Bootstrapping Inflationary Correlators

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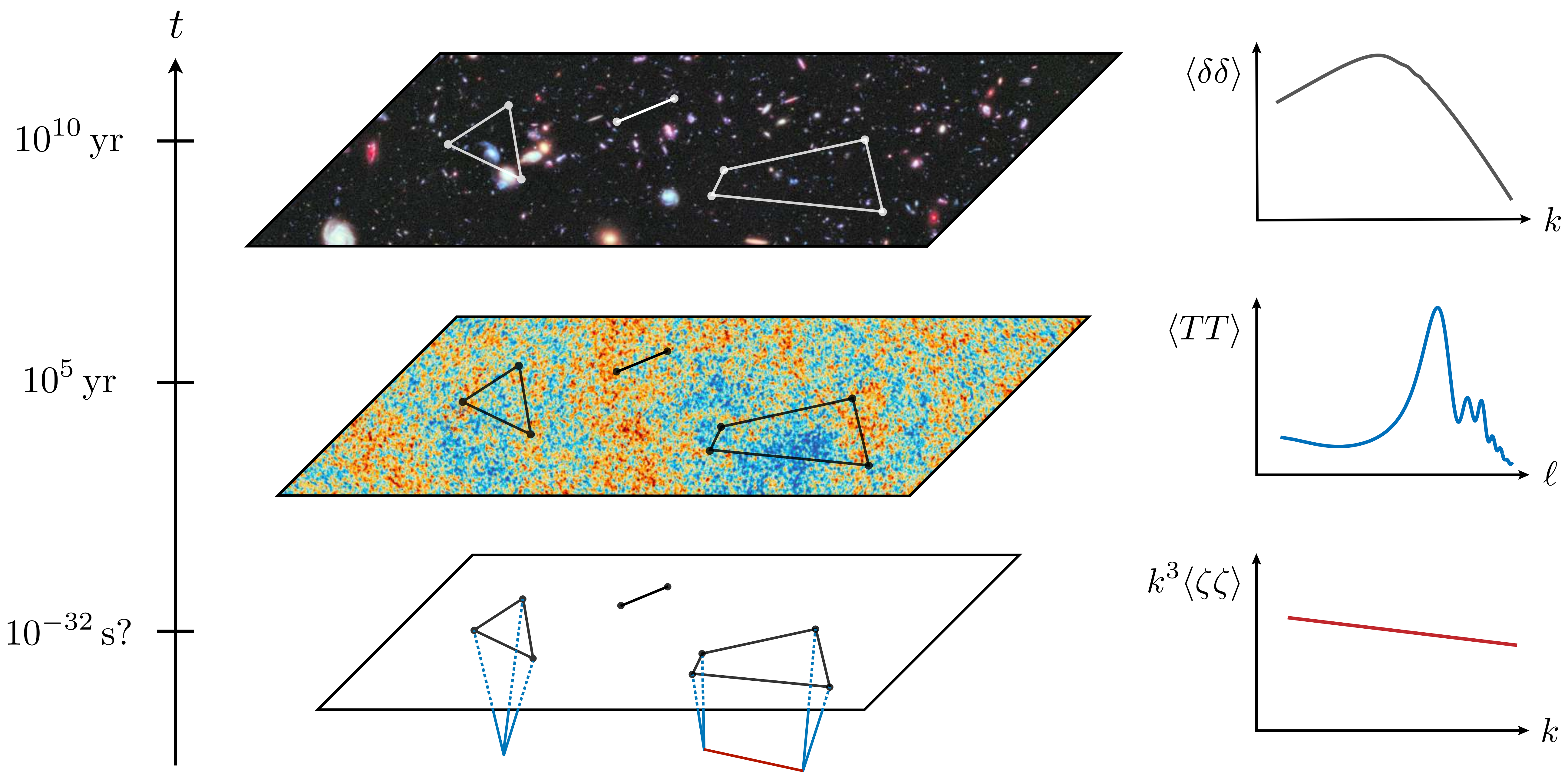
[arXiv:1811.00024, 1910.14051, 2005.04234]

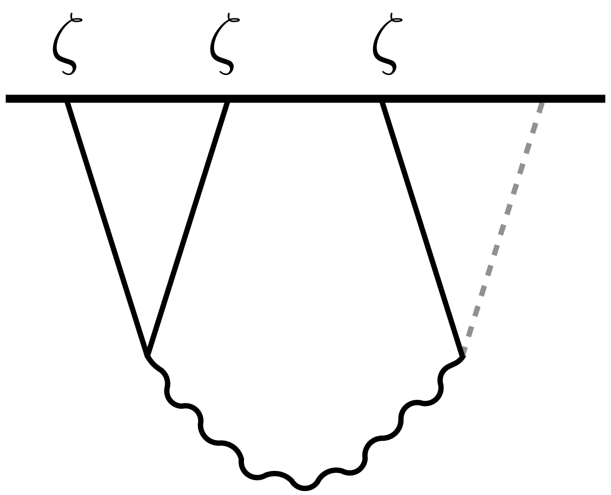
Primordial Fluctuations



New particles can appear as special patterns in cosmological correlations.

Primordial Non-Gaussianity

Feynman diagram calculations are rather complicated.

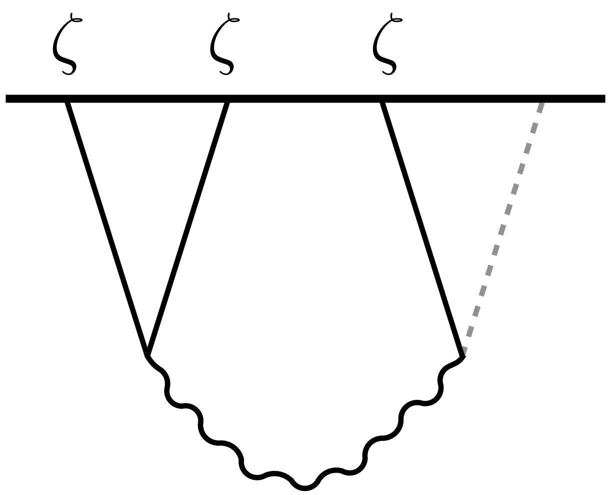


$\leftarrow$

$$
\begin{aligned}
\mathcal{L}_\zeta^{(3)} = {} & 3H^2\alpha^3 - \epsilon H^2\alpha^3 - 9H^2\alpha^2\zeta + 3\epsilon H^2\alpha^2\zeta + \frac{27}{2}H^2\alpha\zeta^2 \\
& - \frac{9}{2}\epsilon H^2\alpha\zeta^2 - \frac{27}{2}H^2\zeta^3 + \frac{9}{2}\epsilon H^2\zeta^3 - 6H\alpha^2\dot{\zeta} + 18H\alpha\zeta\dot{\zeta} \\
& - 27H\zeta^2\dot{\zeta} + 3\alpha(\dot{\zeta})^2 - 9\zeta(\dot{\zeta})^2 - \frac{2}{a^2}H\alpha b_i\partial_i\zeta + \frac{2}{a^2}H\zeta b_i\partial_i\zeta \\
& - \frac{2}{a^2}H\alpha\partial_i\beta\partial_i\zeta + \frac{2}{a^2}H\zeta\partial_i\beta\partial_i\zeta + \frac{2}{a^2}b_i\dot{\zeta}\partial_i\zeta + \frac{2}{a^2}\partial_i\beta\dot{\zeta}\partial_i\zeta \\
& - \frac{1}{a^2}\alpha(\partial_i\zeta)^2 - \frac{1}{a^2}\zeta(\partial_i\zeta)^2 - \frac{1}{a^4}b_j\partial_i\zeta\partial_j b_i - \frac{1}{a^4}\partial_j\beta\partial_i\zeta\partial_j b_i \\
& - \frac{1}{a^4}b_i\partial_j\zeta\partial_j b_i - \frac{1}{a^4}\partial_i\beta\partial_j\zeta\partial_j b_i - \frac{1}{4a^4}\alpha(\partial_j b_i)^2 - \frac{1}{4a^4}\zeta(\partial_j b_i)^2 \\
& - \frac{1}{4a^4}\alpha\partial_j b_i\partial_i b_j - \frac{1}{4a^4}\zeta\partial_j b_i\partial_i b_j + \frac{2}{a^2}H\alpha^2\partial^2\beta - \frac{2}{a^2}H\alpha\zeta\partial^2\beta \\
& + \frac{1}{a^2}H\zeta^2\partial^2\beta - \frac{2}{a^2}\alpha\dot{\zeta}\partial^2\beta + \frac{2}{a^2}\zeta\dot{\zeta}\partial^2\beta - \frac{2}{a^4}b_j\partial_i\zeta\partial_i\partial_j\beta \\
& - \frac{2}{a^4}\partial_j\beta\partial_i\zeta\partial_i\partial_j\beta - \frac{1}{a^4}\alpha\partial_j b_i\partial_i\partial_j\beta - \frac{1}{a^4}\zeta\partial_j b_i\partial_i\partial_j\beta \\
& - \frac{1}{2a^4}\alpha(\partial_i\partial_j\beta)^2 - \frac{1}{2a^4}\zeta(\partial_i\partial_j\beta)^2 + \frac{1}{2a^4}\alpha(\partial^2\beta)^2 + \frac{1}{2a^4}\zeta(\partial^2\beta)^2 \\
& - \frac{2}{a^2}\alpha\zeta\partial^2\zeta - \frac{1}{a^2}\zeta^2\partial^2\zeta - \frac{9}{2}V\alpha\zeta^2 - \frac{9}{2}V\zeta^3
\end{aligned}
$$

Primordial Non-Gaussianity

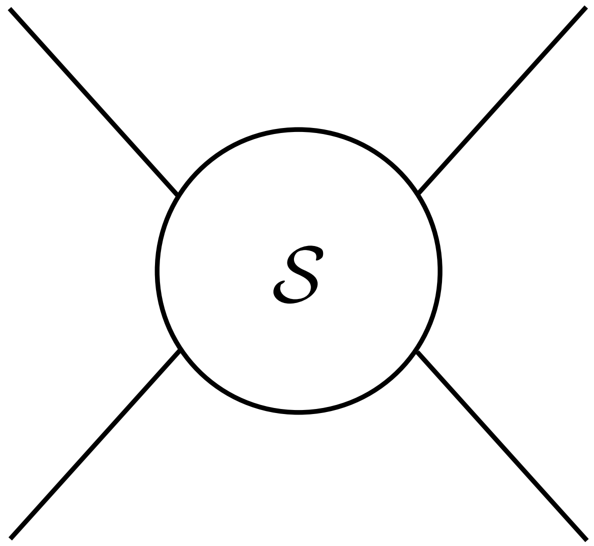
Yet, the final answers are remarkably simple.



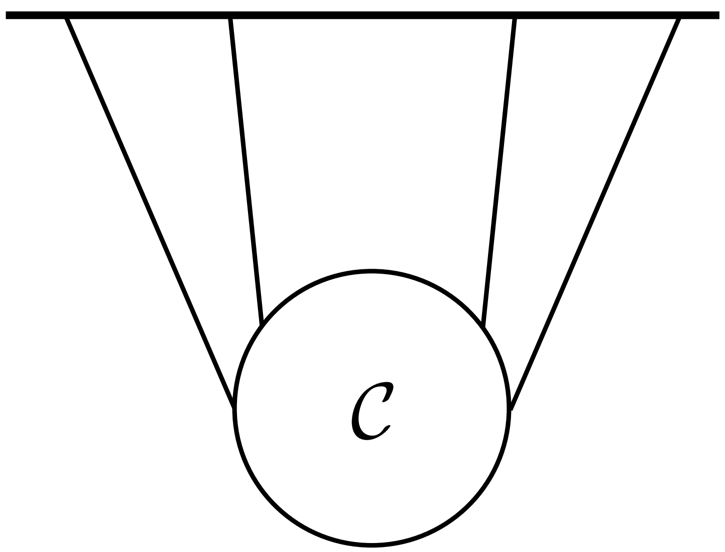
$$= \quad \underbrace{\varepsilon \left[\sum_{n \neq m} k_n k_m^2 + \frac{8}{E} \sum_{n > m} k_n^2 k_m^2 \right]}_{\dfrac{\mathrm{Poly}[k_1, k_2, k_3]}{E}} + \underbrace{(n_s - 1) \sum_n k_n^3}_{\mathrm{Local}}$$

Maldacena [2002]

The Bootstrap



"S-Matrix Bootstrap"



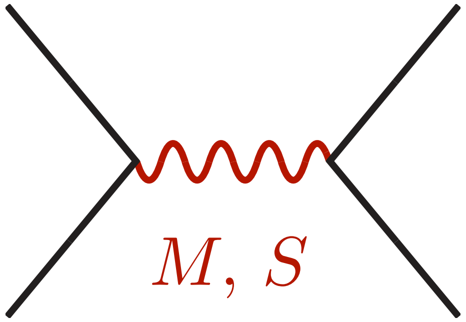
"Cosmological Bootstrap"

Idea: Use consistency principles to determine observables without Feynman diagrams and Lagrangians.

The S-Matrix Bootstrap

In flat space, scattering amplitudes are constrained by:

Lorentz invariance, unitarity, locality.

Simple: 

$$= \frac{g^2}{s - M^2} P_S \left(1 + \frac{2t}{M^2}\right)$$

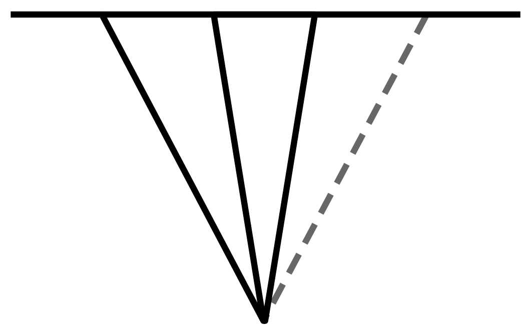
Frontier: Multi-leg, multi-loop: needed for the LHC

The Cosmological Bootstrap

In quasi-de Sitter space, boundary correlators are constrained by:

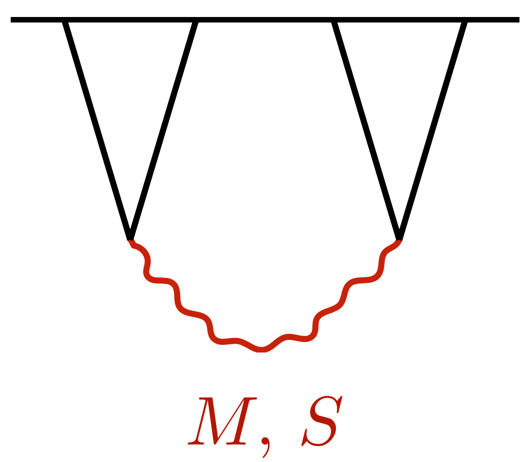
(approximate) conformal symmetry, singularities.

Simple:



$$= \frac{\mathrm{Poly}[k_1, k_2, k_3]}{E^{\#}}$$

Frontier:



needed for next gen. CMB, LSS exp.

Outline

1. The Cosmological Bootstrap
2. Cosmological Collider Physics
3. Outlook

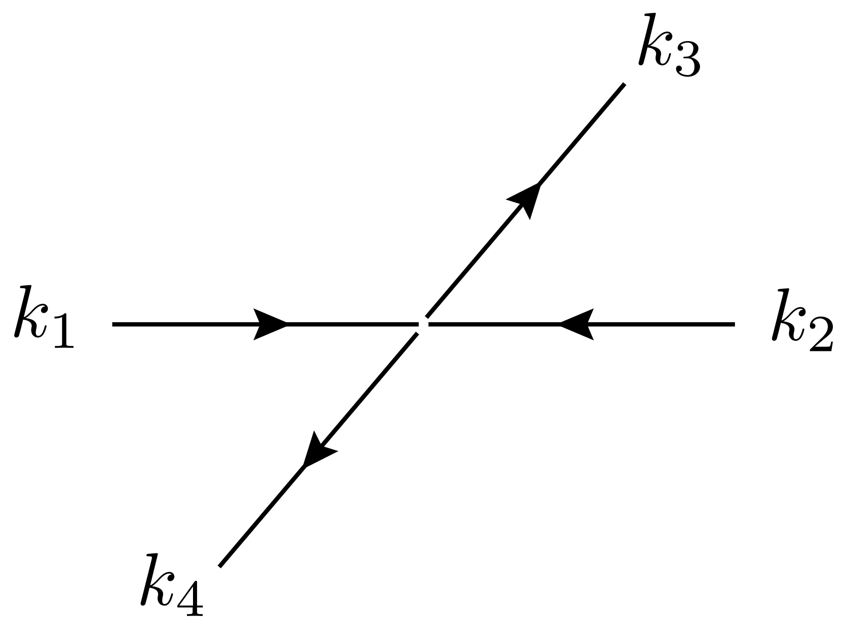
The Cosmological Bootstrap

Arkani-Hamed, Baumann, HL, Pimentel [2018]

Kinematics

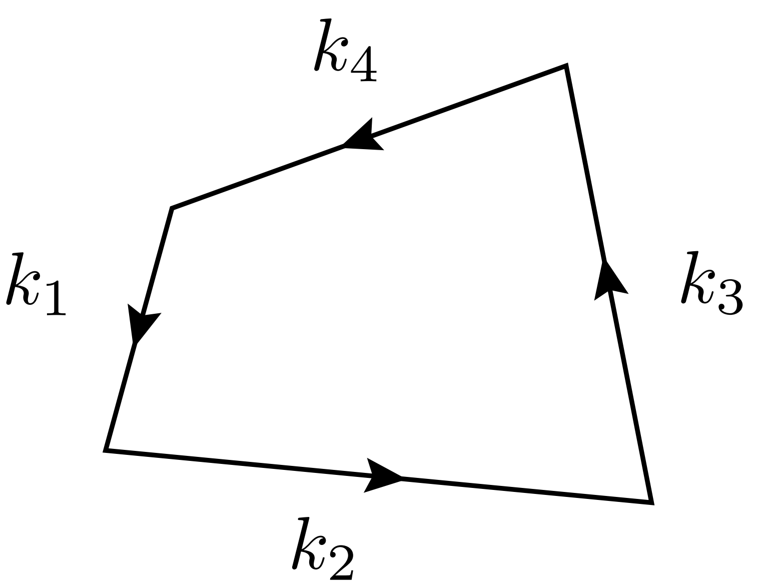
$$\langle \phi_{\vec{k}_1}\phi_{\vec{k}_2}\phi_{\vec{k}_3}\phi_{\vec{k}_4}\rangle = F(\vec{k}_1,\cdots,\vec{k}_4)\delta_{\rm D}(\vec{k}_1+\cdots+\vec{k}_4)$$

Amplitude



$$\delta_{\rm D}(E = |\vec{k}_1| + \cdots + |\vec{k}_4|)$$

Cosmological correlator



$$\frac{A}{E^p} + \cdots$$

Amplitude

Symmetries

Invariance under dilatations and SCTs imply the following constraints:

$$0 = \sum_n \left[1 + \vec{k}_n \cdot \partial_{\vec{k}_n}\right] F$$

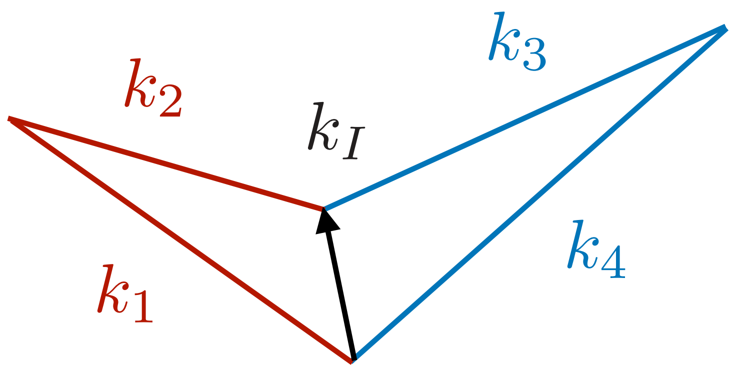
$$0 = \sum_n \left[\vec{k}_n (\partial_{\vec{k}_n} \cdot \partial_{\vec{k}_n}) - 2(\vec{k}_n \cdot \partial_{\vec{k}_n})\partial_{\vec{k}_n} - 2\partial_{\vec{k}_n}\right] F$$

$(m_\phi = \sqrt{2}H)$

"conformal scalars"

The dilatation constraint is solved by

$$\boxed{F = \frac{1}{k_I}\hat{F}(u, v)}$$



$$u \equiv \frac{k_I}{k_1 + k_2} \qquad v \equiv \frac{k_I}{k_3 + k_4}$$

Symmetries

For conformal scalars, the SCT constraints become

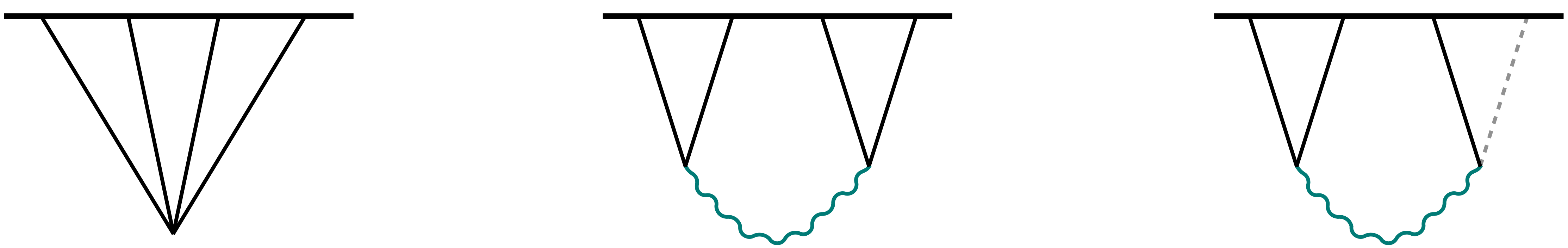
$$(\Delta_u - \Delta_v)\hat{F} = 0$$

$$\Delta_u = u^2(1-u^2)\partial_u^2 - 2u^3\partial_u$$

"hypergeometric"

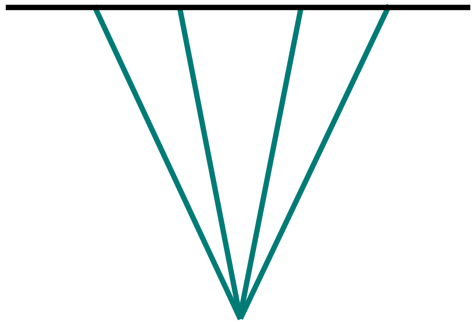
The solutions can be classified according to their analytic structure:

Contact ⟶ Exchange ⟶ Inflationary 3-pt



Dynamics: Contact

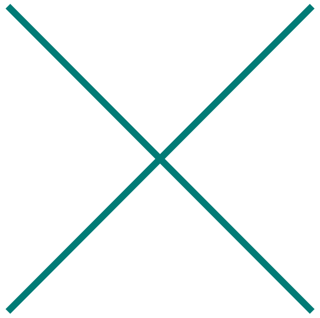
Contact diagrams carry the "simplest possible" analytic structure.



$$= \sum_n c_n \Delta_u^n \left(\frac{1}{E}\right)$$

dS space

flat space

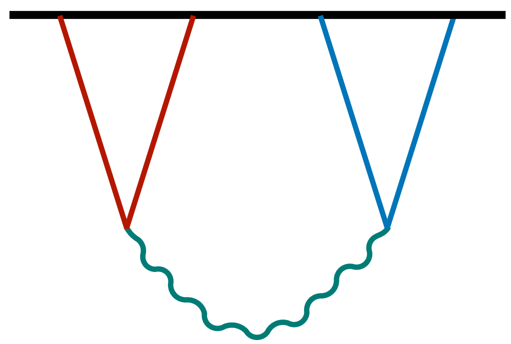


$$= \sum_n c_n s^n$$

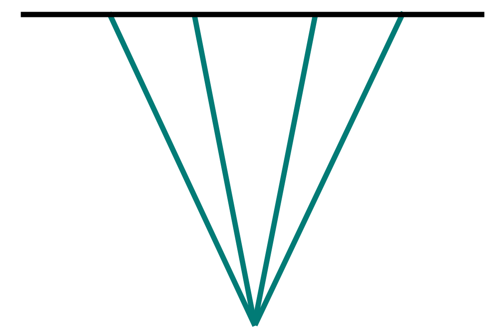
Dynamics: Exchange

For tree exchange, the constraint separates into two ODEs:

$(\Delta_u + M^2)$
$(\Delta_v + M^2)$



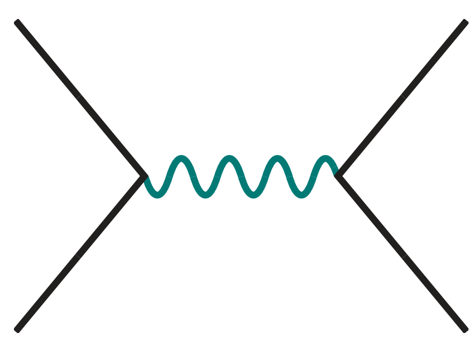
$=$



dS space

flat space

$(s - M^2)$



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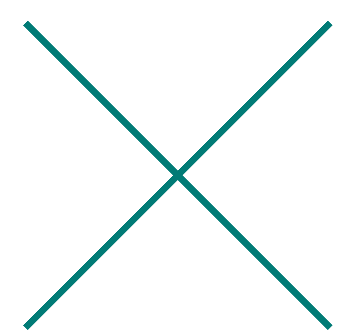
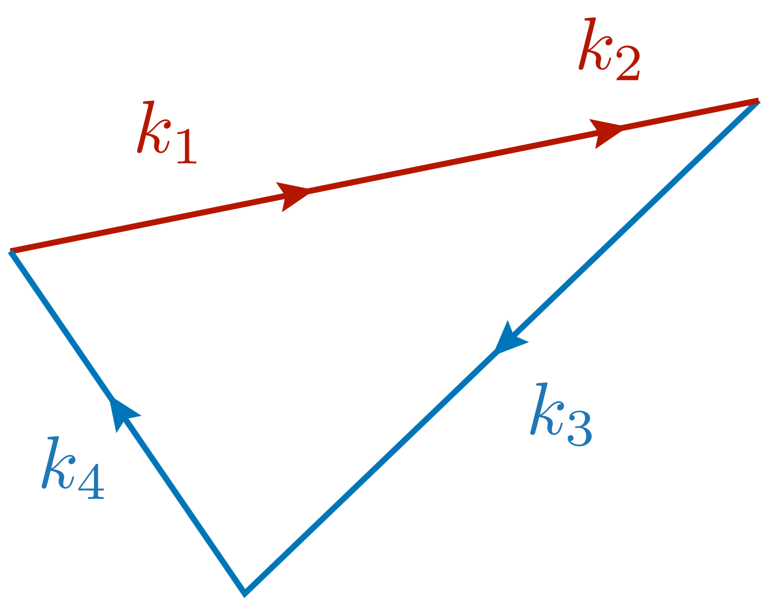


Singularities

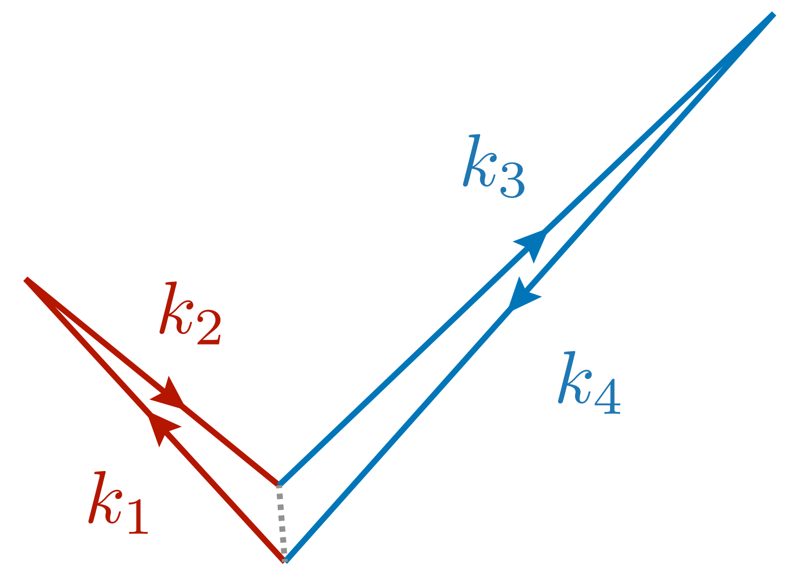
To solve the 2nd order ODE, we impose two boundary conditions:

folded limit



$$\hat{F} \xrightarrow{u,v \to 1} \text{regular}$$

collapsed limit



$$\hat{F} \xrightarrow{u,v \to 0} 3\text{pt} \times 3\text{pt}$$

Solution

For small u, the solution takes the form

$$\left(\mu = \sqrt{\frac{M^2}{H^2} - \frac{9}{4}}\right)$$

analytic

non-analytic

$$\hat{F} = \sum_{n=0}^{\infty} \frac{(-1)^n}{(n+\frac{1}{2})^2 + \mu^2} \left(\frac{u}{v}\right)^{n+1} + \frac{\pi}{\cosh \pi\mu} \frac{\left(\frac{u}{v}\right)^{\frac{1}{2}+i\mu} - c.c.}{2i\mu}$$

$\sim H^2/M^2$

$\sim e^{-\pi M/H}$

EFT expansion

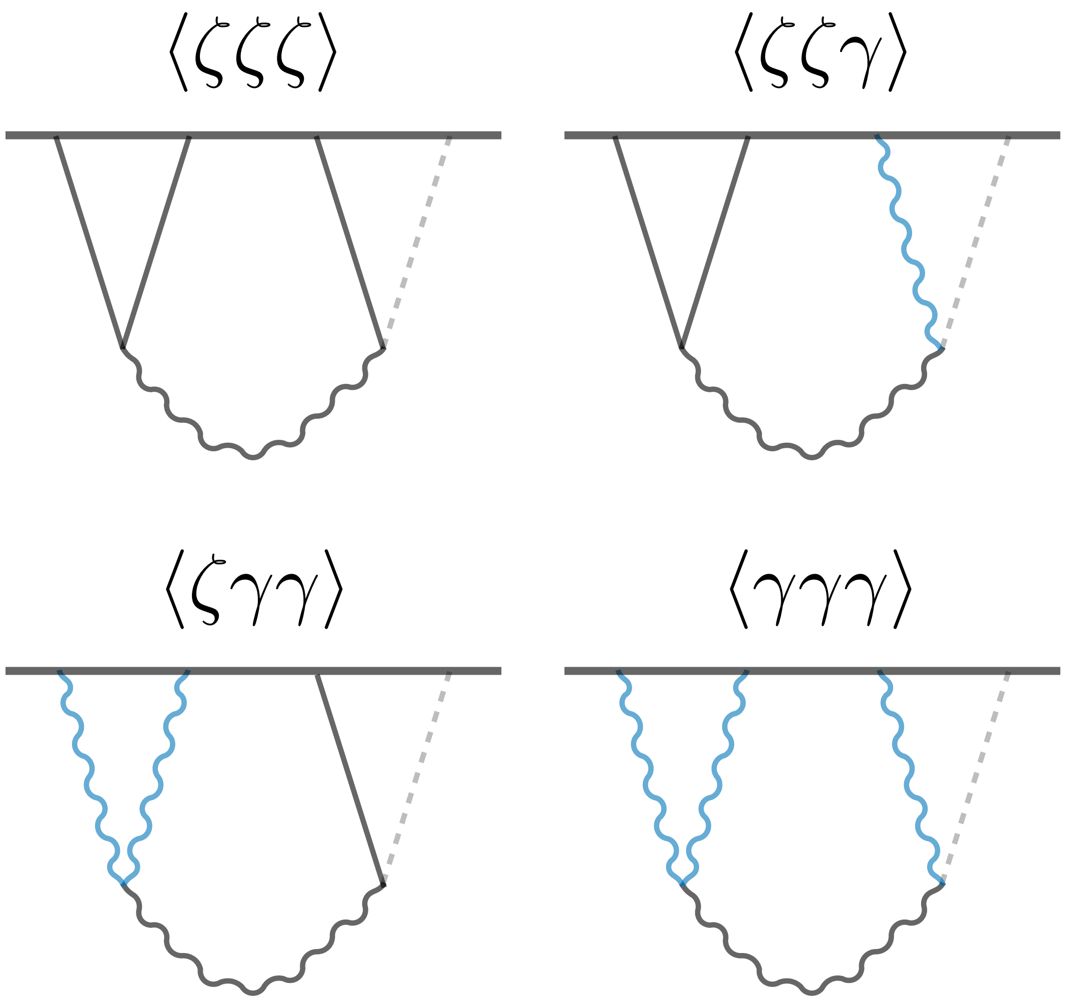
particle production

The full solution can be expressed as a double series expansion.

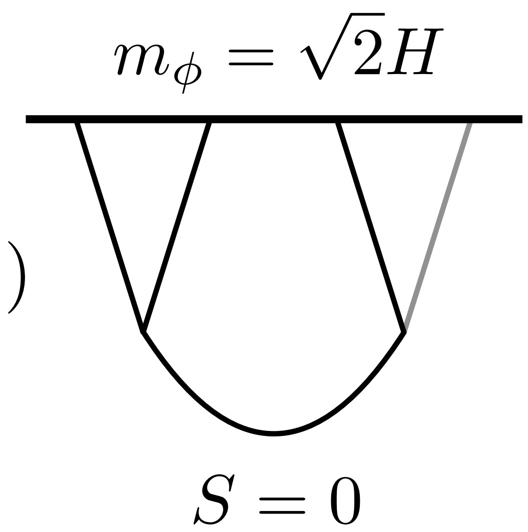
Cosmological Collider Physics

Arkani-Hamed, Baumann, HL, Pimentel [2018]
Baumann, Duaso Pueyo, Joyce, HL, Pimentel [2019, 2020]

Inflationary Correlators

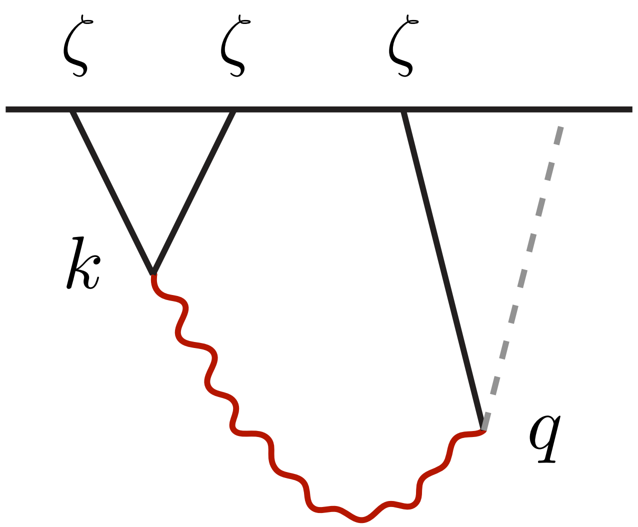


$$= \lim_{k_4 \to 0} \mathcal{D}_{m_i, S_i}(\vec{k}_i, \vec{\partial}_{k_i})$$



Inflationary three-point functions are obtained by acting on the scalar seed with certain differential operators and taking a soft limit.

Squeezed Limit



$$\xrightarrow{q \ll k} \quad \frac{\varepsilon}{\cosh \pi\mu} \left[e^{i\delta} \left(\frac{q}{k}\right)^{\frac{3}{2}+i\mu} + c.c. \right] P_S(\cos\theta)$$

dilution | oscillations

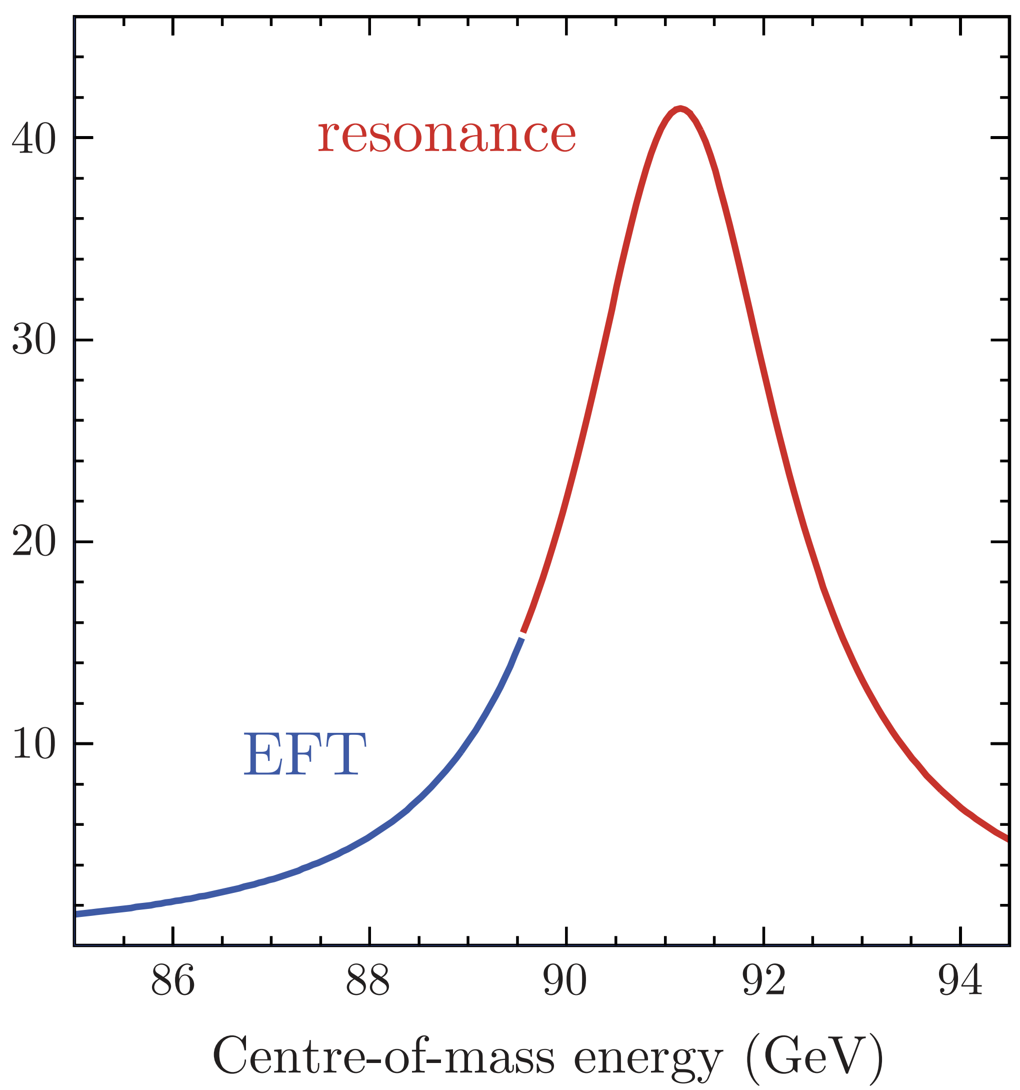
particle production | phase | angular dependence

The squeezed limit provides a clean "detection channel" for extra particles.

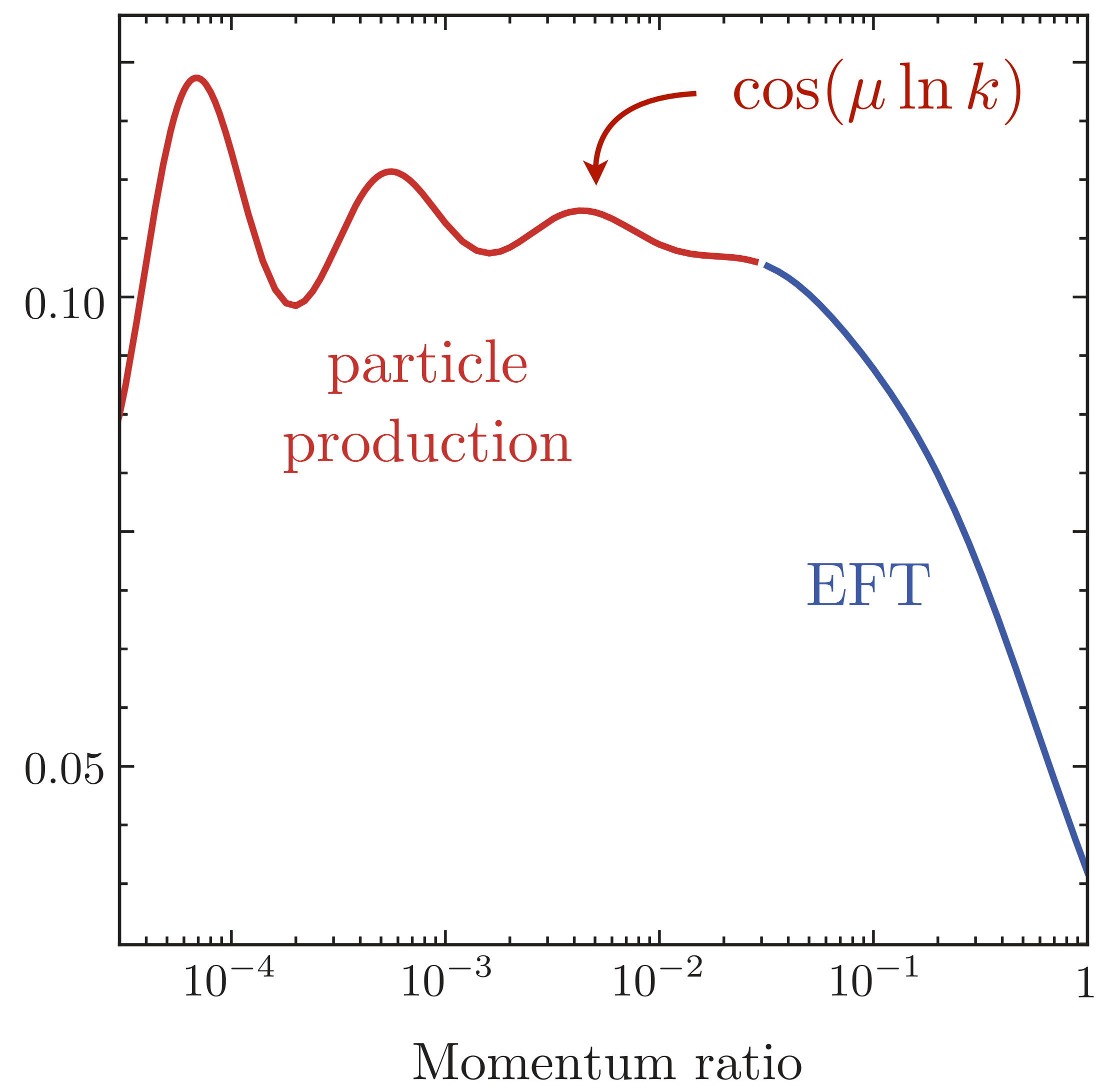
See e.g. Chen, Wang [2009], Baumann, Green [2011], Noumi, Yamaguchi, Yokoyama [2013], Arkani-Hamed, Maldacena [2015], HL, Baumann, Pimentel [2016], ...

Particle Spectroscopy

Amplitude



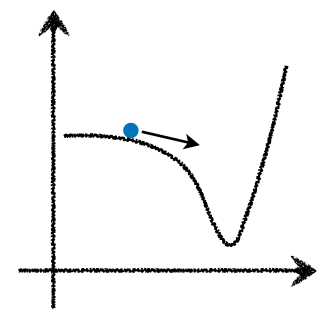
Cosmological correlator



Outlook

Amplitudes/CFT Meet Cosmology

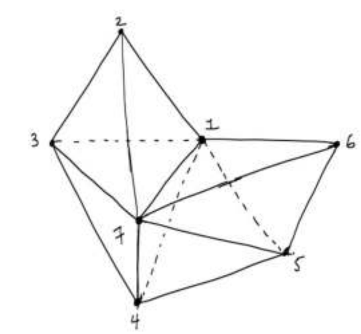
Inflation



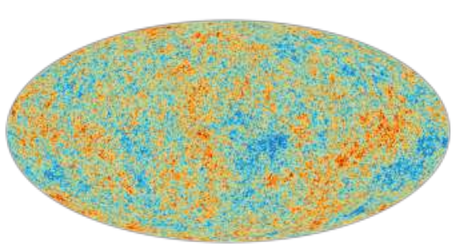
(A)dS/CFT

Cosmological bootstrap

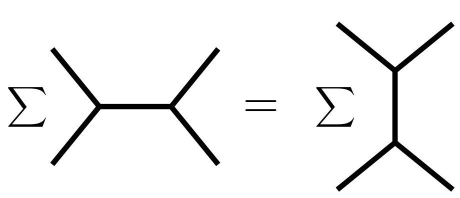
Scattering amplitudes



Observational cosmology



Conformal bootstrap



Open Problems

On the formal side, we would like to understand:

- On-shell formulation of spinning correlators
- Analytic structure of correlators beyond tree level
- Space of consistent theories in dS/inflation

On the phenomenological side, we hope to:

- Generalize the formalism to more general inflationary scenarios
- Derive new observational targets
- Build efficient non-Gaussianity templates