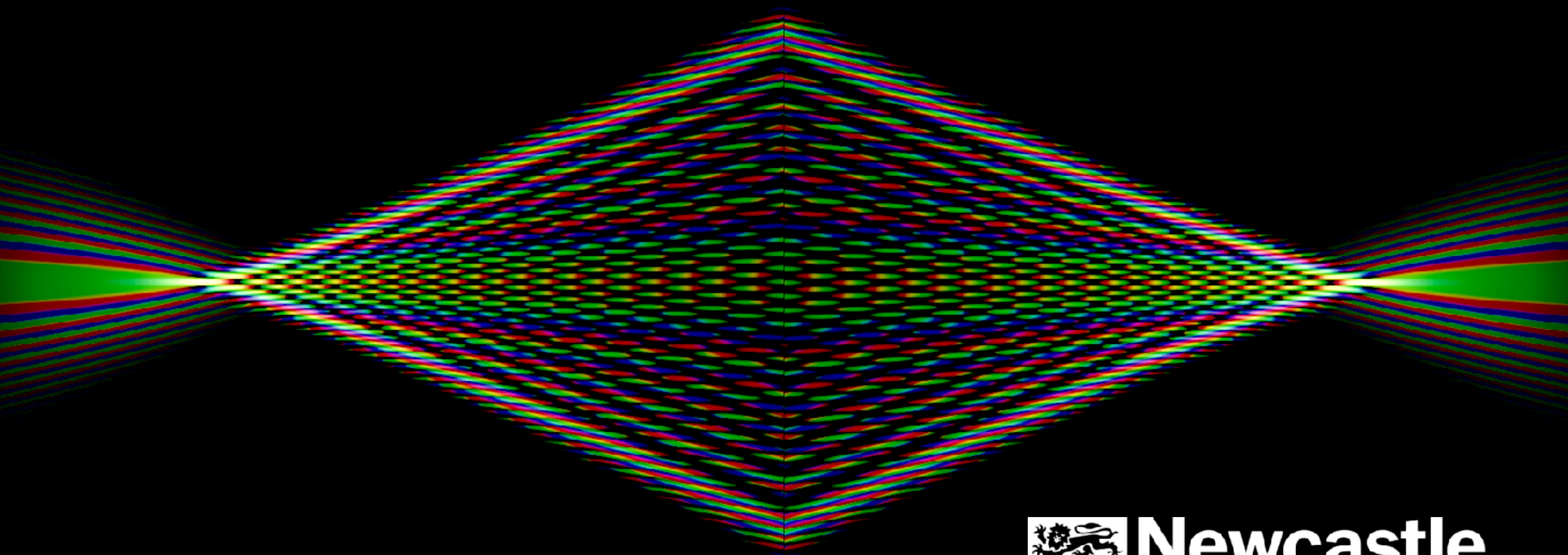
A Semiclassical Path to the Cosmic Web



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with Oliver Hahn, Cornelius Rampf & Mateja Gosenca

Newcastle University

Cosmo from Home, August 2020

Newcastle University

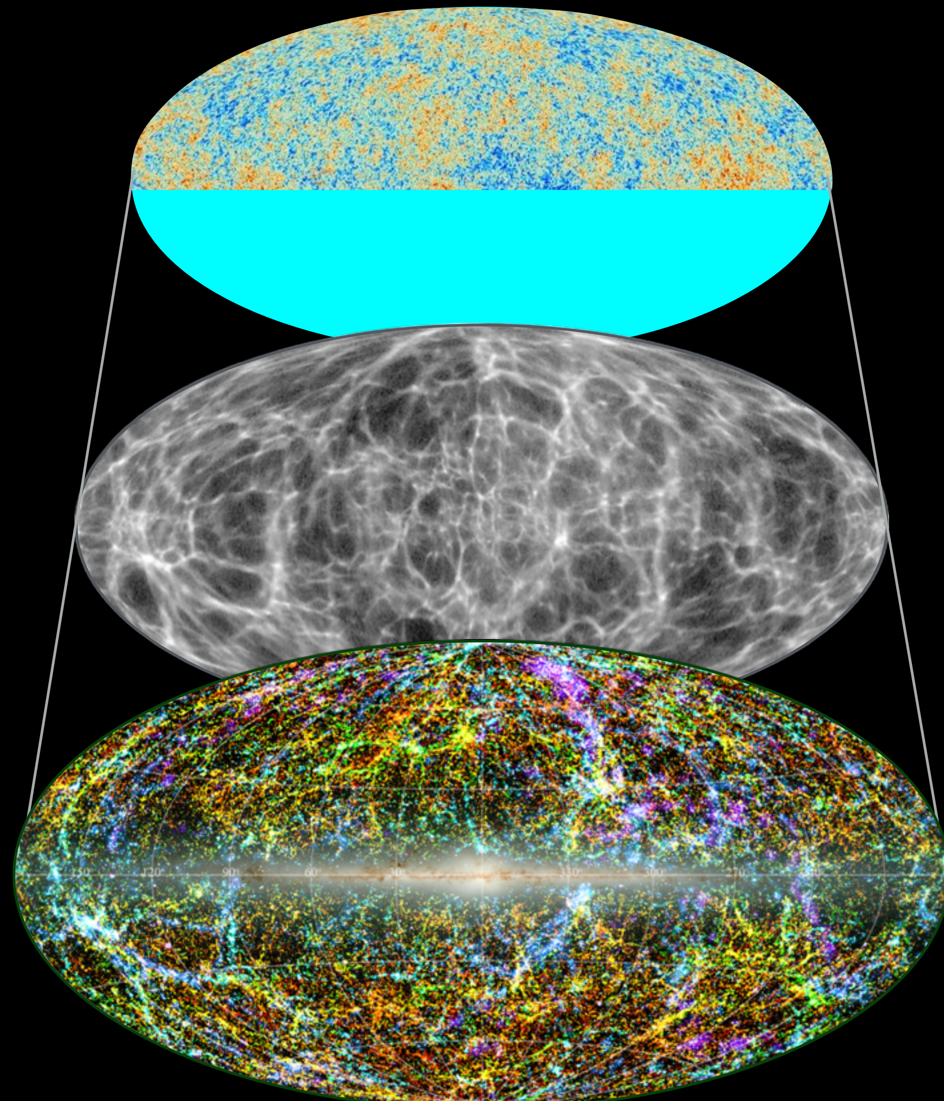
Cosmology & Quantum Gravity

info: blogs.ncl.ac.uk/cosmology/

some colleagues here this week

Cosmic Laboratory

beginning
nearly
uniform



CMB
early universe

COSMIC WEB
dark matter

today
rich
structure

COSMIC WEB
galaxies

Cosmic Laboratory

BIG QUESTIONS

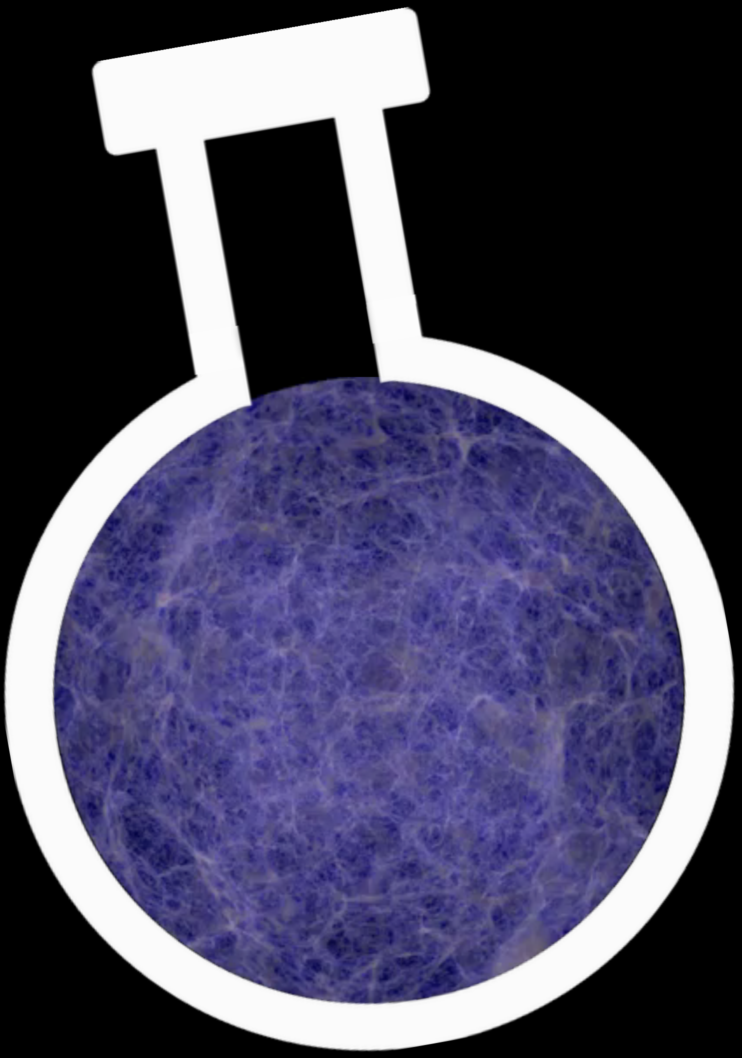
dark matter

early universe

dark energy & gravity

ANSWERS?

nonlinear clustering



Cosmic Web Challenge

numerical	analytical
N particles	2 fields
small scales	large scales
limited power	limited accuracy
limited sampling	limited features

Cosmic Web Challenge

NUMERICAL

N PARTICLES

ANALYTICAL

2 FIELDS

1 Wave Function

perturbation theory
& initial conditions

Classical Dynamics

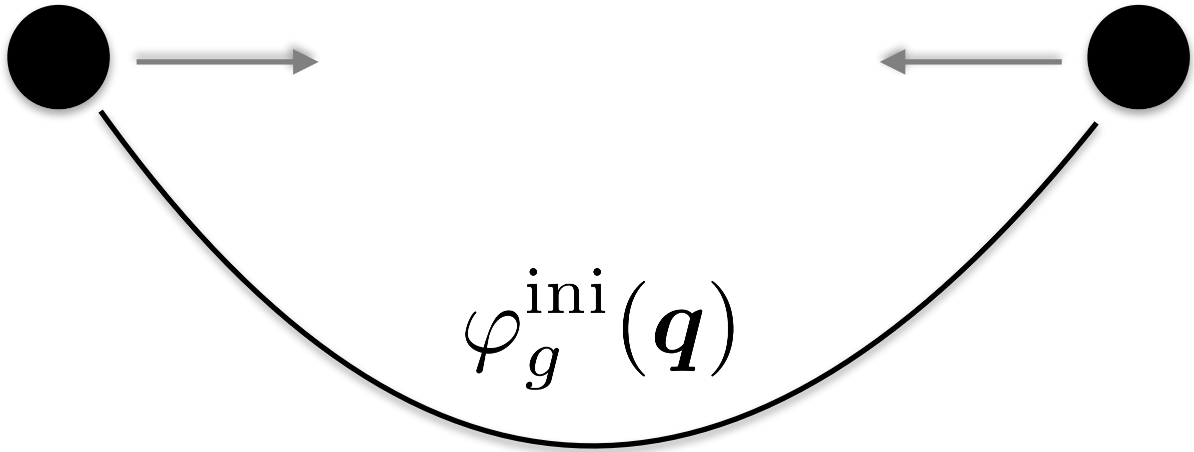
Approximate: Shoot Particles

follow initial gravitational potential

$$\boldsymbol{v}(\boldsymbol{q}, a) = -\boldsymbol{\nabla}\varphi_g^{\mathrm{ini}}(\boldsymbol{q})$$

Zel'dovich approx

$$\boldsymbol{x}(\boldsymbol{q}, a) = \boldsymbol{q} - a\boldsymbol{\nabla}\varphi_g^{\mathrm{ini}}(\boldsymbol{q})$$



Free Propagation

classical action: displacement × velocity

$$S_0(\boldsymbol{x}, \boldsymbol{q}, a) = \frac{1}{2}(\boldsymbol{x} - \boldsymbol{q}) \cdot \frac{\boldsymbol{x} - \boldsymbol{q}}{a}$$

background expansion

Free Propagation

transition amplitude

$$\psi_0(\boldsymbol{x}, a) = N \int d^3q \exp\left[\frac{i}{\hbar} S_0(\boldsymbol{x}, \boldsymbol{q}, a)\right] \psi_0^{\mathrm{ini}}(\boldsymbol{q})$$

Schrödinger equation

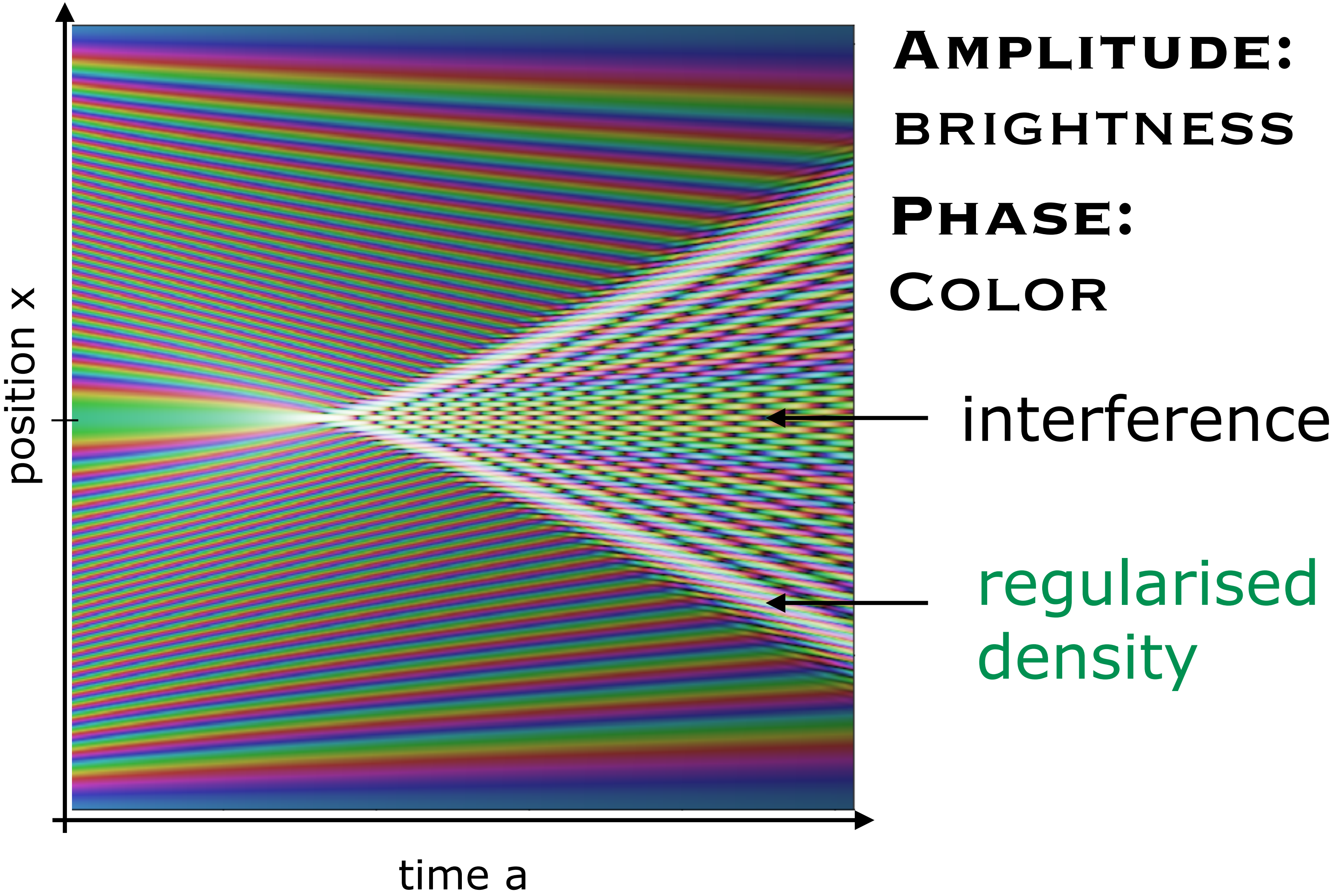
$$i\hbar\partial_a\psi_0 = -\frac{\hbar^2}{2}\nabla^2\psi_0$$

small parameter

free-particle approx
Coles & Spencer '03

CU, Rampf & Hahn '20
arXiv:1812.05633

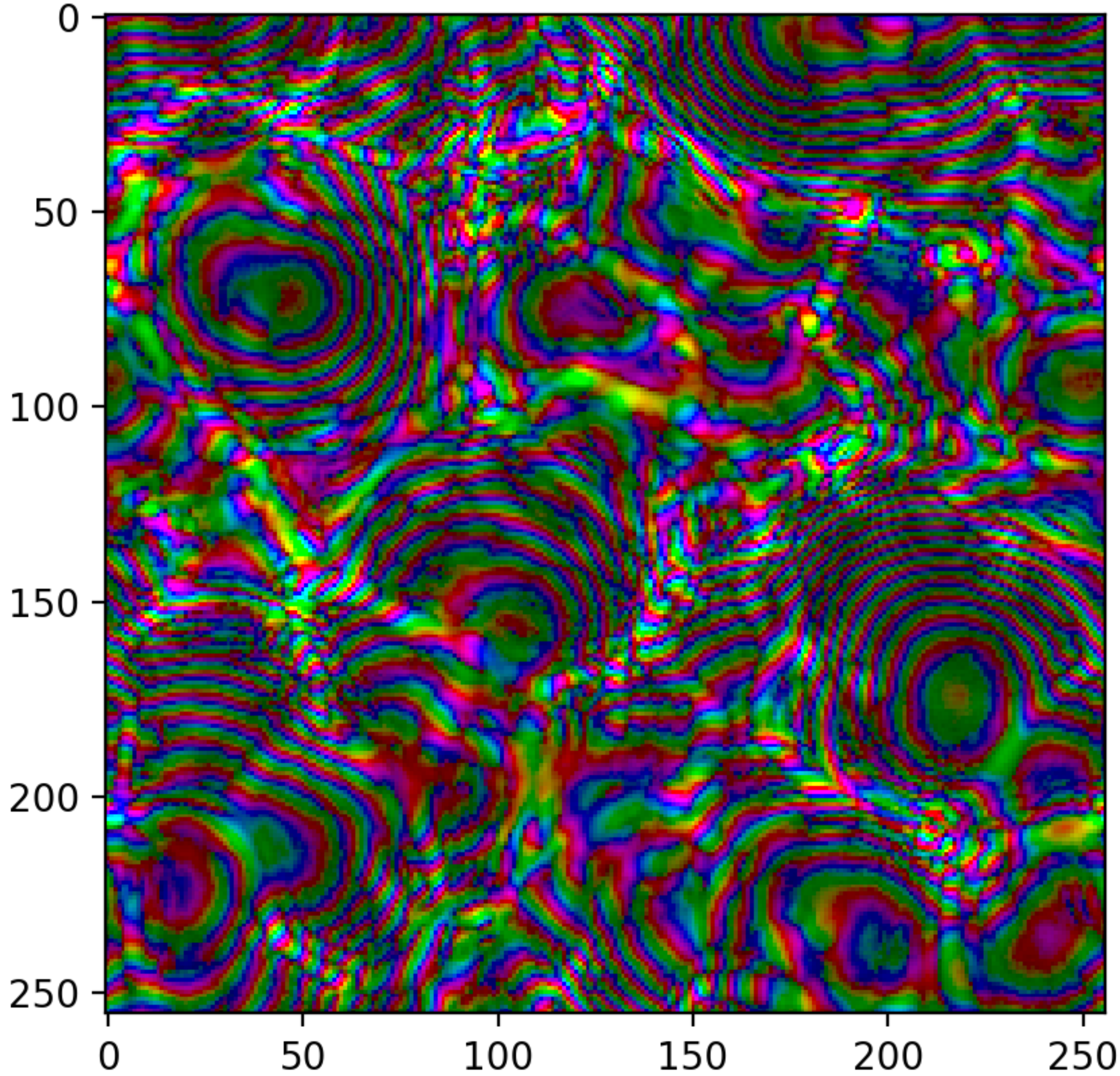
Sine-Wave Example



Amplitude: brightness

Phase: Color

Random Cosmo ICs



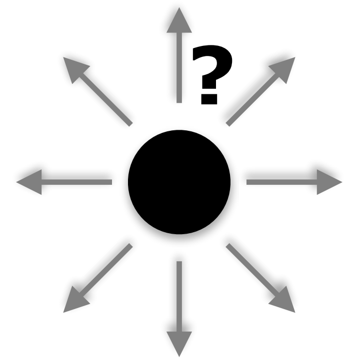
Propagator PT

Interactive Propagation

$$i\hbar\partial_a\psi = -\frac{\hbar^2}{2}\nabla^2\psi + V_{\rm eff}(\boldsymbol{x},a)\psi$$

↑

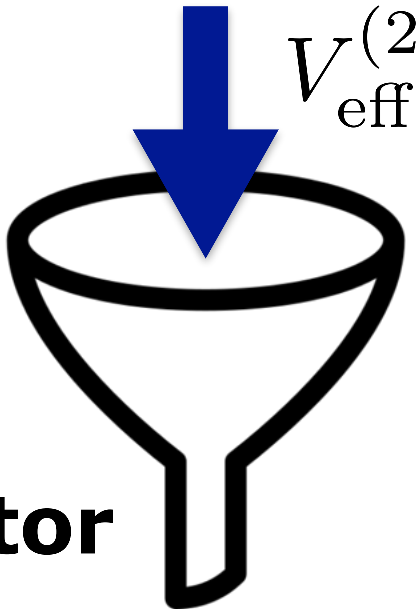
perturbative fluid: tidal effects



$$V_{\rm eff}^{(2)}(\boldsymbol{x}) = \frac{3}{7}\nabla^{-2}\left[\left(\nabla^2\varphi_{\rm g}^{\rm (ini)}\right)^2 - \left(\nabla_i\nabla_j\varphi_{\rm g}^{\rm (ini)}\right)^2\right]$$

Propagator PT

Schrödinger eq.



$V_{\rm eff}^{(2)}(\boldsymbol{x})$

propagator

$$\left(S_0 + S_{\rm tid}\right)(\boldsymbol{x}, \boldsymbol{q}, a)$$

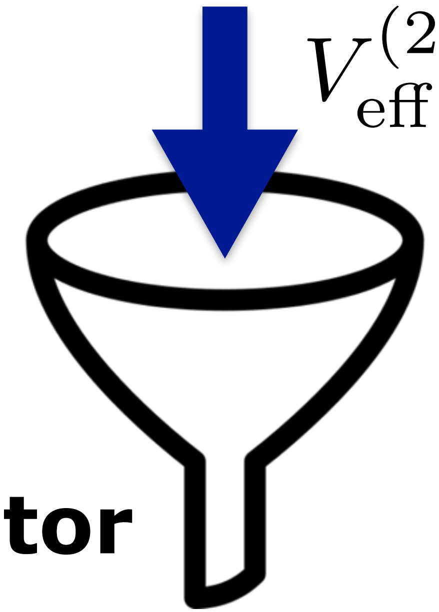
$$S_{\rm tid} \simeq -\frac{a}{2}\left[V_{\rm eff}^{(2)}(\boldsymbol{q}) + V_{\rm eff}^{(2)}(\boldsymbol{x})\right]$$

kick+drift+kick

Propagator PT

$$V_{\mathrm{eff}}^{(2)}(\boldsymbol{x})$$

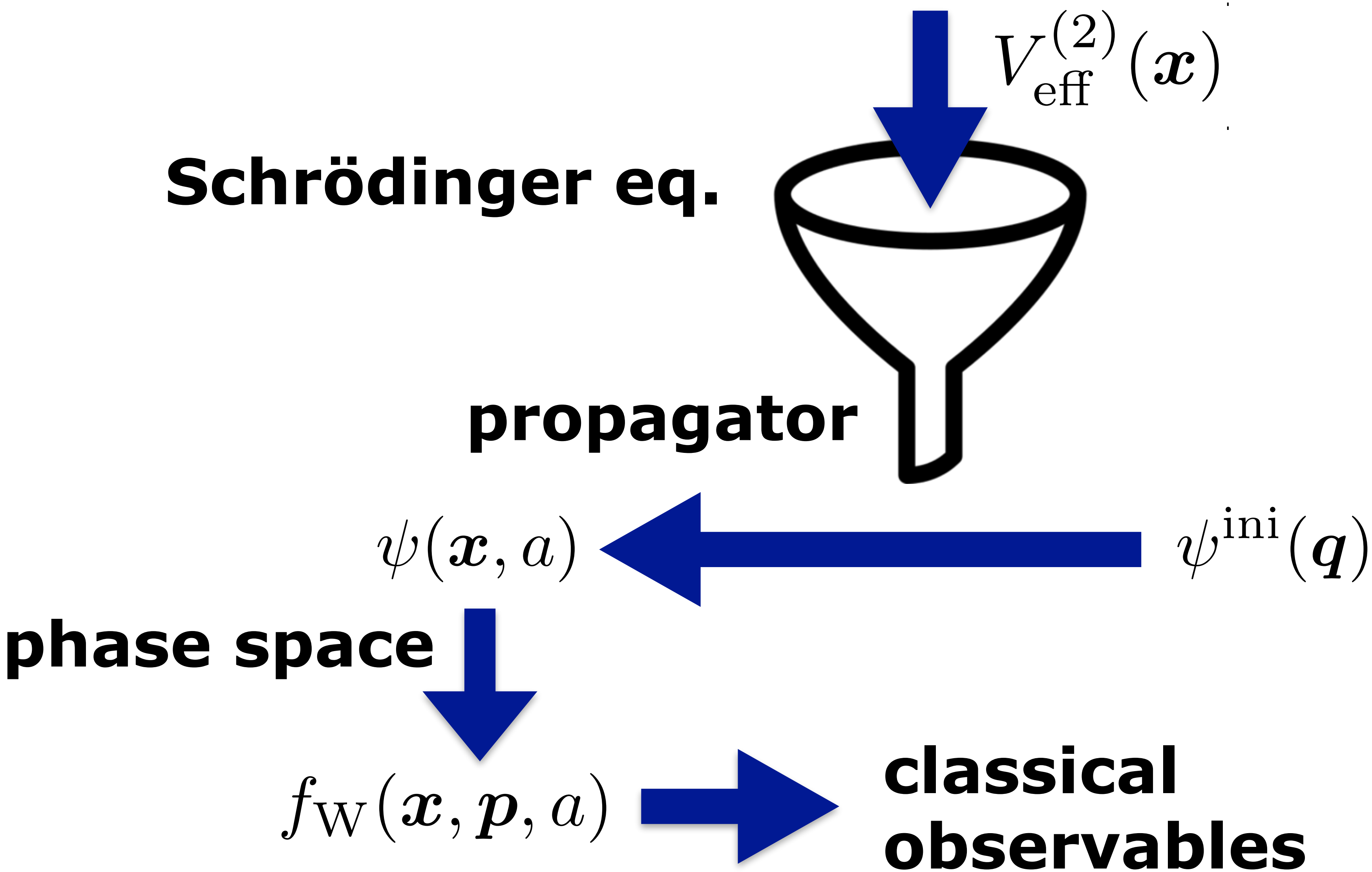
Schrödinger eq.



propagator

$$\psi(\boldsymbol{x}, a) \propto \int d^3q \exp\left[\frac{i}{\hbar} S(\boldsymbol{x}, \boldsymbol{q}, a)\right] \psi^{\mathrm{ini}}(\boldsymbol{q})$$

Propagator PT



Classical Observables

Phase-Space Distribution

coarse-grained Wigner $\bar{f}_W[\psi, \hbar \to 0]$

$$f_{\mathrm{W}}(\boldsymbol{x},\boldsymbol{p}) = \int \frac{\mathrm{d}^3 x'}{(2\pi)^3} \exp\left[\frac{-\mathrm{i}\boldsymbol{p}\cdot\boldsymbol{x}'}{a^{3/2}}\right] \psi(\boldsymbol{x} + \tfrac{\hbar}{2}\boldsymbol{x}')\, \bar{\psi}(\boldsymbol{x} - \tfrac{\hbar}{2}\boldsymbol{x}')$$

phase-space info in wave function

Lagrangian Fluid

displacement: 2LPT

velocity beyond $\boldsymbol{v}^L(\boldsymbol{q}) = \dot{\boldsymbol{\xi}}(\boldsymbol{q})$

$$\boldsymbol{v}(\boldsymbol{q}) = -\boldsymbol{\nabla}\varphi_g^{(\mathrm{ini})} - a\boldsymbol{\nabla}V_{\mathrm{eff}}^{(2)}$$

$$+\frac{a^2}{2}\boldsymbol{\nabla}\nabla V_{\mathrm{eff}}^{(2)} \cdot \nabla\varphi_g^{(\mathrm{ini})}$$

vorticity conserver

Vorticity Conservation

Eulerian $\nabla_x \times \boldsymbol{v} = 0$

before shell-crossing

Vorticity Conservation

Lagrangian: Cauchy invariants

$$\varepsilon_{ijk} x_{l,j} \dot{x}_{l,k} = 0$$

2LPT

$$= \mathcal{O}(a^2)$$

2PPT

$$= \mathcal{O}(a^3)$$

Classical Observables

Eulerian Fluid

density & velocity

$$\rho(\boldsymbol{x}) = |\psi(\boldsymbol{x})|^2$$

$$\boldsymbol{v}(\boldsymbol{x}) = \frac{i\hbar}{2} \frac{\psi \boldsymbol{\nabla} \bar{\psi} - \bar{\psi} \boldsymbol{\nabla} \psi}{|\psi|^2}$$

application: Lyman-α

Porqueres et al. arXiv:2005.12928

Propagator PT ~ LPT in Eulerian space

PPT Initial Conditions

ICs for Eulerian Hydro Sims

2 fluids: DM & baryons

relative density $\delta_{bc}^{\rm ini} = \delta_{b}^{\rm ini} - \delta_{c}^{\rm ini}$

neglect decaying modes

Rampf, CU, Hahn
& Hahn, Rampf, CU
fresh on arXiv

$\Rightarrow$ 1 propagator for 2 wave functions

see Oliver Hahn’s Talk “Initial conditions for Cosmological Simulations: The next generation”

Conclusion

Large-scale structure = cosmic laboratory

Challenge: nonlinear clustering

analytical methods & numerical simulations

Tool: semiclassical physics

correspondence classical ⇄ quantum

Schrödinger eq. with effective potential

Propagator PT: best of Lagrangian & Eulerian

Applications: Lyman-α & initial conditions