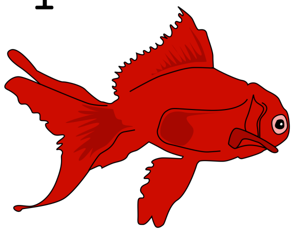

Poisson isn't a red herring: modified gravity on non-linear scales

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red herring

a fact, idea, or subject that takes people's attention away from the central point being considered:

$$\nabla^2\Phi = 4\pi G\rho$$

Laplacian (k^2 in Fourier space)

Gravitational Potential

$\propto$

Density

Based on arXiv: 2004.13051, chosen as editor's suggestion for PRD

Credits: Definition from Cambridge online dictionary, image from wikipedia

Plan

Why modified gravity?
(To provoke observers)

Why model independent modified gravity?
(To provoke theorists)

Punchline/Result
(You can all pause the video after this point and make coffee)

Some physical reasoning behind the result
(Not too technical, I promise)

Closing remarks
(Where I point out some caveats of this approach)

Model-independent modified gravity: Why?

Why modified gravity?

- Not ruled out
- Dark sector (only gravitational evidence)
- WIMPs not found, tensions, Cosmological constant problem…
- Quantum gravity: where does GR break down? Is the IR limit exactly GR?
- Good to test accepted theories in new regimes

Why *model-independent* modified gravity?

- None of the models are really well motivated
- We might not have the right answer
- Horndeski isn't really general
- Acts as null test of standard cosmology
- Testing gravity is a good idea independently of dark sector motivation (PPN)

Aside: The story of Urbain Le Verrier, Neptune and Vulcan is a fascinating parable. Neptune was the first "dark matter"; GR was "modified gravity"

The state of play

Model-independent modified gravity

- Well understood on large (linear) scales
- Model independence well-formulated on large (linear) scales

$$k^2\Phi = 4\pi G\mu(a,k)\bar{\rho}\Delta$$

Relativistic version of Poisson equation

$$\Psi = \eta(a,k)\Phi$$

Potentials equal in Newtonian limit and many GR situations

- **No* formulation of model independence on small scales**
- Modified N-body codes for some specific models (small scales)

Future surveys (Euclid etc) will have majority of data on non-linear scales

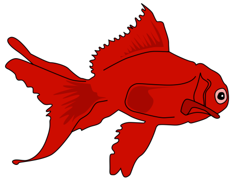
*: Cool work by Hassani & Lombriser (arXiv:2003.05927); Clifton & Sanghai (arXiv:1803.01157)
But note these are *a priori* restricted to certain parts of theory space

Is this a problem?

Not just an elephant in the room…

… it is a whole safari.

Clue or red herring?



Poisson equation appears in lots of different contexts:

Linear GR, Newtonian limit, modified gravity…

Maybe we can just use this?

Punchline

Can derive parameterised Poisson & "slip" (ratio of potentials) equations

Valid on all scales (density not required to be small)

$$\frac{1}{c^2}k^2\tilde{\phi}_P = -\frac{1}{c^2}4\pi G a^2\bar{\rho}\mu\left(a,\vec{k}\right)\left(\tilde{\delta}+\frac{\dot{a}}{a}\frac{3}{c^2k^2}\tilde{\theta}\right)$$

$$\tilde{\psi}_P = \eta\left(a,\vec{k}\right)\tilde{\phi}_P.$$

Can be used to analyse (& combine) data from any survey on any scales

+ Algorithm to determine whether a modified gravity theory fits into approach

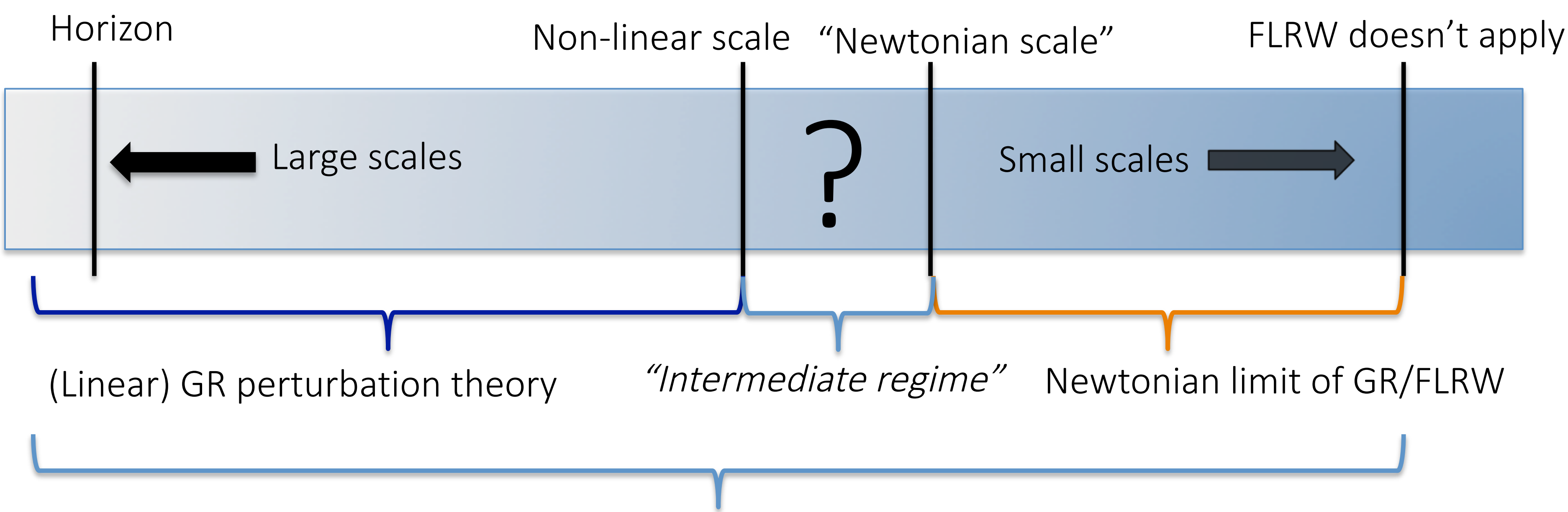
(Note: $\mu(a,k),\eta(a,k)$ sweep a lot of complications under the carpet)

Pause the video, get some strong caffeine, and then I’ll highlight some of the physical reasoning and steps in the argument.

Gravity in cosmology

General Relativity

- Gravity studied in 2 limits:
  large scales (relativistic perturbation theory) & small scales (Newtonian)
- Little *explicit* discussion of possible "intermediate regime" between these
- Ideally want a single framework spanning all scales, including both limits



Single approach: Full GR (very complicated)? New tool?

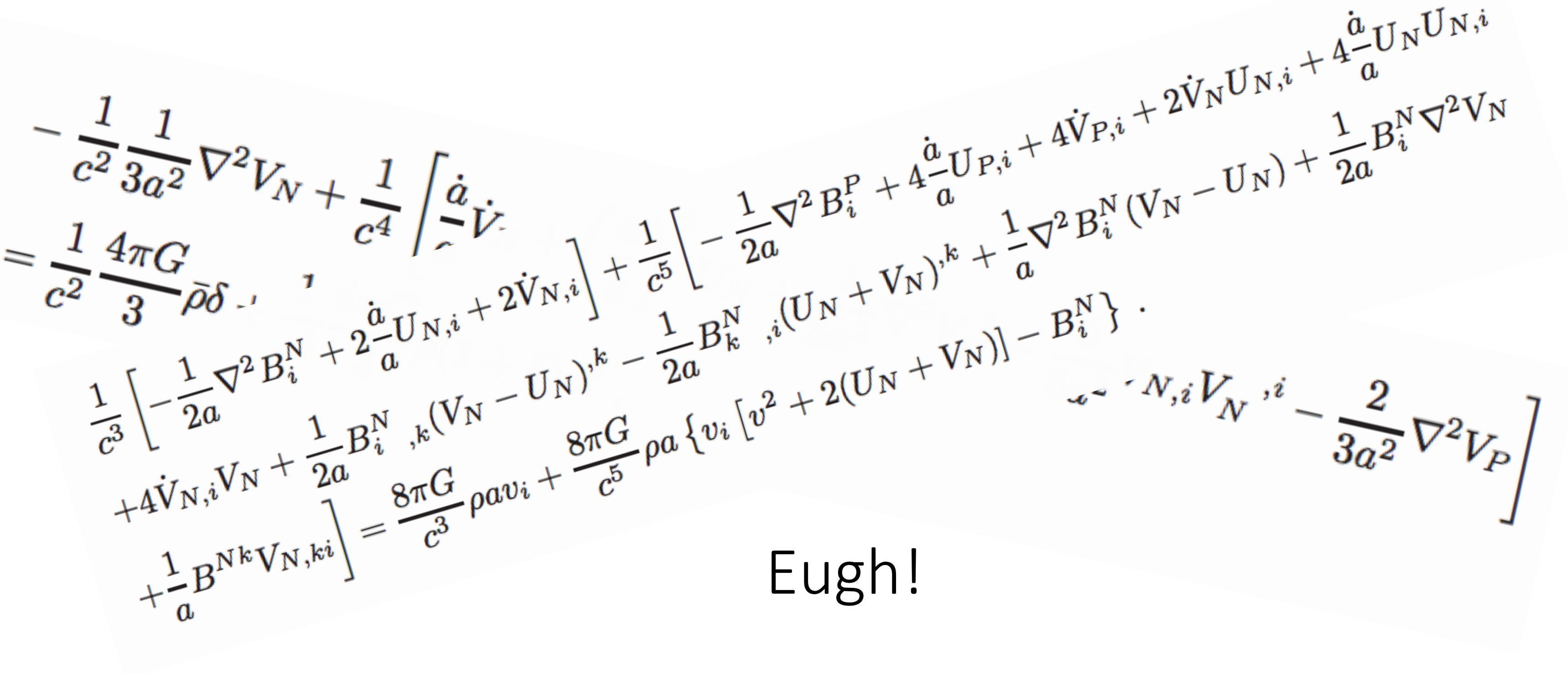
Post-Friedmann formalism (Milillo et al 2015; arXiv:1502.02985)

Post-Friedmann formalism (full “1PF” equations)

- Post-Newtonian-like expansion in powers of speed of light c
- Crucial: doesn’t require small density contrast
- *Not* solved order-by-order: Expanded to order c^{-4} and “resummed”

➡ Contains linear perturbations + Newtonian limit

➡ Can describe intermediate regime



Eugh!

Post-Friedmann formalism (full "1PF" equations)

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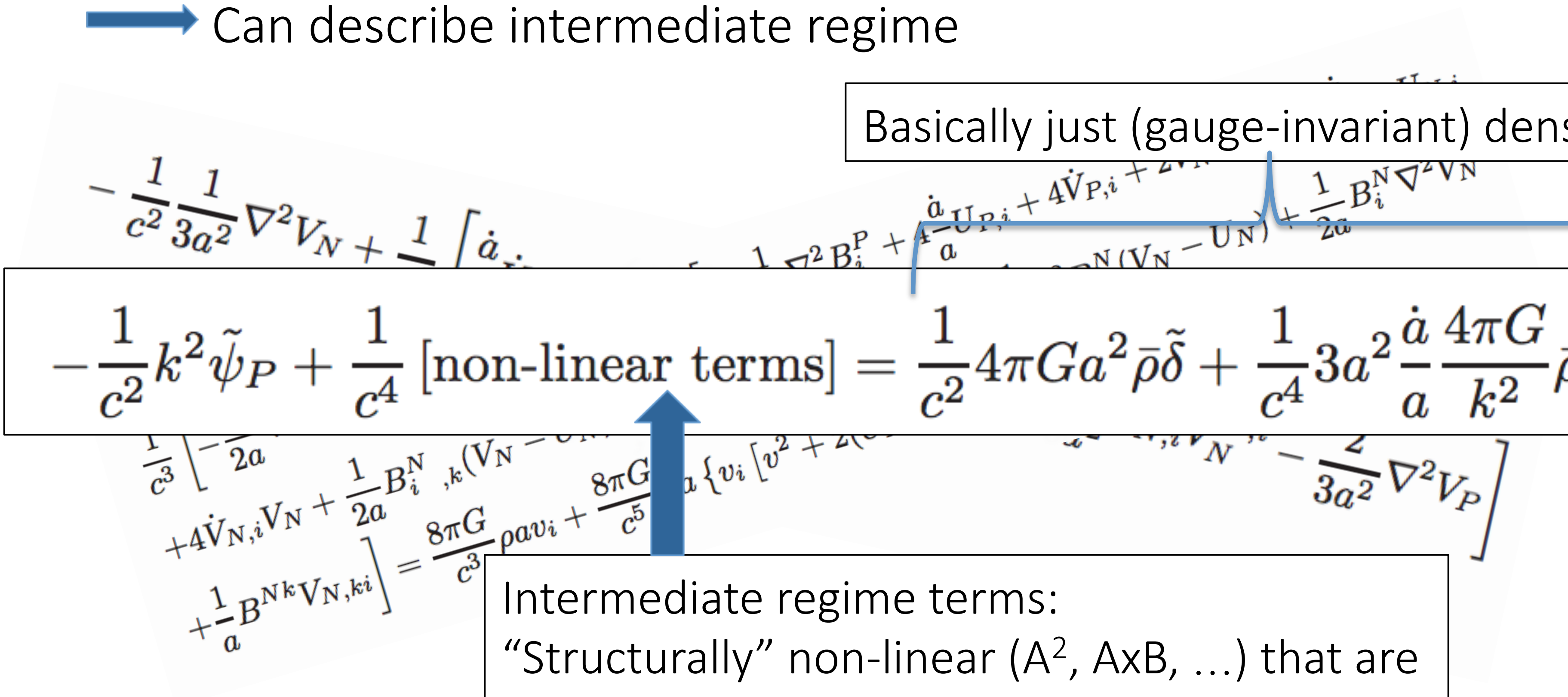
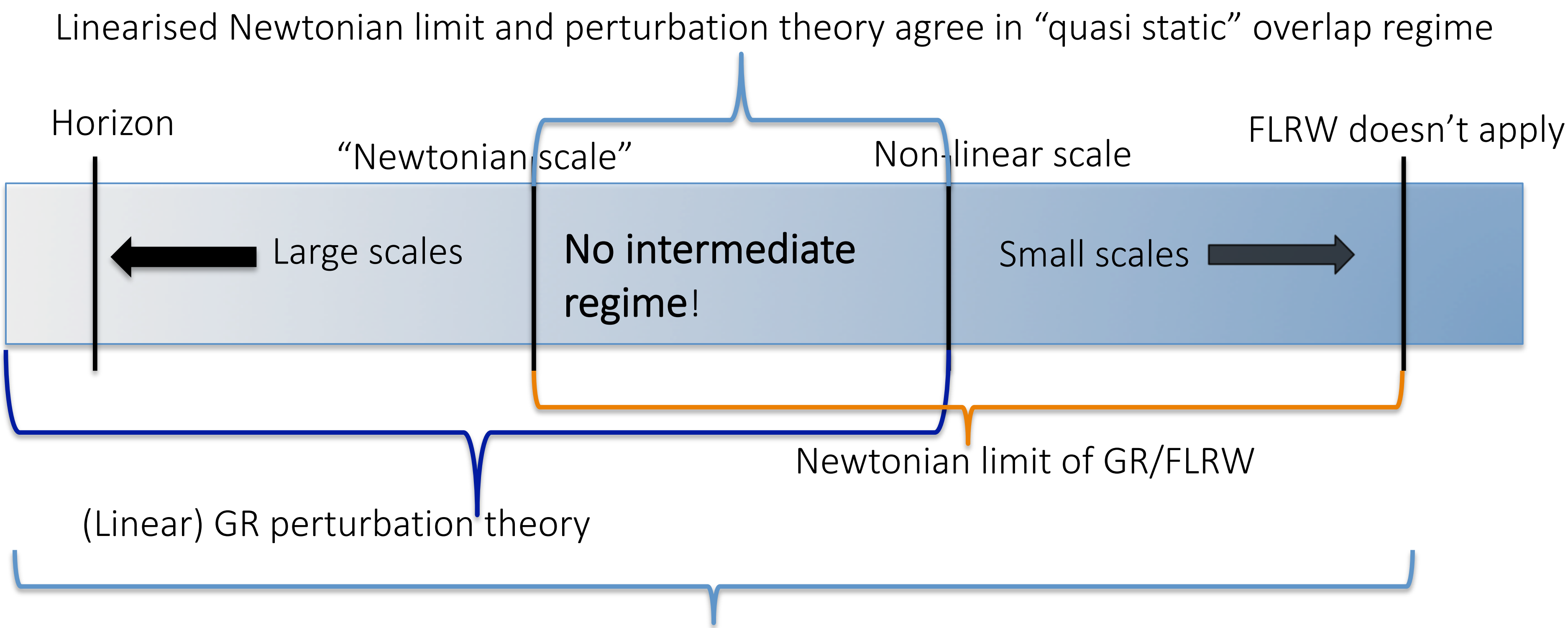


Basically just (gauge-invariant) density

$$-\frac{1}{c^2}k^2\tilde{\psi}_P + \frac{1}{c^4}\,[\text{non-linear terms}] = \frac{1}{c^2}4\pi G a^2\bar{\rho}\tilde{\delta} + \frac{1}{c^4}3a^2\frac{\dot{a}}{a}\frac{4\pi G}{k^2}\bar{\rho}\tilde{\theta}$$

Intermediate regime terms:
"Structurally" non-linear (A^2, AxB, …) that are higher order in Newtonian approximation

Late time GR+LCDM



Use post-Friedman, drop all intermediate regime terms: "*simple* 1PF"

- Simple equations that apply on *all* scales
- Poisson contains the respective Newtonian and linear theory equations

$$\frac{1}{c^2}k^2\tilde{\psi}_P = -\frac{1}{c^2}4\pi G a^2\bar{\rho}\tilde{\delta} - \frac{1}{c^4}3a^2\frac{\dot{a}}{a}\frac{4\pi G}{k^2}\bar{\rho}\tilde{\theta}$$

$$\frac{1}{c^2}k^2\tilde{\psi}_P = -\frac{1}{c^2}4\pi G a^2\bar{\rho}\tilde{\delta} - \frac{1}{c^4}3a^2\frac{\dot{a}}{a}\frac{4\pi G}{k^2}\bar{\rho}\tilde{\theta}$$

If there is

- A good Newtonian limit on small scales
- No intermediate regime

Then gravity is described on ALL scales by this equation.

(Plus a gravitational slip equation).

→ Can turn these conditions into a set of checks to see if this applies to any particular modified gravity theory

Back to modified gravity

1. Is the weak field metric appropriate on all scales in the theory?

2. Can the vector and tensor perturbations be ignored at first order?

3. No terms in linear Newtonian limit that aren't in linear perturbative limit?

4. Is quasi-static approximation good (sub-horizon & ignore time derivatives)?
   (Usually ensures linear perturbation theory matches linearised Newtonian)

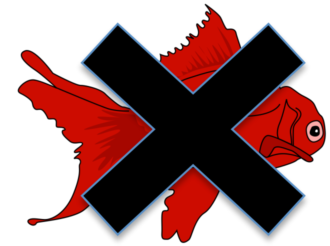
5. Is the Newtonian approximation a good description of all non-linear scales?
   (There is an explicit test for this using N-body simulations; see paper)

Theories that satisfy these can be described by the parameterised "simple 1PF" Poisson+slip equations on all scales

$$\frac{1}{c^2}k^2\tilde{\phi}_P = -\frac{1}{c^2}4\pi G a^2 \bar{\rho}\mu\left(a,\vec{k}\right)\left(\tilde{\delta}+\frac{\dot{a}}{a}\frac{3}{c^2k^2}\widetilde{\theta}\right)$$

(Theories that don't probably struggle to create a sufficiently LCDM-like universe)

Closing remarks



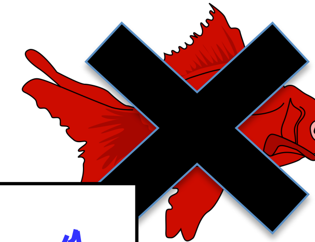
- Poisson (& slip) equation that apply on *all* scales (density can be large)
  Can be used to analyse (& combine) data from any survey…
  … but more development required…

- $\mu(a,k), \eta(a,k)$ Sweep a lot of complications under the carpet
  See paper for possible functional forms to investigate
  Lombriser work: $\mu(a,k), \eta(a,k)$ to match known theories such as f(R)

- Time and space "pixel approach" for $\mu(a,k), \eta(a,k)$: LCDM null test
  Good if no deviation detected.
  Key issue: hard to infer theory if deviation found

- Simulation work ongoing to explore phenomenology
  (with Sankar Srinivasan, PhD student at Manchester)

Closing remarks



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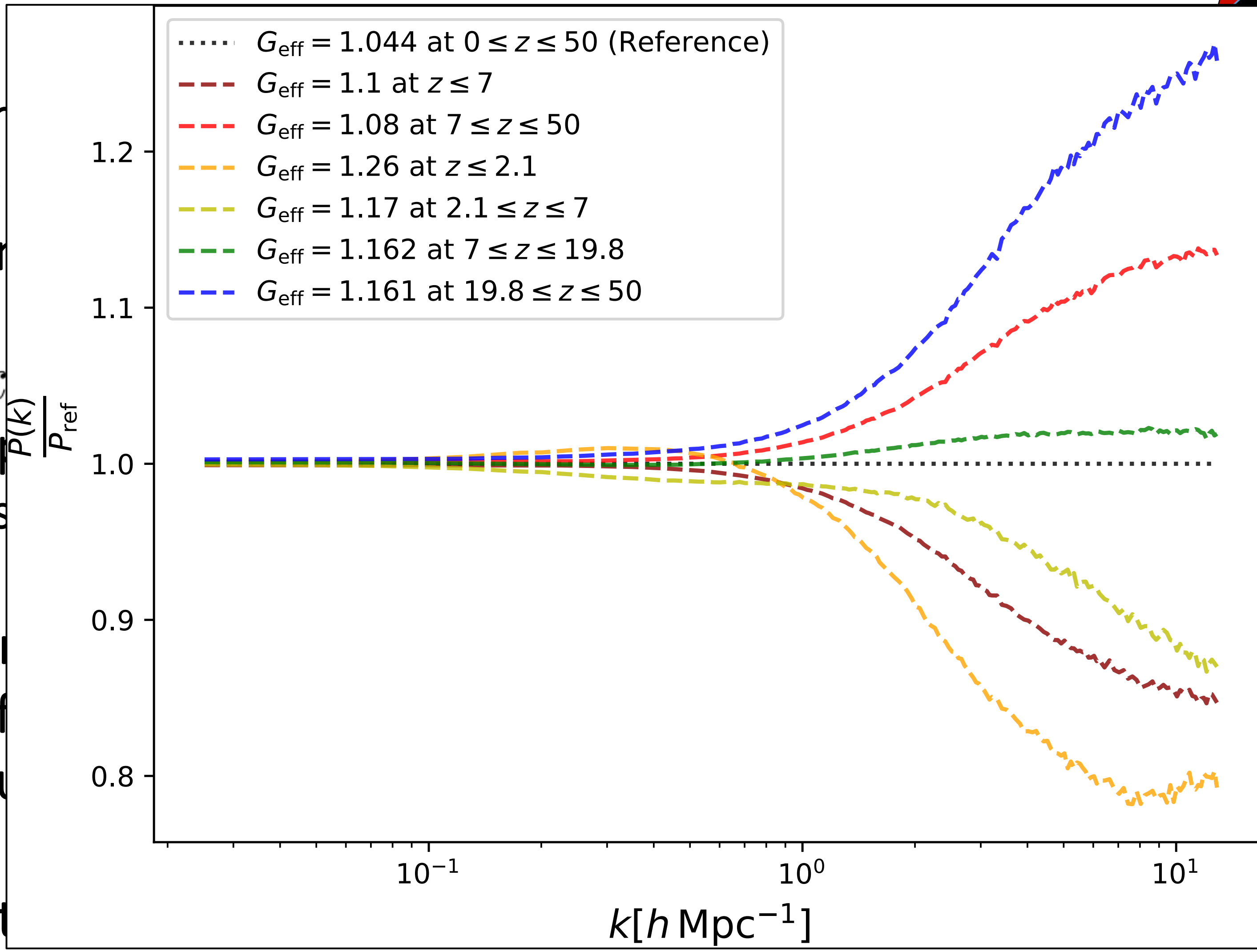


Thanks!

Bonus Content

Complete "simple 1PF" gravity & matter equations

$$\frac{1}{c^2}k^2\tilde{\psi}_P = -\frac{1}{c^2}4\pi G a^2\bar{\rho}\tilde{\delta} - \frac{1}{c^4}3a^2\frac{\dot{a}}{a}\frac{4\pi G}{k^2}\bar{\rho}\tilde{\theta}$$

$$\tilde{\psi}_P = \tilde{\phi}_P$$

$$\frac{dv_i}{dt} + \frac{\dot{a}}{a}v_i - \frac{\phi_{P,i}}{a} = 0$$

$$\left[\frac{dv_i}{dt} + \frac{\dot{a}}{a}v_i - \frac{\phi_{P,i}}{a} = \frac{1}{c^2}\frac{1}{a}\frac{d}{dt}(a\omega_{Pi})\right]$$

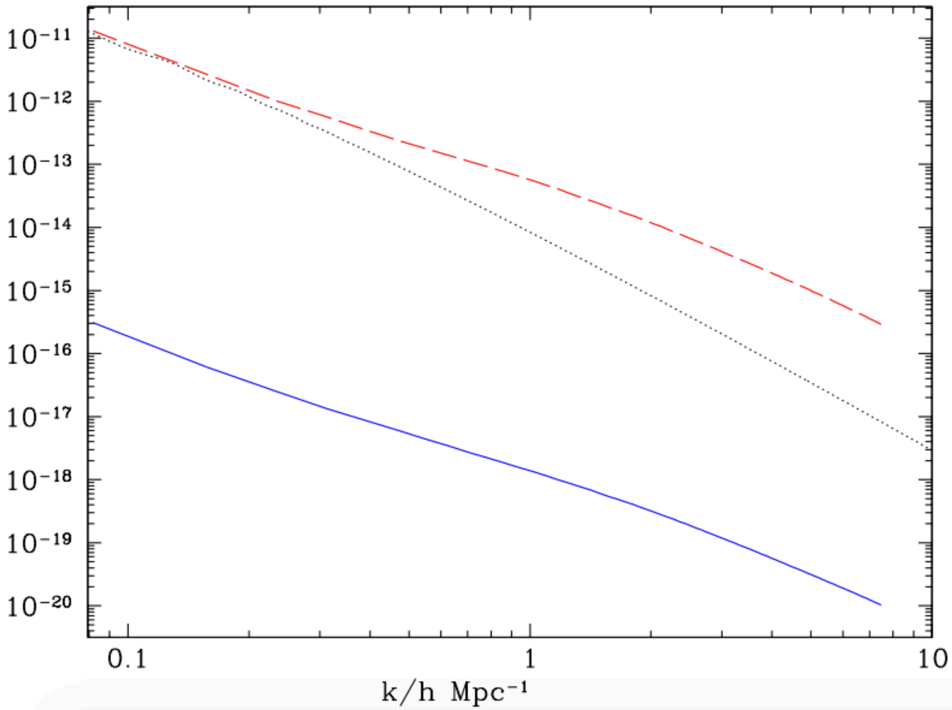
$$\frac{d\delta}{dt} + \frac{v^i_{,i}}{a}(1+\delta) - \frac{3}{c^2}\frac{d\psi_P}{dt} = 0.$$

Vector required for consistency of Einstein equations; needs to be small

$$\frac{1}{2c^3a^2}k^2\tilde{\omega}_{Pi} = \left[\frac{1}{c^3}8\pi G a\widetilde{\rho v_i}\right]|_v - \frac{1}{c^5}8\pi G\bar{\rho}a\tilde{\omega}_{Pi}$$

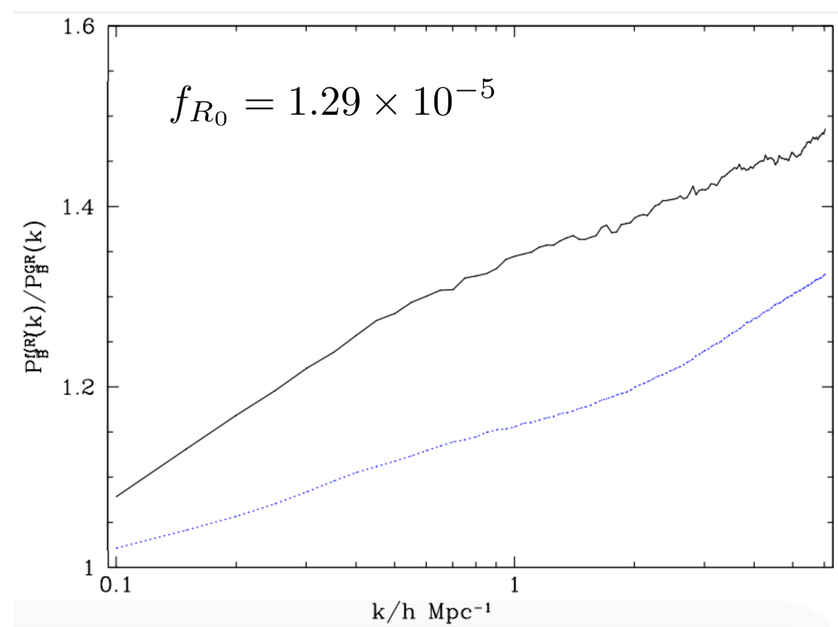
Vector potential

$$\frac{1}{c^3}\nabla^2 B_i^N = -\frac{16\pi G\bar{\rho}a^2}{c^3}\left[(1+\delta)v_i\right]|_v$$



LCDM power spectra of scalar (red) and vector (blue) potentials
(arXiv: 1306.1562)
(arXiv: 1501.00799)

Ratio of vector potential power spectra in f(R) and GR at z=0 (black) and z=1 (blue)
(arxiv: 1503.07204)



DE vs MG, and ubiquity of Poisson equation
(sketch only!)

$$\mathcal{L} = (\phi)R + \mathcal{L}_\phi + \mathcal{L}_m$$

(Jordan frame)

$$\phi = \phi^{(0)}(t) + \frac{1}{c^2}\phi + \ldots$$

Assuming a weak field metric (i.e linear limit or Newtonian approx), for simple DE or MG, one arrives at (schematically)

$$\nabla^2\Psi = \frac{4\pi G}{\phi^{(0)}}\rho + f(\phi)$$

Gory post-Friedmann Details

$$g_{00} = -\left[1 - \frac{2U_N}{c^2} + \frac{1}{c^4}\left(2U_N^2 - 4U_P\right)\right]$$
$$g_{0i} = -\frac{aB_i^N}{c^3} - \frac{aB_i^P}{c^5}$$
$$g_{ij} = a^2\left[\left(1 + \frac{2V_N}{c^2} + \frac{1}{c^4}\left(2V_N^2 + 4V_P\right)\right)\delta_{ij} + \frac{h_{ij}}{c^4}\right]$$

Metric

$$\phi_P := -(U_N + \frac{2}{c^2}U_P)$$
$$\psi_P := -(V_N + \frac{2}{c^2}V_P)$$

Re-summed potentials

Four velocity

$$u^i = \frac{1}{c}\frac{v^i}{a}u^0,$$
$$u^0 = 1 + \frac{1}{c^2}\left(U_N + \frac{1}{2}v^2\right) + \frac{1}{c^4}\left[\frac{1}{2}U_N^2 + 2U_P + v^2V_N + \frac{3}{2}v^2U_N + \frac{3}{8}v^4 - B_i^Nv^i\right]$$
$$u_i = \frac{av_i}{c} + \frac{a}{c^3}\left[-B_i^N + v_iU_N + 2v_iV_N + \frac{1}{2}v_iv^2\right],$$
$$u_0 = -1 + \frac{1}{c^2}\left(U_N - \frac{1}{2}v^2\right) + \frac{1}{c^4}\left[2U_P - \frac{1}{2}U_N^2 - \frac{1}{2}v^2U_N - v^2V_N - \frac{3}{8}v^4\right].$$

Stress-energy tensor

$$T^0{}_0 = -c^2\rho - \rho v^2 - \frac{1}{c^2}\rho\left[2(U_N + V_N)v^2 - B_i^Nv^i + v^4\right]$$
$$T^0{}_i = c\rho a v_i + \frac{1}{c}\rho a\left\{v_i[v^2 + 2(U_N + V_N)] - B_i^N\right\},$$
$$T^i{}_0 = -c\frac{1}{a}\rho v^i - \frac{1}{c}\frac{1}{a}\rho v^2 v^i,$$
$$T^i{}_j = \rho v^i v_j + \frac{1}{c^2}\rho\left\{v^iv_j[v^2 + 2(U_N + V_N)] - v^iB_j^N\right\},$$
$$T^\mu{}_\mu = T = -\rho c^2.$$