Daniel Vagie

University of Oklahoma

17 August 2020

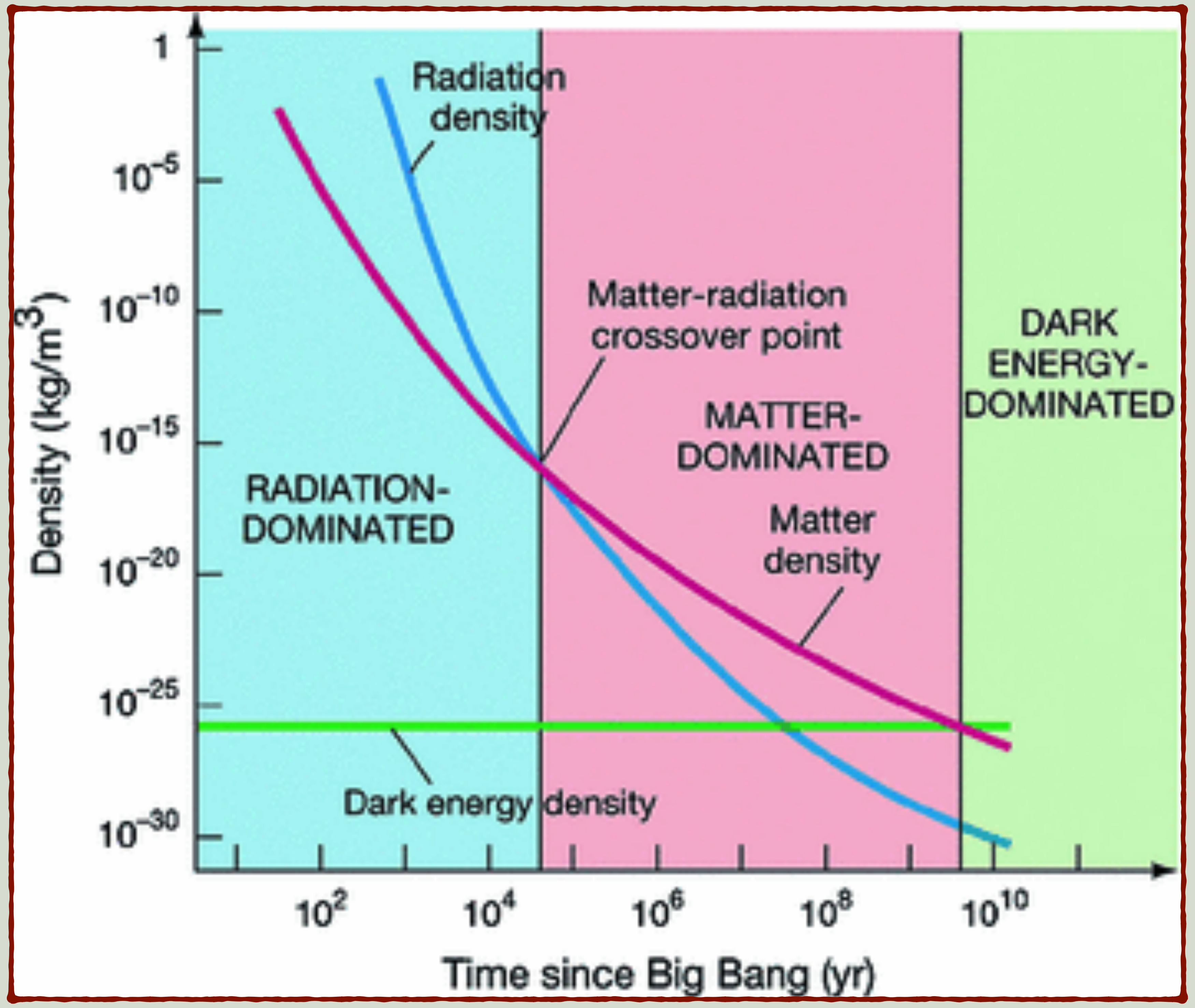
Based on work by
Huaike Guo, Kuver Sinha, DV, Graham White

Phase Transitions in an Expanding Universe: Stochastic Gravitational Waves in Standard and Non-Standard Histories

arXiv:2007.08537

Introduction

De Angelis A., Pimenta M. (2018)

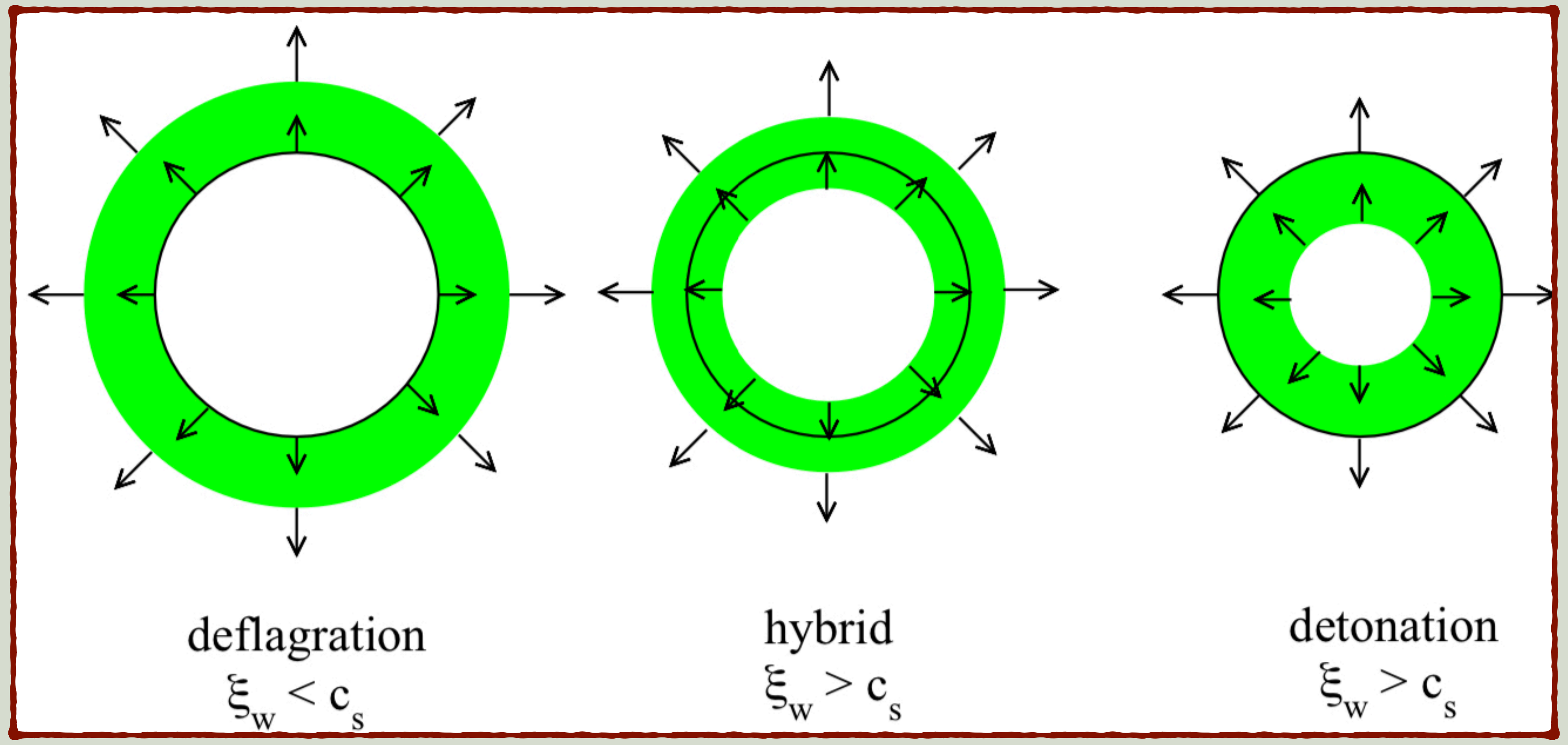


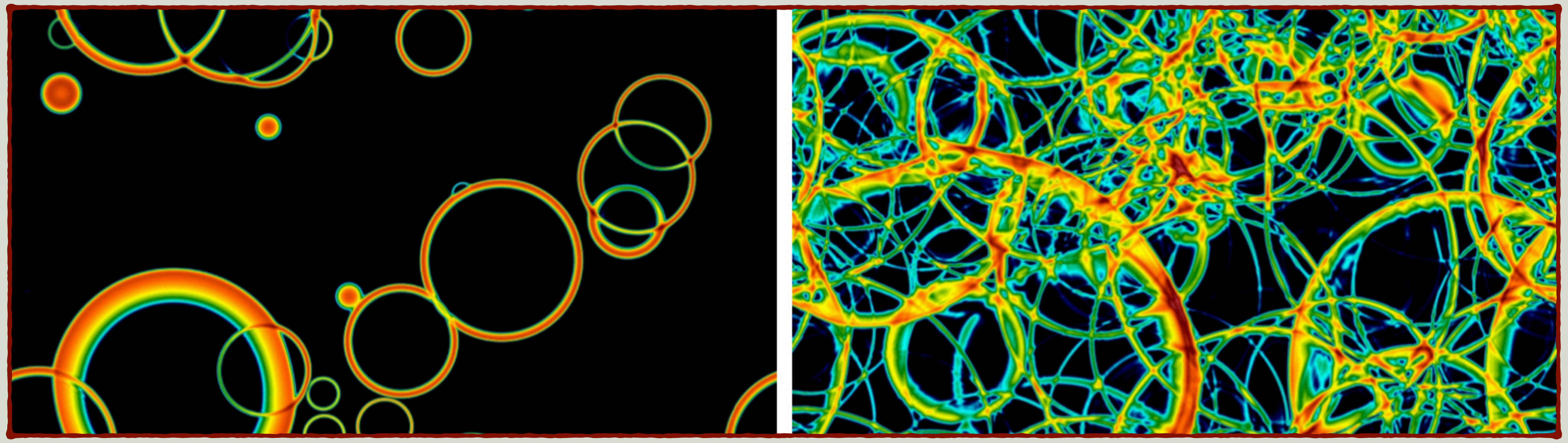
- Standard view of cosmology suggests RDE -> MDE -> Today
- Theoretical motivation for a modified expansion history
- Gravitational waves produced during PT
- Typically assumed PT happened during RDE with equations in Minkowski spacetime
- Sound Shell model (Hindmarsh 2019) provides the best model for the acoustic GWs
- Equations in an expanding universe can be rescaled to have Minkowski form
- Suppression in GW spectrum not seen before

Acoustic Gravitational Waves

- Important source of GWs
- Colliding sound shells
- Compression waves surrounding the expanding bubbles of the stable phase propagate long after the phase transition
- Computable from relativistic hydrodynamics
- Detectable at LISA

arXiv:1004.4187





arXiv:1705.01783

Gravitational Waves in Expanding Universe

- FLRW metric: $ds^2 = -dt^2 + a^2(\delta_{ij} + h_{ij}(\mathbf{x}))d\mathbf{x}^2$
- Conformal time: $dt = a\, d\eta$
- GW sourced by T.T. part of perturbed energy momentum tensor
- Einstein equation describes time evolution of each Fourier component of GWs
- Solve by method of Green's function

$$h_q'' + 2\frac{a'}{a}h_q' + q^2 h_q = 16\pi G a^2 \pi_q^T$$

$$\mathscr{P}_{GW} = \frac{d\Omega_{GW}}{d\ln k} = \frac{1}{24\pi^2 H^2} k^3 P_{\dot{h}}(t,k)$$

Spectral Density of $\dot{h}$

$$\langle \dot{h}_{ij}(t, \mathbf{q}_1)\dot{h}_{ij}(t, \mathbf{q}_2)\rangle = (2\pi)^3\delta^3(\mathbf{q}_1 + \mathbf{q}_2)P_{\dot{h}}(q_1, t)$$

$$h'_{ij}(\eta, \mathbf{q}) = (16\pi G)\int_{\tilde{\eta}_0}^{\tilde{\eta}} d\tilde{\eta}' \frac{\partial G\left(\tilde{\eta}, \tilde{\eta}'\right)}{\partial \tilde{\eta}} \frac{a^2(\eta')\pi^T_{ij}\left(\eta', \mathbf{q}\right)}{q}$$

- Average over the random processes generating the GWs
- GW power spectrum depends on 2 point correlator of the T.T. energy momentum tensor
- Model correlator with Sound Shell model
- $G\left(\tilde{\eta}, \tilde{\eta}'\right)$ determined by expansion history

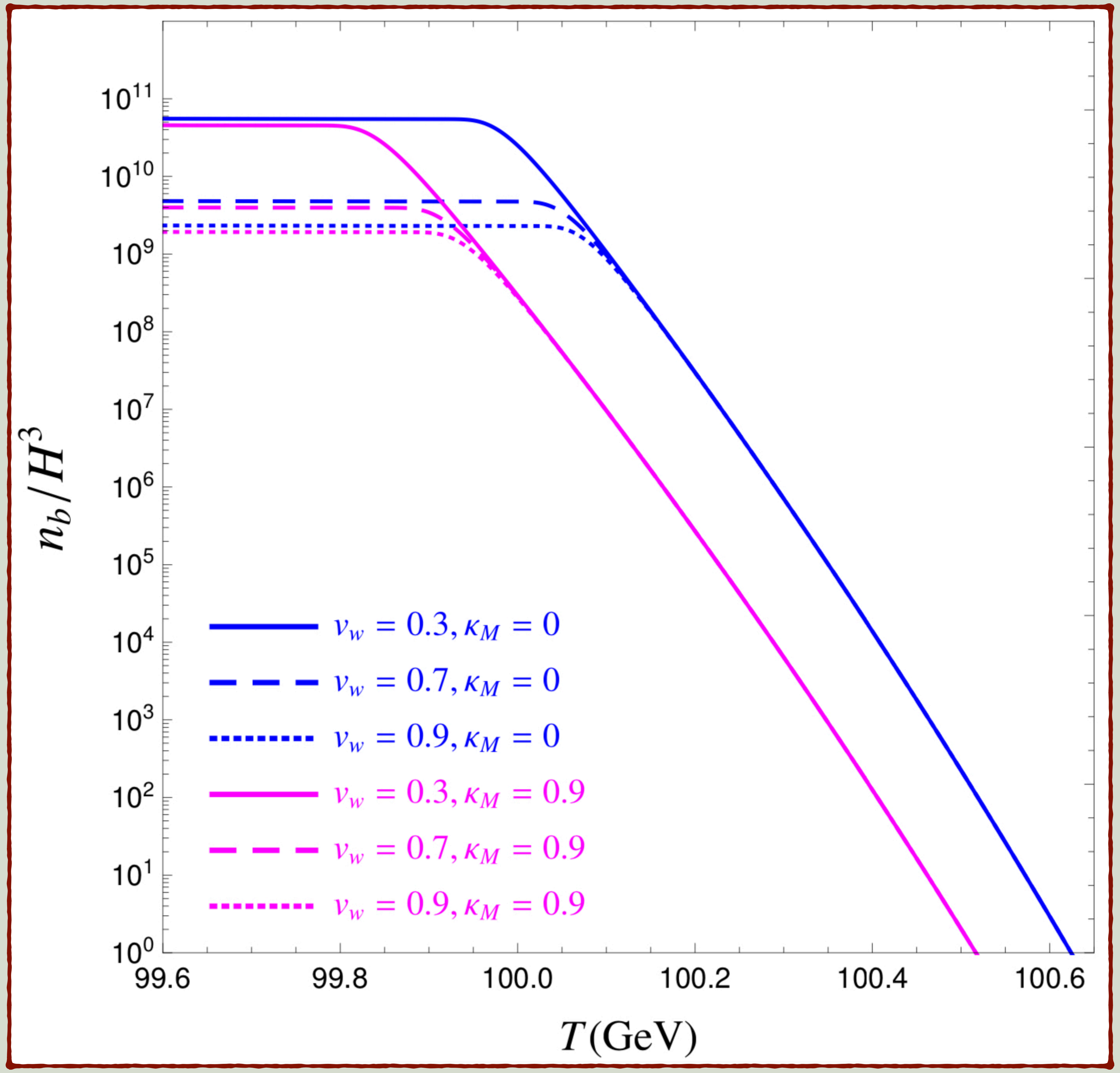
Bubble Number Density

- Evolution of N_b/V

$$\frac{d\left[n_b a^3(t)\right]}{dt} = p(t)g\left(t_c, t\right) a^3(t)$$

- Nucleation temperature found when $n_b/H^3 = 1$
- Analytical approximation often used using RDE

$$\frac{S_3(T_n)}{T_n} \sim 140$$

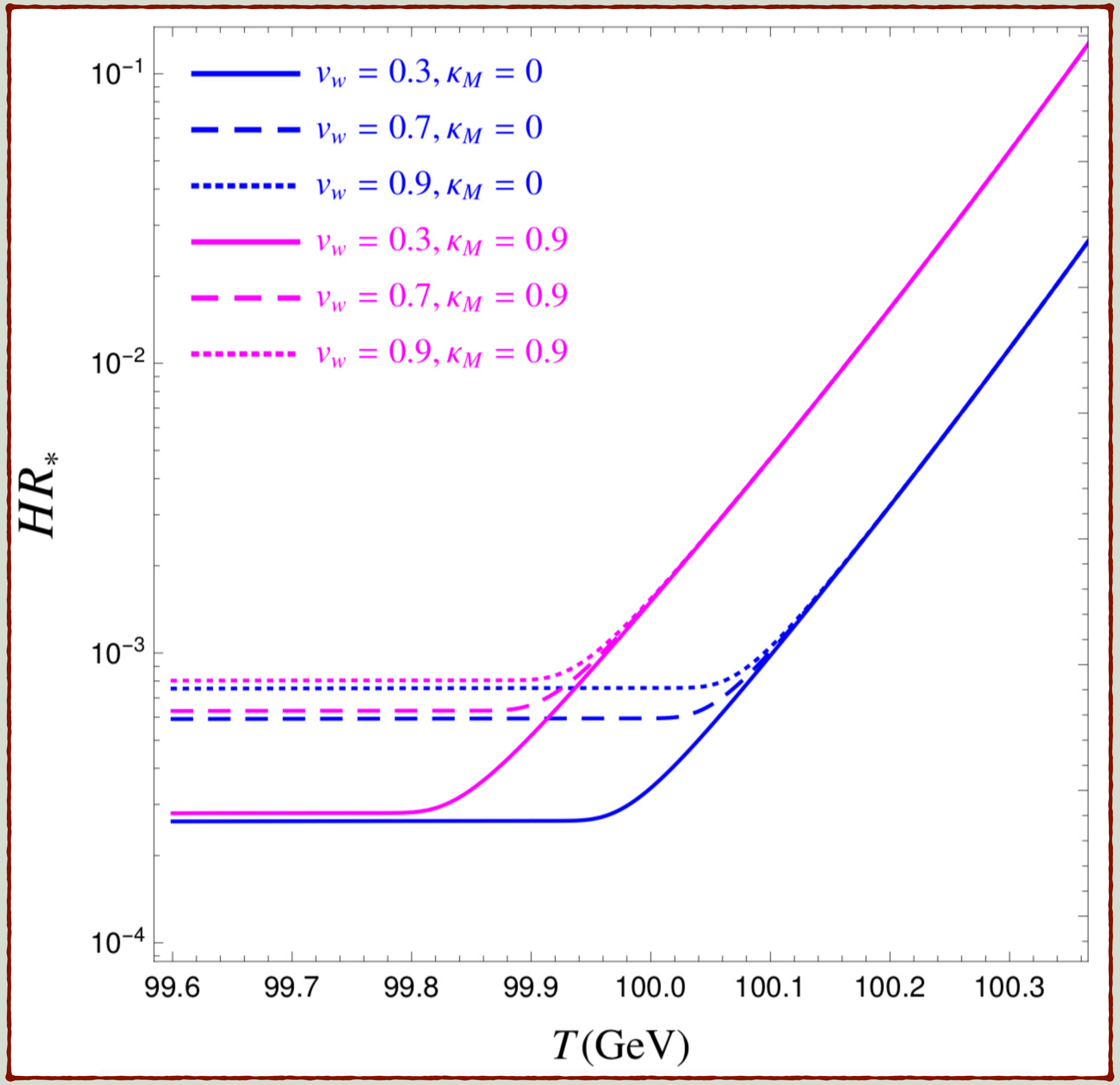


Mean Bubble Separation and Inverse Time Duration

- Peak frequency given by R_* [1,2]
- Asymptotic value after bubbles have disappeared
- $R_{*c} = 1/n_{b,c}^3$ at T_f when $g = e^{-1}$
- Analytically given in Minkowski spacetime [3]

$$R_* = \frac{(8\pi)^{1/3} v_w}{\beta(v_w)}$$

- Satisfied in comoving coordinates $\beta \to \beta_c$ and $R_* \to R_{*c}$
- $2\,\%$ deviation in β from S_3



1. arXiv:1504.03291
2. arXiv:1704.05871
3. arXiv:1909.10040

Bubble Lifetime Distribution

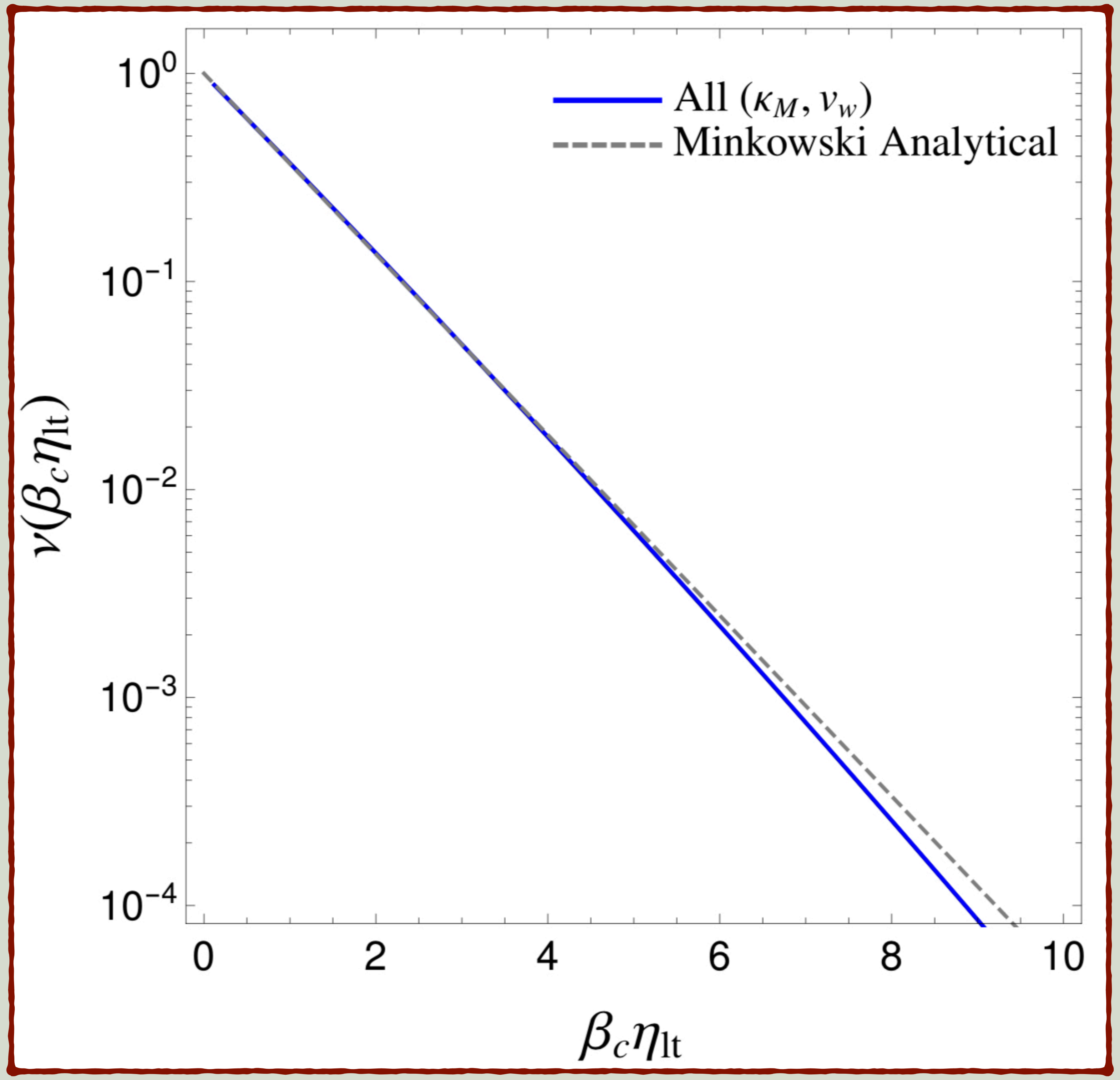
- Distribution of bubble lifetime for all the bubbles ever formed and destroyed during the entire process of the phase transition.
- Derived analytically in Minkowski spacetime [1]

$$\nu\left(\tilde{T}\right) \propto e^{-\tilde{T}} \text{ or } \tilde{T}^2 e^{-\tilde{T}^3}$$

- Related to the time $n_{b,c}(r)\,|_{t_f}$ becomes a constant
- Expanding universe

$$\tilde{n}_{b,c}(\eta_{lt}) = v_w \int_{t_c}^{t_f} dt' p(t') a^3(t') \mathscr{A}_c(t(t', v_w \eta_{lt}))$$

$$= \frac{\beta_c}{R_{*c}^3} \nu\left(\beta_c \eta_{lt}\right)$$



1. arXiv:1909.10040

Fluid Velocity Field

- Dominate source of GWs is the local velocity field from the sound waves in plasma as bubbles propagate and and interact with surrounding fluid
- Fluid velocity field is the linear superposition of single-bubble contributions (Sound Shell Model)
- E.O.M of fluid same as Minkowski spacetime in expanding universe with $v = v(\eta)$
- Computed from the convolution of power spectrum sourced by the velocity field
- Sound Shell Model → Velocity power spectrum

<u>Before collision</u>

$$v^i(\eta < \eta_c, \mathbf{x}) = \frac{1}{2}\int \frac{d^3q}{(2\pi)^3}\left[\tilde{v}^i_{\mathbf{q}}(\eta)e^{i\mathbf{q}\cdot\mathbf{x}} + \tilde{v}^{i*}_{\mathbf{q}}(\eta)e^{-i\mathbf{q}\cdot\mathbf{x}}\right]$$

<u>After collision</u>

$$v^i(\eta > \eta_c, \mathbf{x}) = \int \frac{d^3q}{(2\pi)^3}\left[v^i_{\mathbf{q}}e^{-i\omega\eta+\mathbf{q}\cdot\mathbf{x}} + v^{i*}_{\mathbf{q}}e^{i\omega\eta-i\mathbf{q}\cdot\mathbf{x}}\right]$$

Velocity Spectral Density

- Velocity field, after most bubbles collide, is obtained by adding all the individual bubble contributions
- Velocity profile becomes initial condition for freely propagating sound waves
- Total number of bubbles nucleated within a Hubble volume with co-moving size V_c is N_b
- Velocity field follows a Gaussian distribution to a good approximation

$$v_{\mathbf{q}}^{i} = \sum_{n=1}^{N_b} v_{\mathbf{q}}^{i(n)}$$

- Randomness removed by doing an ensemble average

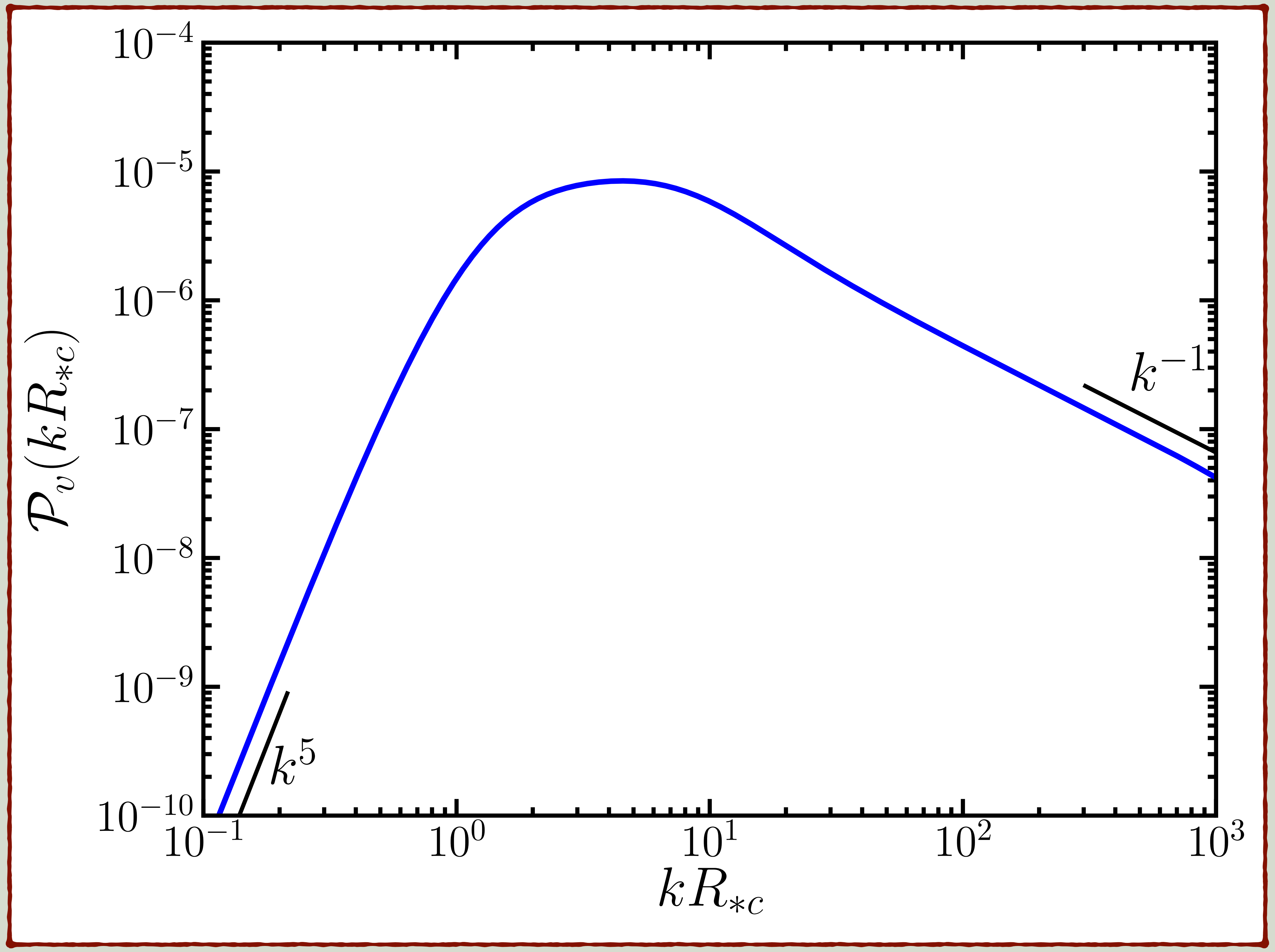
$$\langle v_{\mathbf{q}}^{i} v_{\mathbf{q}}^{j*} \rangle = \hat{q}^i \hat{q}^j (2\pi)^3 \delta^3(\mathbf{q}_1 - \mathbf{q}_2) \underbrace{\frac{1}{R_{*c}^3 \beta_c^6} \int d\tilde{T} \tilde{T}^6 \nu(\tilde{T}) \left| A\left(\frac{q\tilde{T}}{\beta_c}\right) \right|^2}_{\equiv P_v(q)}$$

Dimensionless Velocity Power Spectrum

- $\mathscr{P}_v$ should go as k^5 and k^{-1}
- Contains information on the shape of fluid shells, fluid shell thickness, wall speed, and peak amplitude

$$\mathscr{P}_v = 2\frac{(qR_{*c})^3}{2\pi^2 R_{*c}^3} P_v\left(qR_{*c}\right)$$

$$\beta_c = (8\pi)^{1/3}\frac{v_w}{R_{*c}}$$



$$\alpha_n = 0.0046,\ v_w = 0.92,\ a = 1$$

Einstein Equation

$$y(\eta) = a(\eta)/a_s$$

$$(\kappa_M y + 1 - \kappa_M)\frac{d^2 h_q}{dy^2} + \left[\frac{5}{2}\kappa_M + \frac{2(1-\kappa_M)}{y}\right]\frac{dh_q}{dy} + \tilde{\tilde{q}}^2 h_q = \frac{16\pi G a(y)^2 \pi_q^T(y)}{(a_s H_s)^2}$$

$\kappa_m = 0$

$$G(\tilde{y}, \tilde{y}_0) = \frac{\tilde{y}_0 \sin(\tilde{y} - \tilde{y}_0)}{\tilde{y}}$$

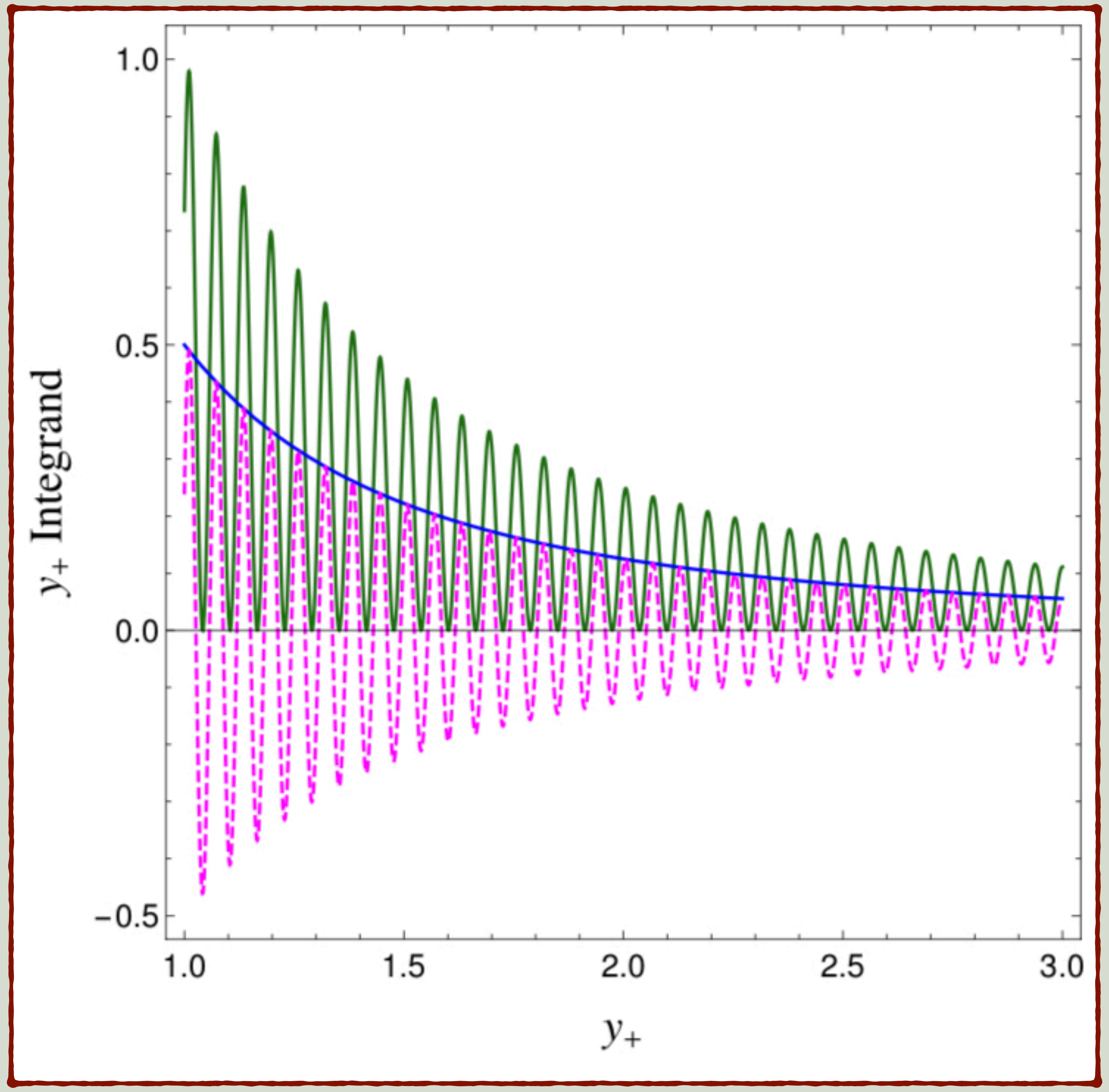
$0 < \kappa_m < 1$

$$G(\tilde{y}, \tilde{y}_0) = \frac{(\lambda\lambda_0 + 1)\sin(\lambda - \lambda_0) - (\lambda - \lambda_0)\cos(\lambda - \lambda_0)}{\lambda^3/2}$$

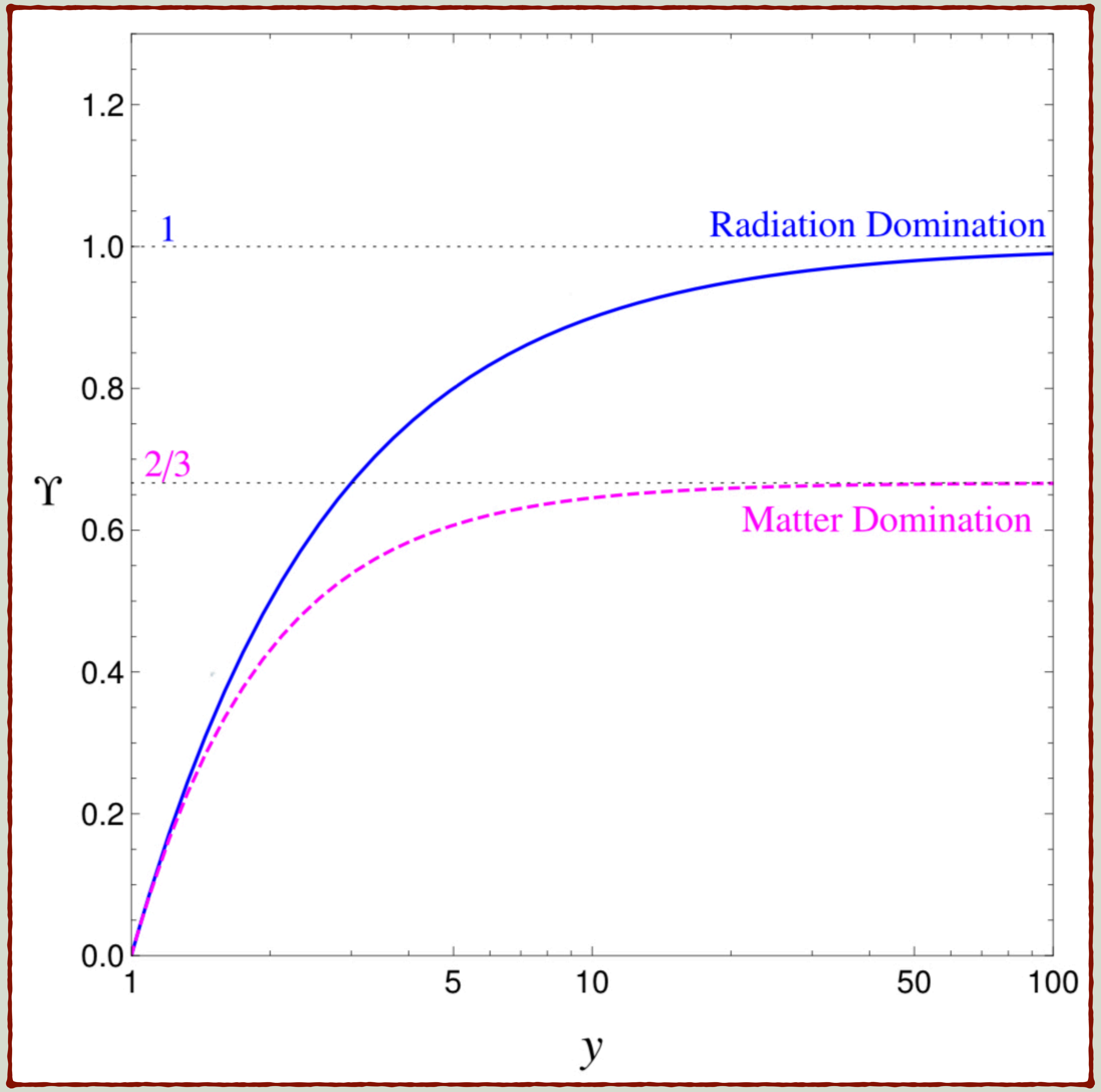
GW Spectral Density

$$P_{h'} = [16\pi G\,(\bar{\tilde{\epsilon}} + \bar{\tilde{p}})\,\bar{U}_f^2]^2 L_f^3 \int_{\tilde{y}_s}^{\tilde{y}} d\tilde{y}_1 \int_{\tilde{y}_s}^{\tilde{y}} d\tilde{y}_2 \left(\frac{\partial \tilde{y}}{\partial \tilde{\eta}}\right)^2 \frac{\partial G(\tilde{y},\tilde{y}_1)}{\partial \tilde{y}} \frac{\partial G(\tilde{y},\tilde{y}_2)}{\partial \tilde{y}} \times \frac{a_s^8}{a^2(\tilde{y}_1)a^2(\tilde{y}_2)} \frac{\tilde{\Pi}^2(kL_f, k\eta_1, k\eta_2)}{k^2}.$$

- Expansion history in partial derivative terms $\tilde{y}$ and $G\left(\tilde{y}, \tilde{y}_{1,2}\right)$
- Velocity Power Spectrum
- Neglect oscillatory terms in y_+ integrand
- Suppression factor from y_+ integrand



Suppression Factor



$$\left[\int dy_+ \cdots\right] = \frac{1}{2}\Upsilon(y)\cos\left(\tilde{\tilde{k}}y_-\right)$$

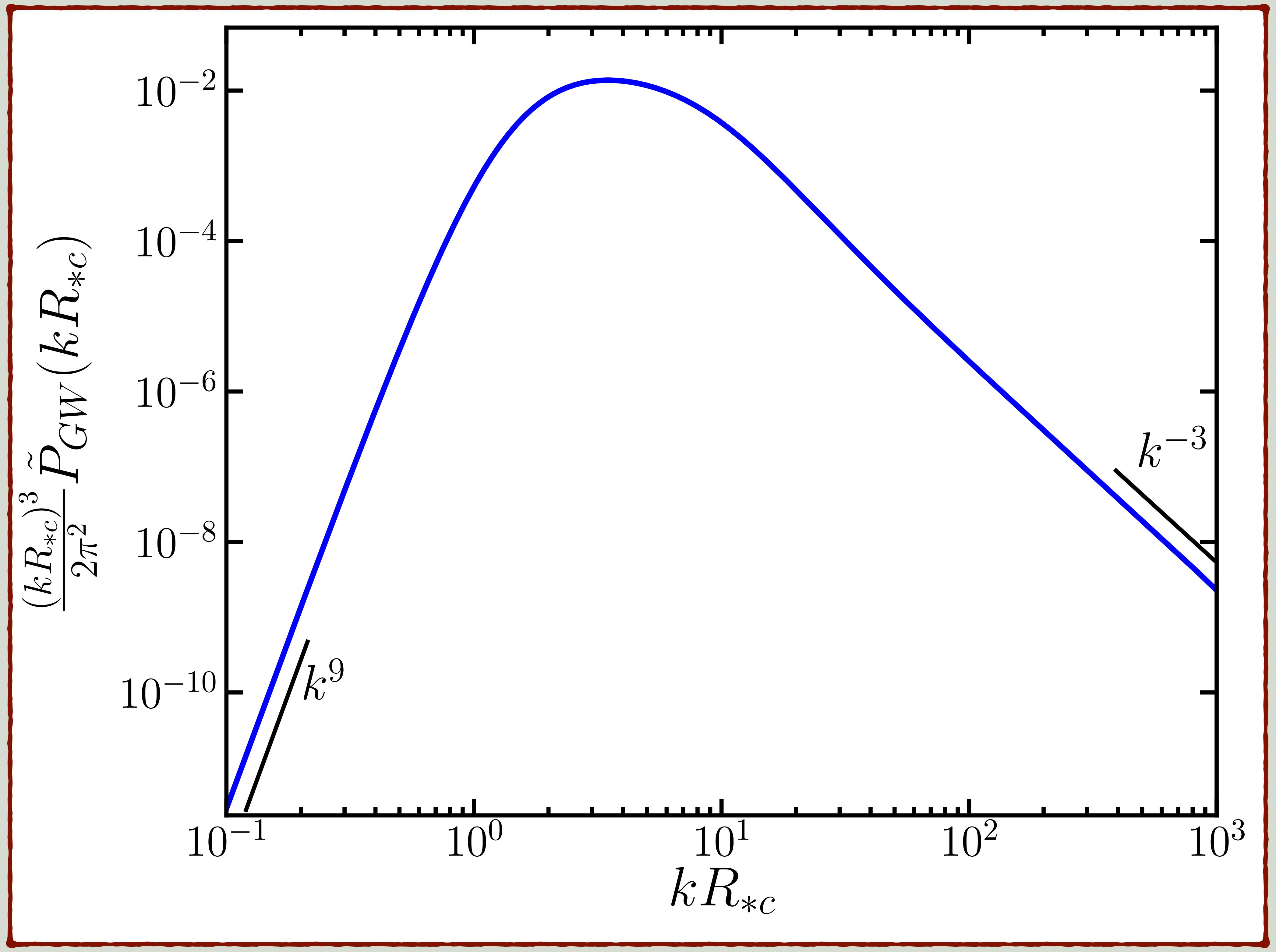
If the asymptotic value can be reached depends on how long the source lasts

Suppression in GW Power Spectrum

- $\mathscr{P}_{GW}$ should go as k^9 and k^{-3}
- MInkowski form
- $\Upsilon(y)$ suppression factor dependent on cosmological history

$$\mathscr{P}_{GW}\left(y, kR_{*c}\right) = 3\Gamma^2 \bar{U}_f^4 \left(H_s a_s R_{*c}\right) \frac{\left(kR_{*c}\right)^3}{2\pi^2} \tilde{P}_{gw}\left(kR_{*}c\right) \times$$

$$\Upsilon(y) \begin{cases} 1 \\ \dfrac{\left(1-\kappa_m\right)^2}{\kappa_m y + 1 - \kappa_m} \end{cases}$$



$\alpha_n = 0.0046,\ v_w = 0.92,\ a = 1$

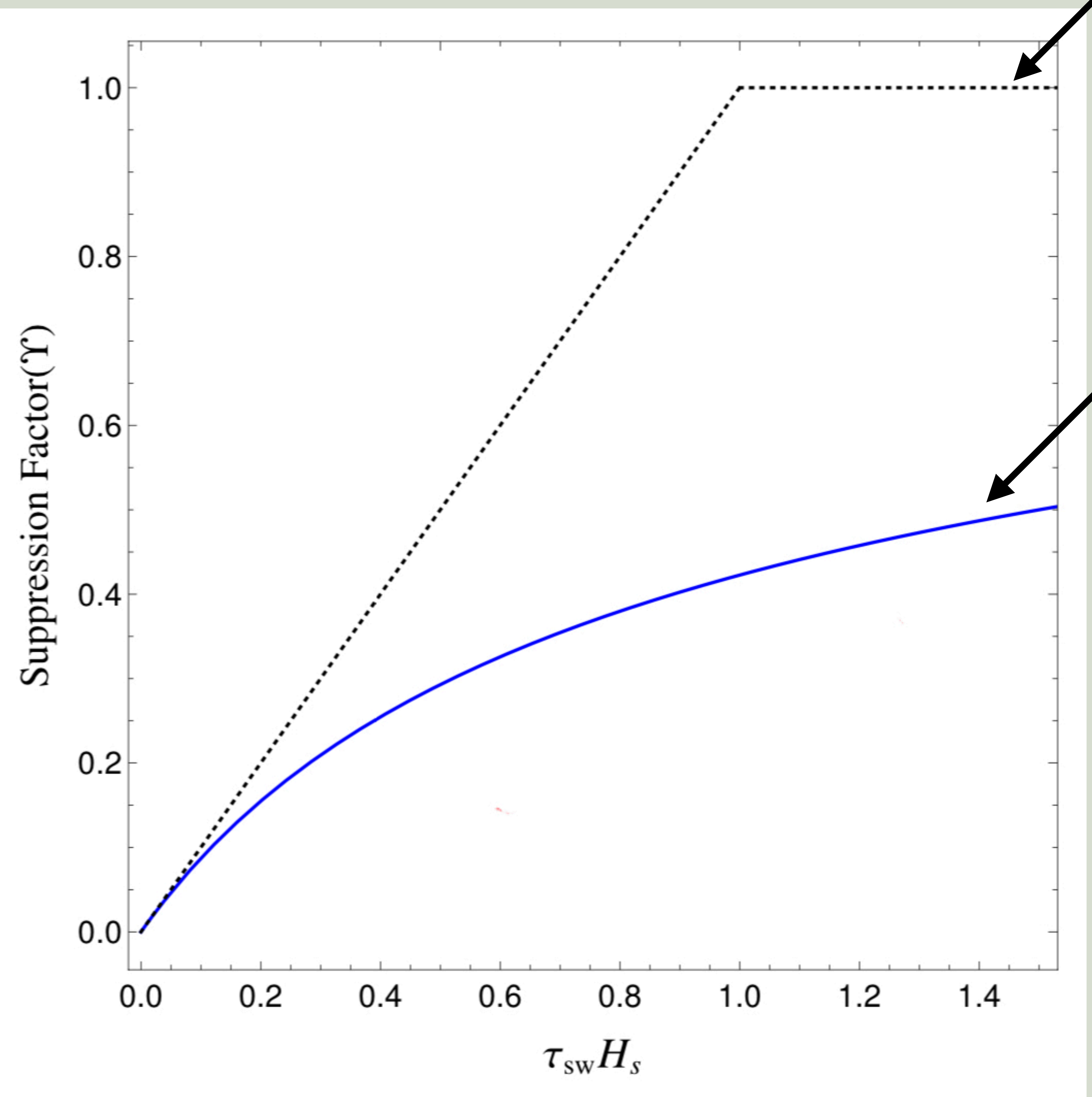
Lifetime of the Source

- Asymptotic value $\Upsilon^{RD} \to 1$ after many Hubble times

- Hard to reach due to non-linear shocks and turbulence which disrupts sound wave source

$$\tau_{SW} H_S \approx \frac{H_S R_*}{\bar{U}_f}$$

arXiv:1903.09642v3



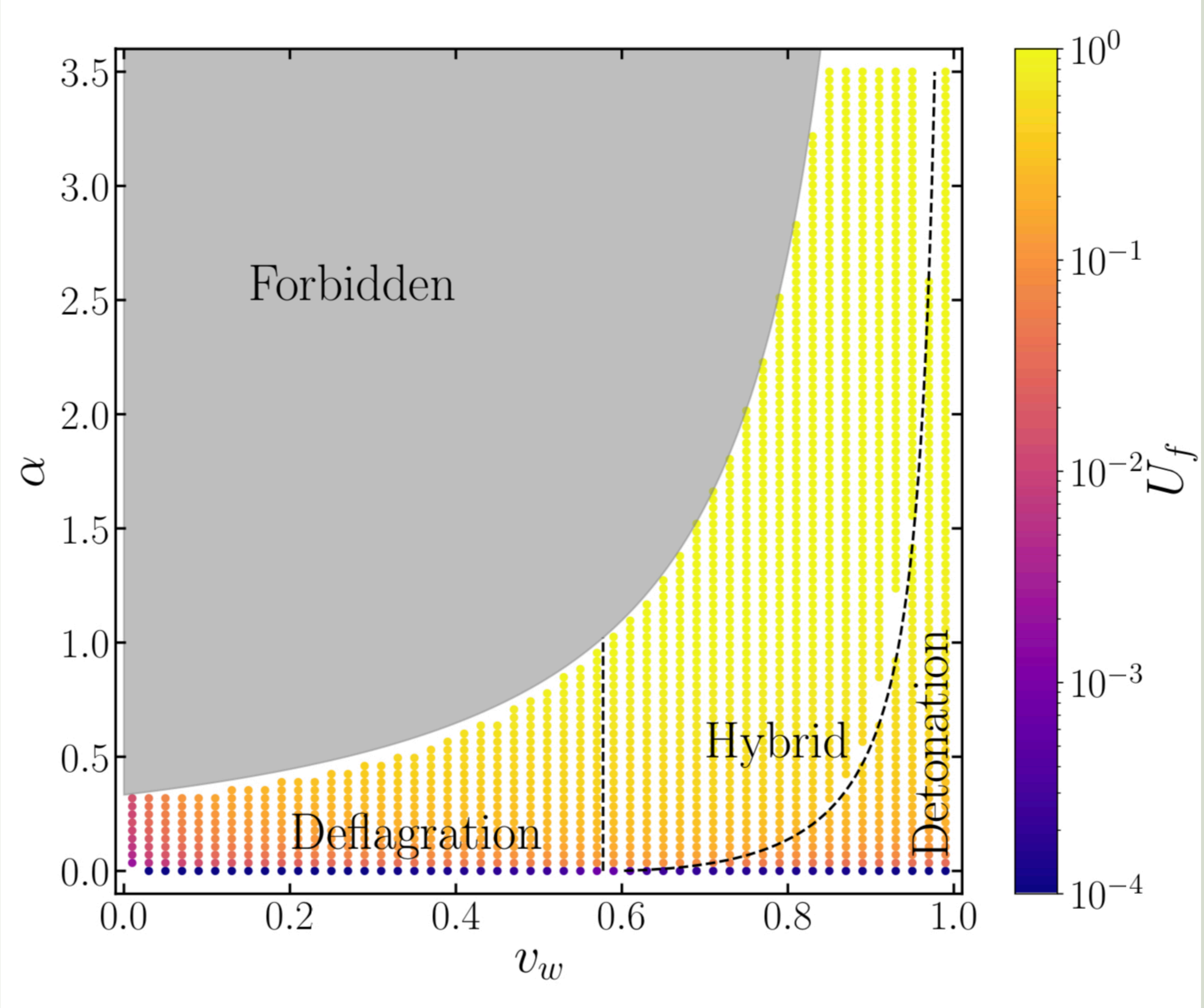
$$\Upsilon^{RD} = 1 - \frac{1}{\sqrt{1 + 2\tau_{SW} H_s}}$$

Peak Gravitational Waves Today

RDE

$$h^2\Omega_{GW} = 8.5 \times 10^{-6}\left(\frac{100}{g_s(T_e)}\right)\Gamma^2\bar{U}_f^4\left[\frac{H_s}{\beta(v_w)}\right] v_w \times \Upsilon$$

- Adiabatic index: $\Gamma \sim 4/3$
- D.O.F of entropy when GW production ends: $g_s\left(T_e\right)$
- β from R_* or $\frac{d\left(S_3/T\right)}{dT}$



Conclusion

- Detailed analysis of PTs in an expanding universe
- Sound Shell model can be used to calculate the GW spectrum for various cosmological histories
- Suppression found in spectrum
- Change in spectral form for early MDE vs RDE