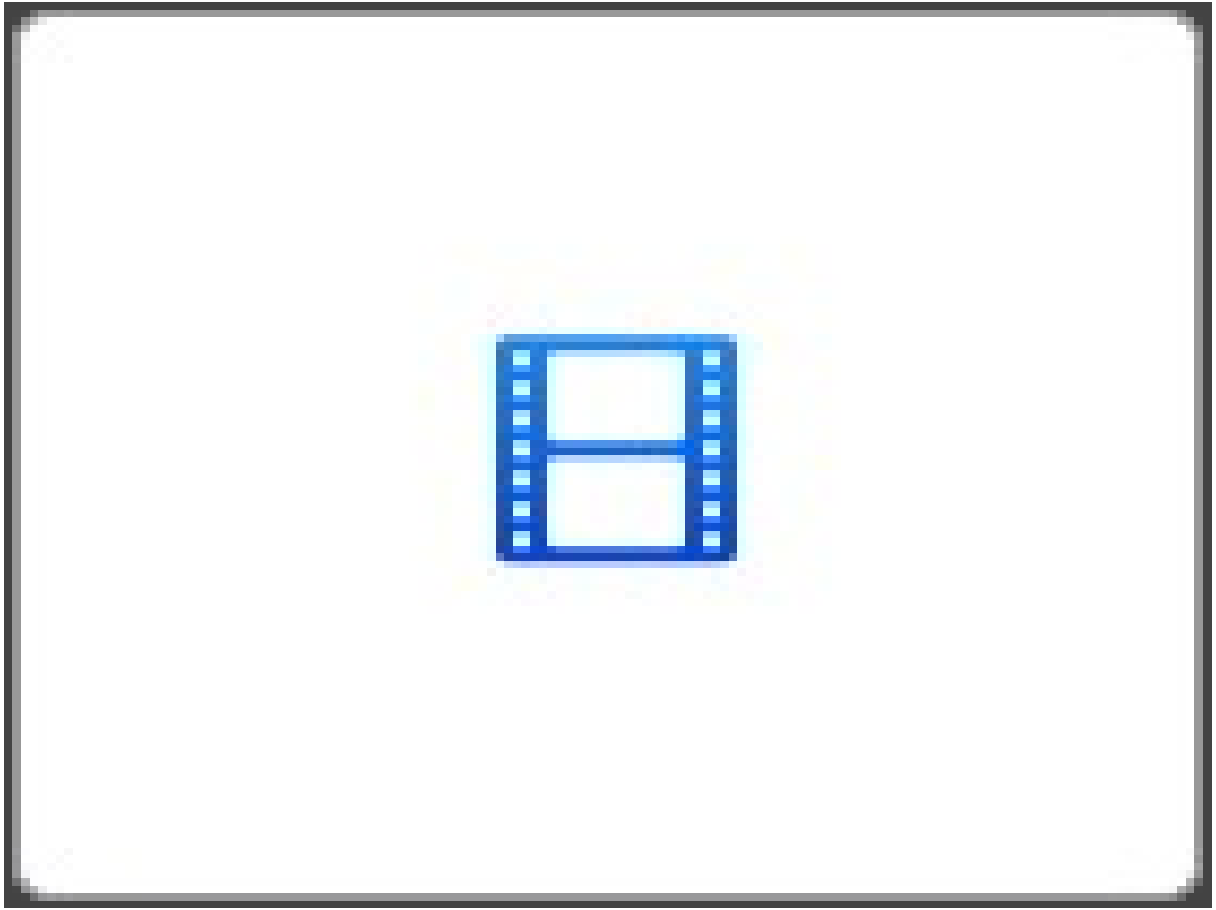
  + Linear Kaiser: coherent inflow into galaxy clusters

  + Fingers of God (FoG) blur our vision!

➔ Redshift space

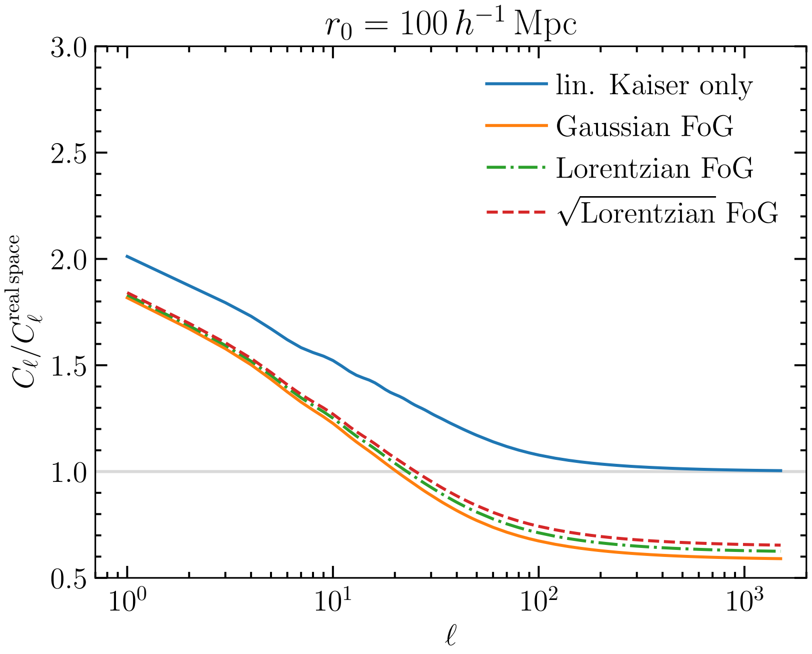


Trilobite? Rib cage?



$$\frac{\partial j_\ell(kr)}{\partial(kr)} \simeq -\frac{\partial j_\ell(kr)}{\partial \ell}$$

Non-linear effects bleed into all transverse scales



Δr = 8 Mpc/h

- FoG lead to suppression on all transverse scales
- FoG is less for thicker bins

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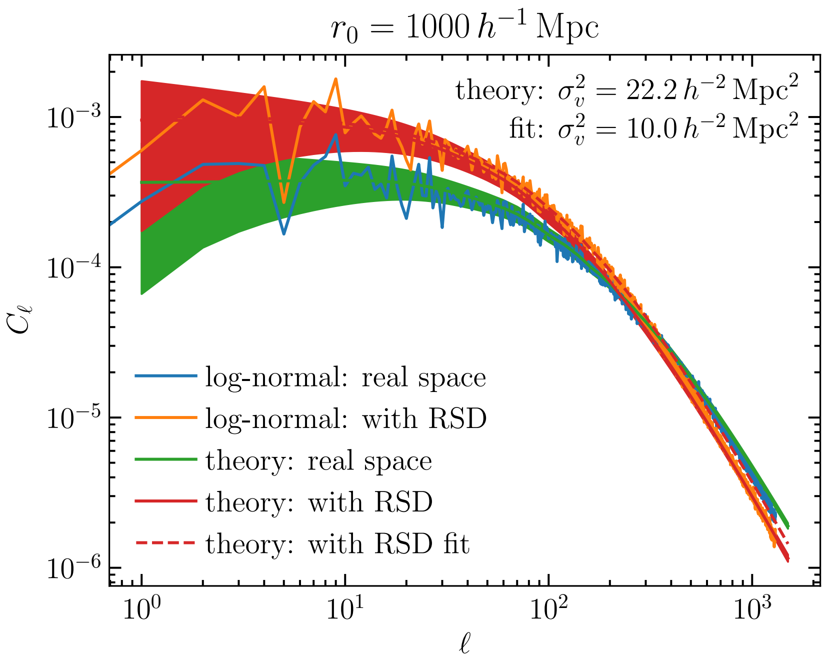
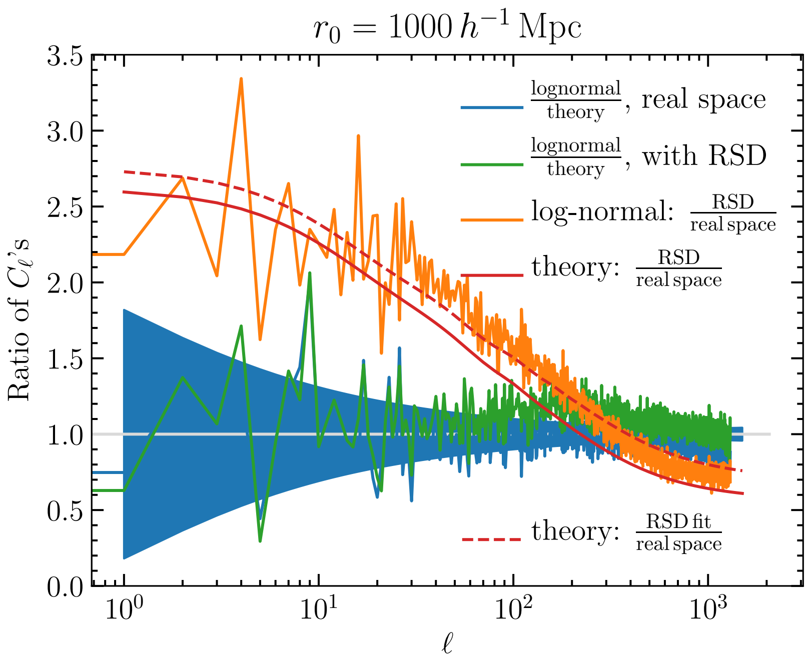
Comparison with Log-normal Simulation





Bin width $8\,h^{-1}\,\mathrm{Mpc}$

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Conclusion and Future Outlook

- Spherical Harmonic (SH) projection cleanly separates radial and angular effects
- SH projection is 2-FAST
- Fingers-of-God and non-linear Kaiser can be done in spherical harmonic space
- Thin bins lead to non-linear effects on all scales

- Future:
  - Include more non-linear terms
  - Apply to data!