

# constraining $M_\nu$ with *the bispectrum*

Changhoon Hahn



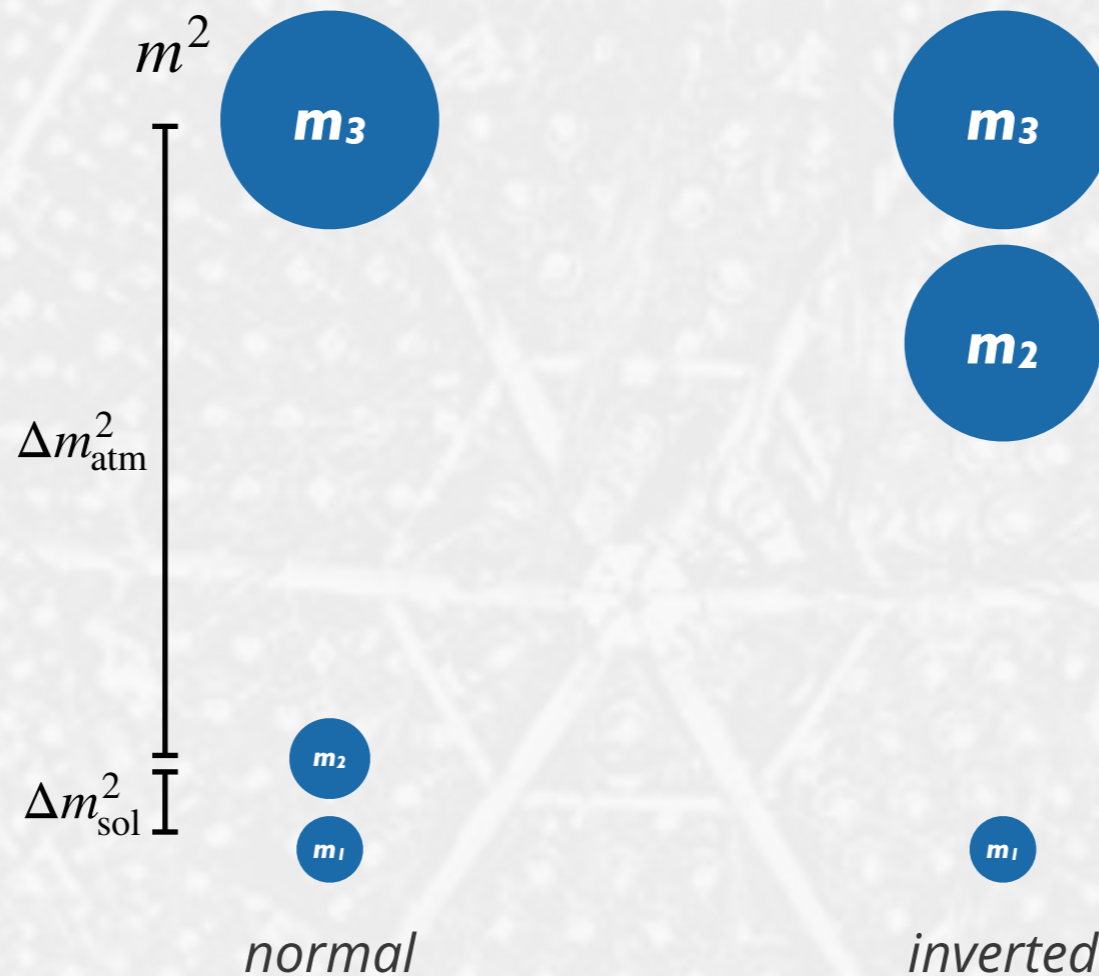
arXiv:1909.05273

arXiv:1909.11107

*neutrino oscillation experiments* established a **lower bound**  
on the sum of neutrino masses  $M_\nu \gtrsim 0.06 \text{ eV}$  🏆

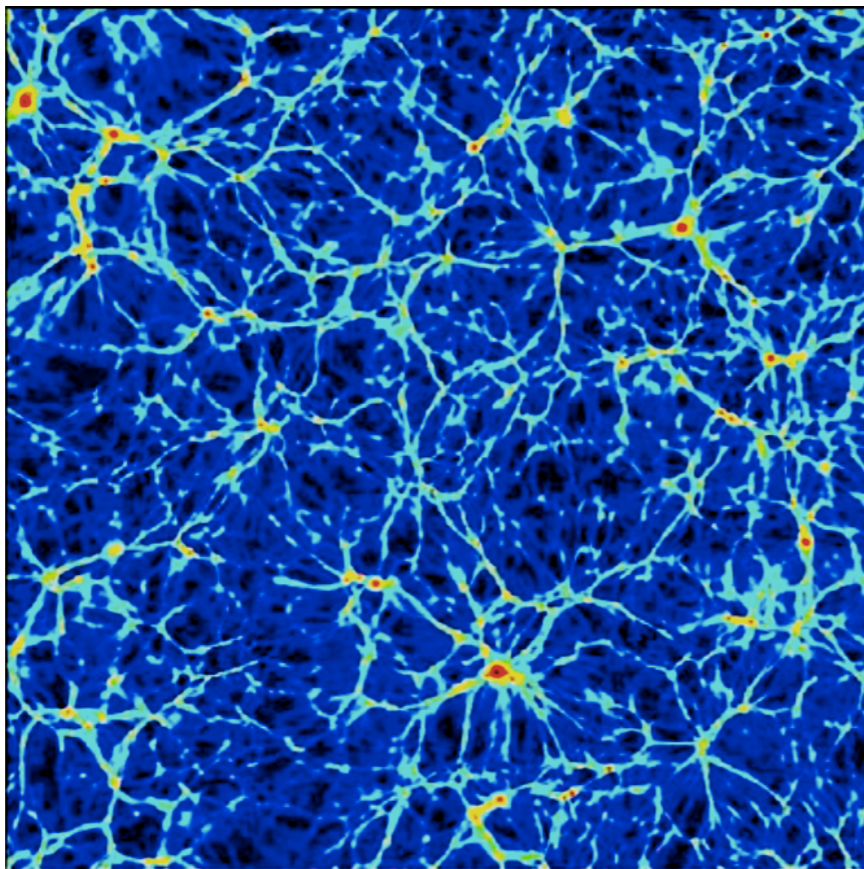
**but they don't measure  $M_\nu$**

precise  $M_\nu$  can distinguish between “*normal*” and “*inverted*” *hierarchies* and reveal physics **beyond the Standard Model**

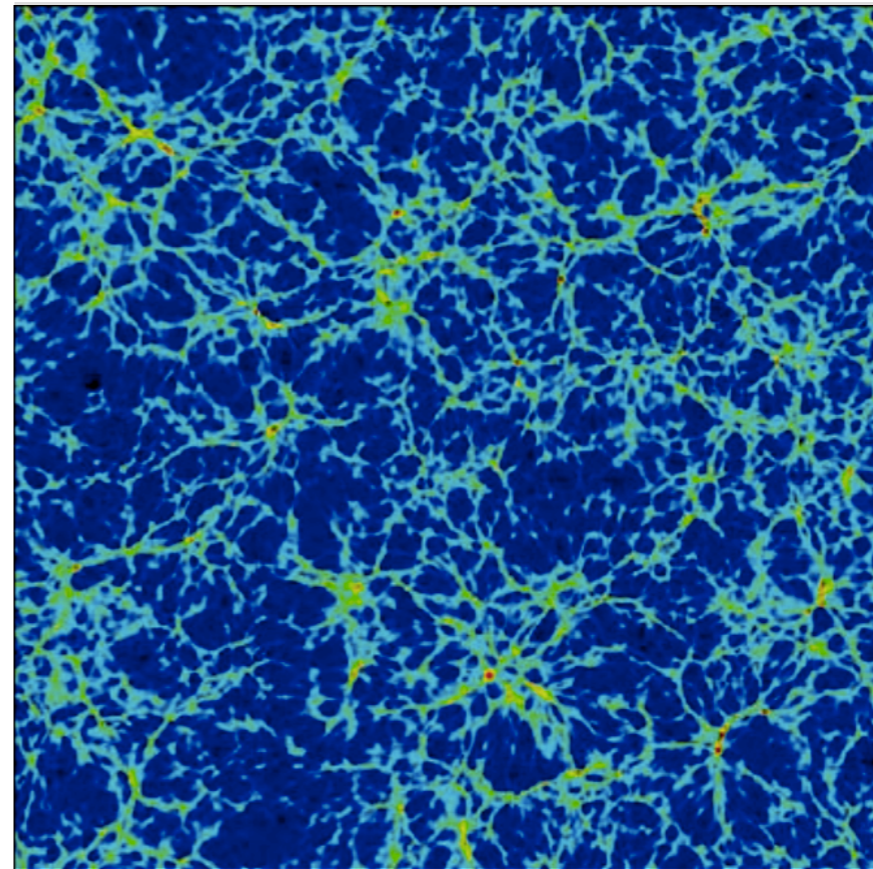


upcoming laboratory experiments are ***not sensitive enough***

$M_\nu > 0$  eV **suppresses the growth of structure** on *small scales* below their free-streaming scale

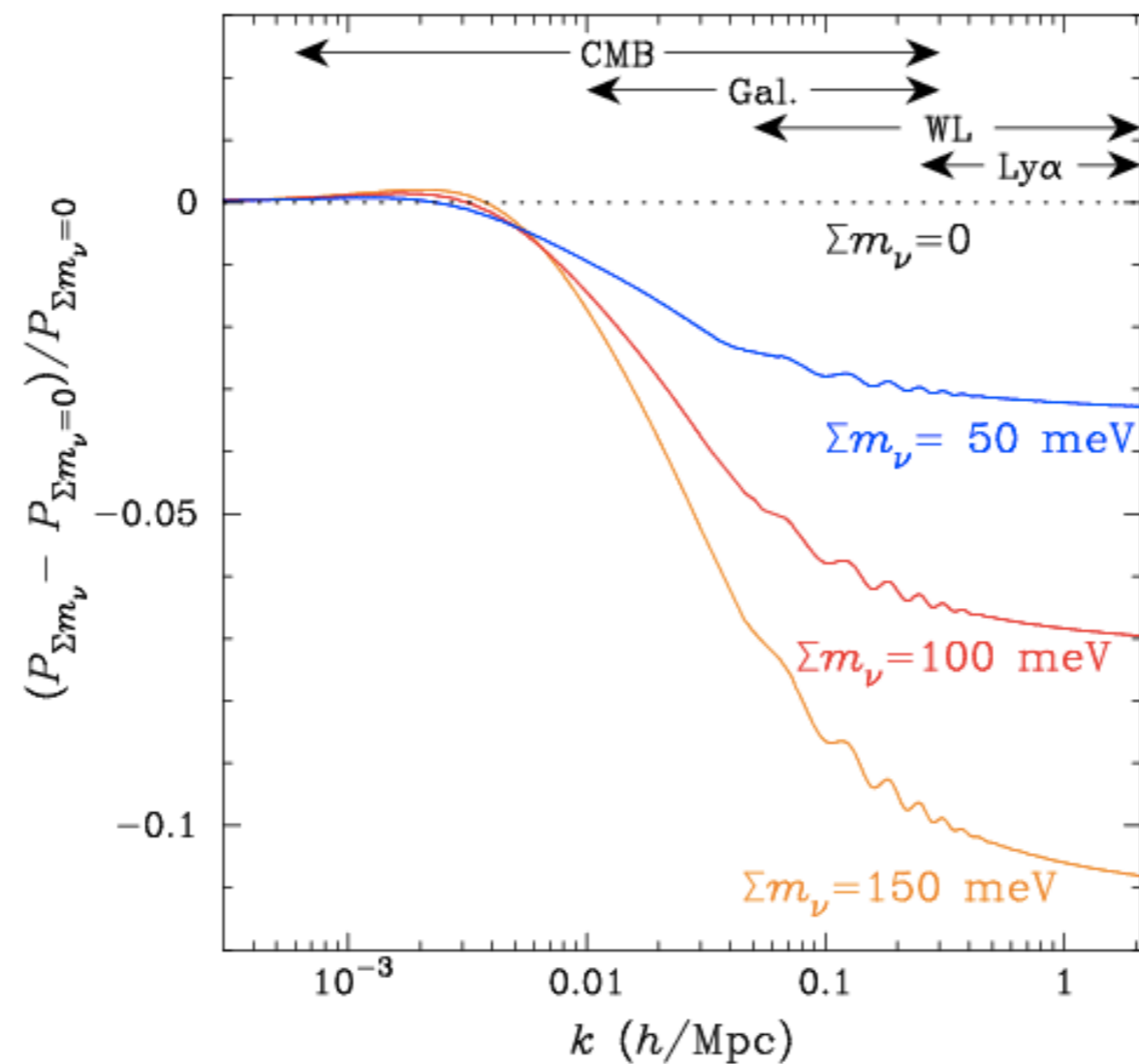


$M_\nu = 0$  eV



$M_\nu = 1.9$  eV

$M_\nu > 0$  eV **suppresses the growth of structure** on *small scales* below their free-streaming scale



*best constraints* currently come from cosmology: **CMB + LSS**

$$M_\nu < 0.13 \text{ eV (95\%, Planck TT+lowE+lensing+BAO)}$$

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$$\underline{M_\nu} < 0.13 \text{ eV (95\%, Planck TT+lowE+lensing+BAO)}$$

*KATRIN 1.1 eV upper limit (Aker+2019)\**

*\*not a fair comparison but still...*

*CMB* measures  $A_s e^{-2\tau}$  and thus **heavily rely on  $\tau$  constraints**

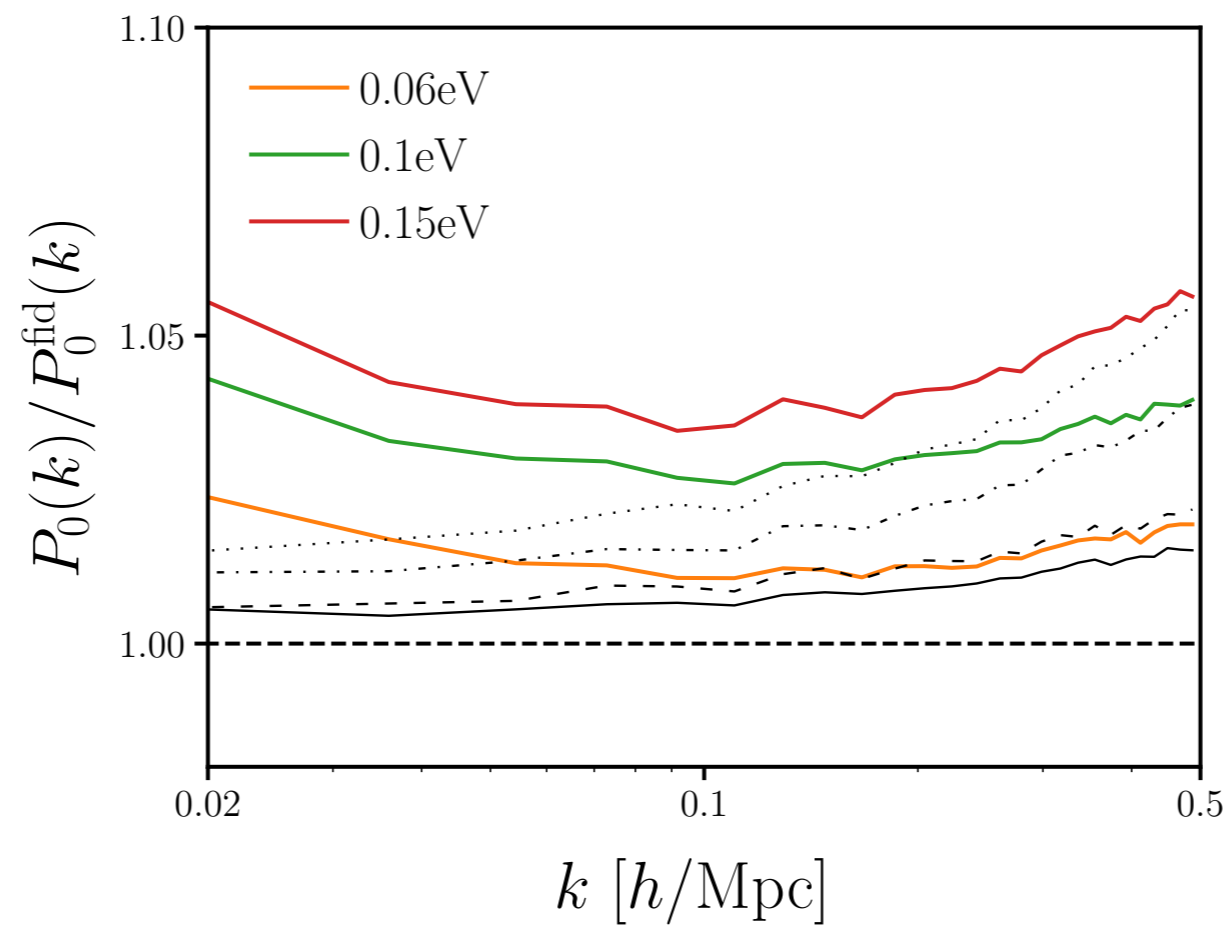
but upcoming ground-based experiments (*CMB-S4*)  
**will not directly constrain  $\tau$  ... *LiteBIRD?*, *LiteCOrE?***



*imprint of  $M_\nu$*  can also be measured from **galaxy clustering**

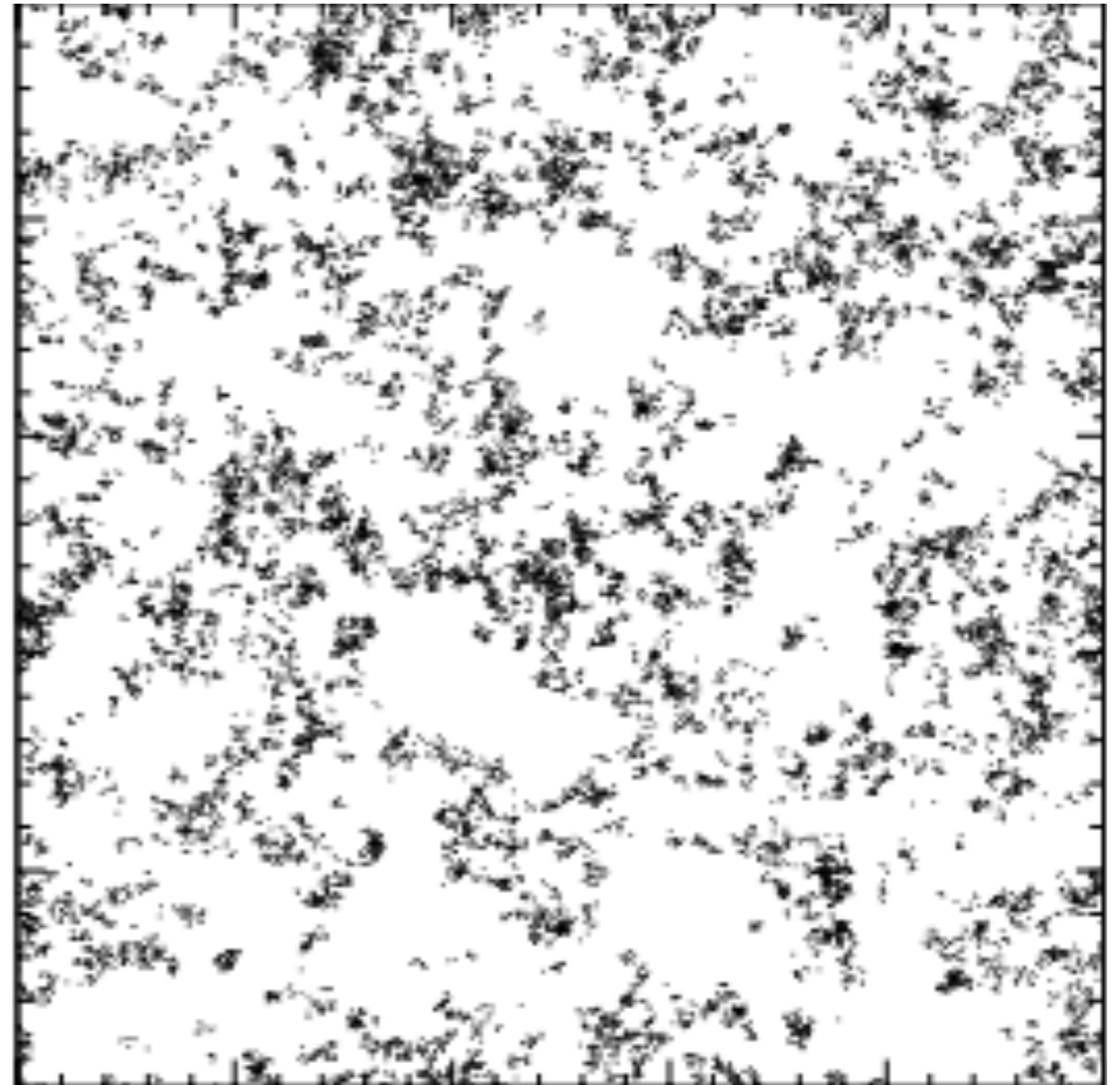
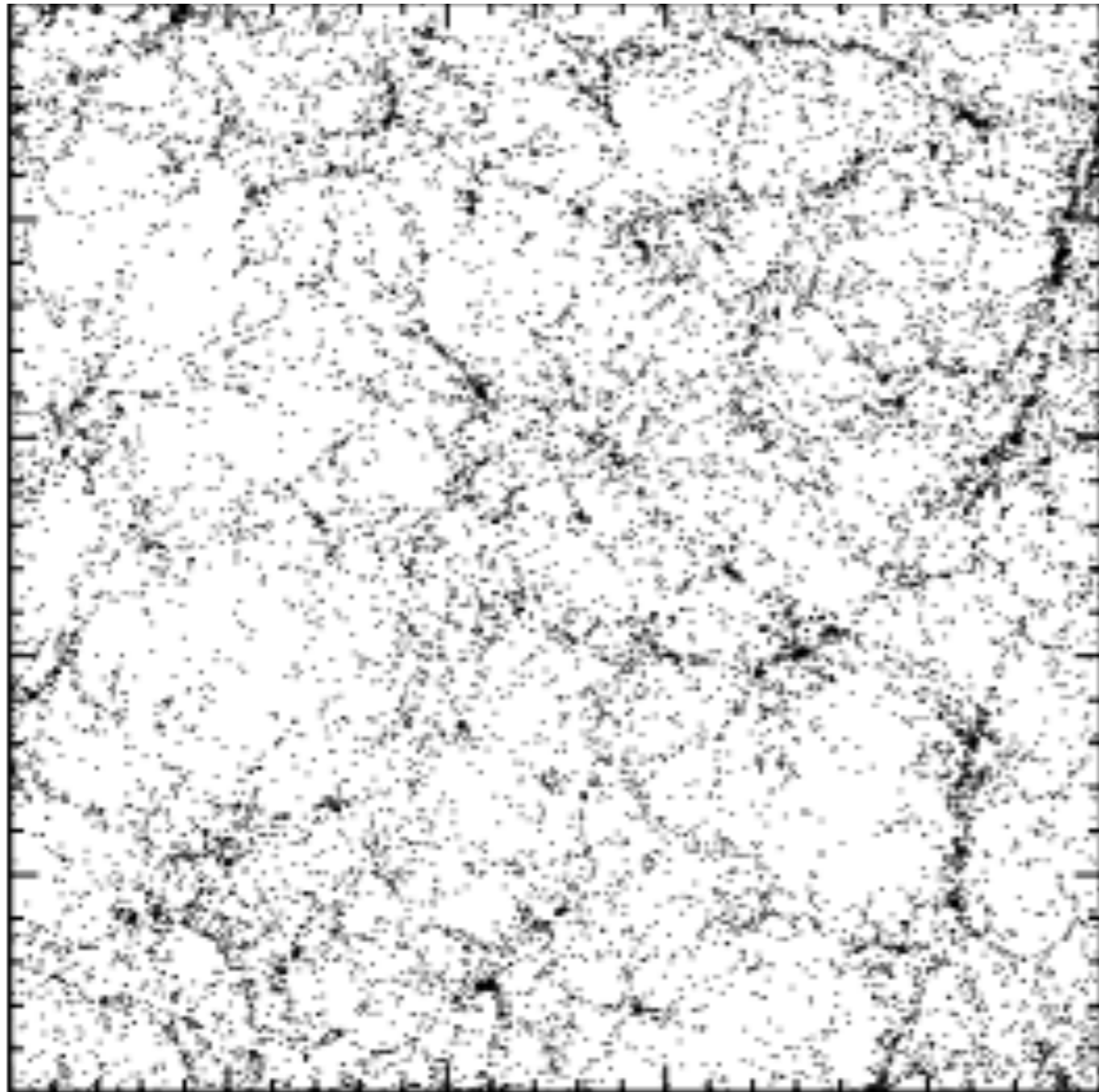
upcoming surveys: *PFS, DESI, Euclid, WFIRST*

$M_\nu - \sigma_8$  **degeneracy is a major limitation** for the redshift-space *power spectrum*

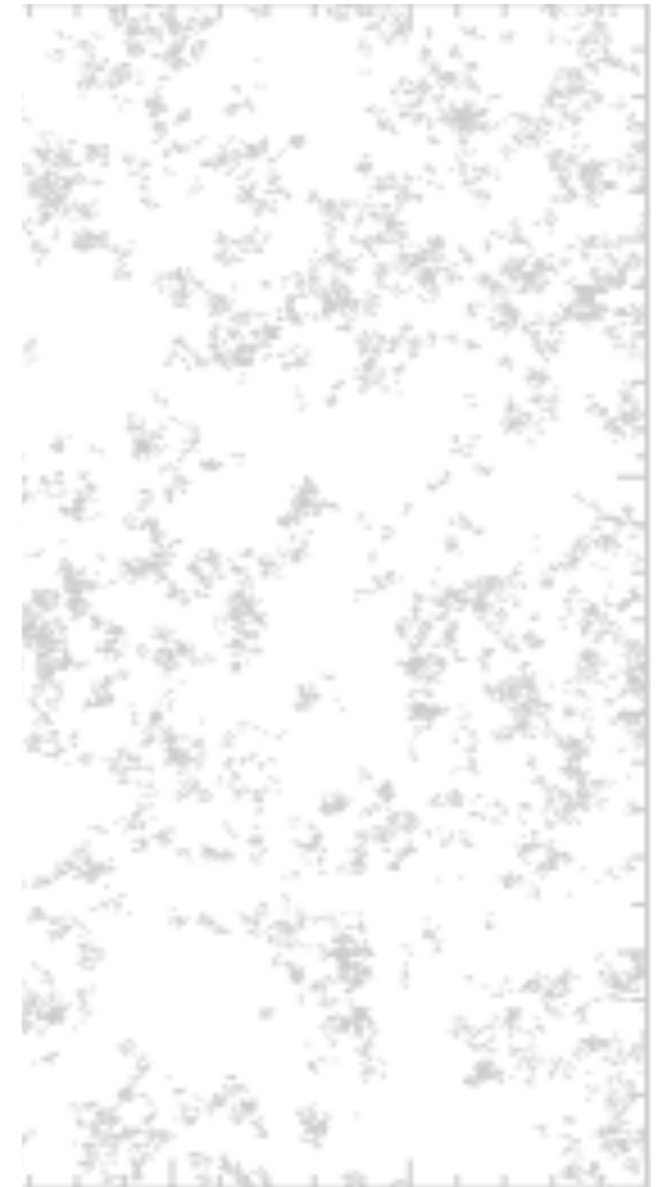
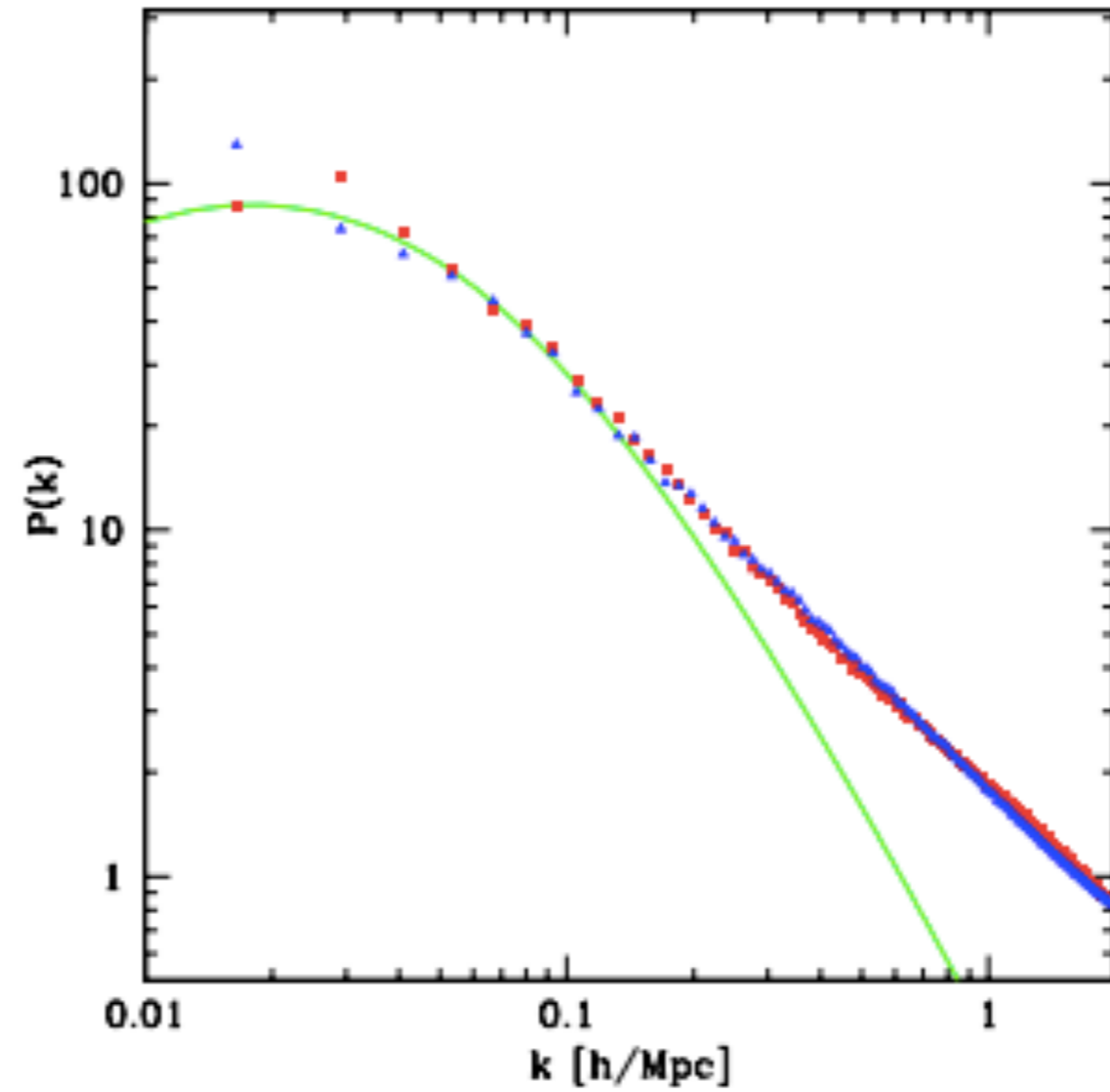


information in the *nonlinear regime* cascades from the power spectrum to **higher-order statistics**

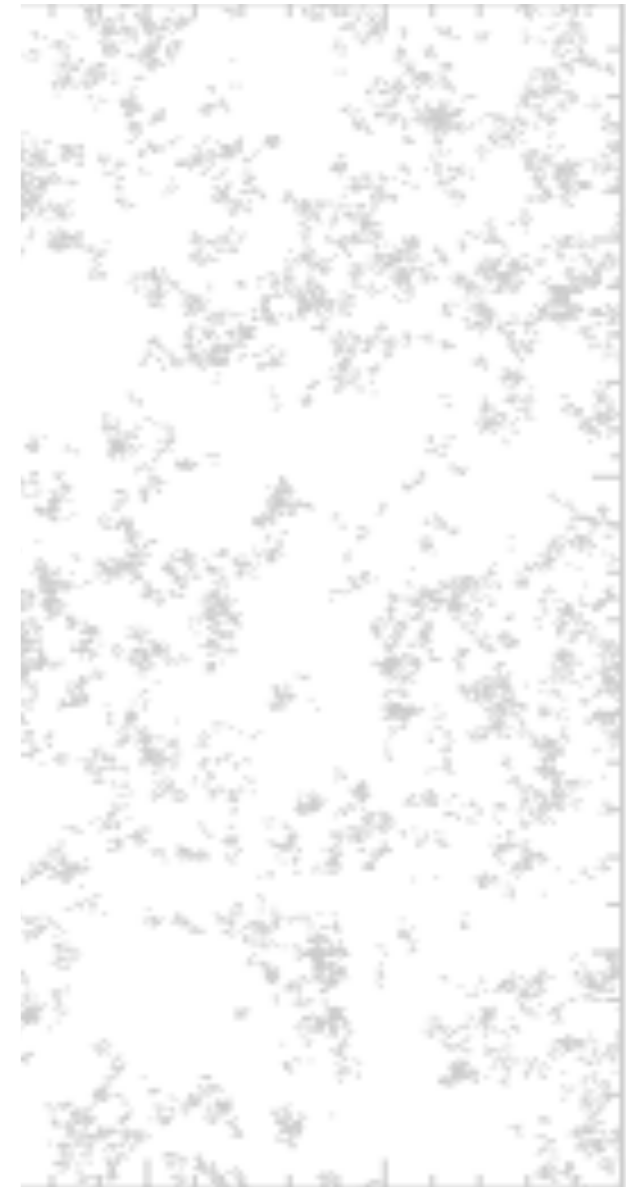
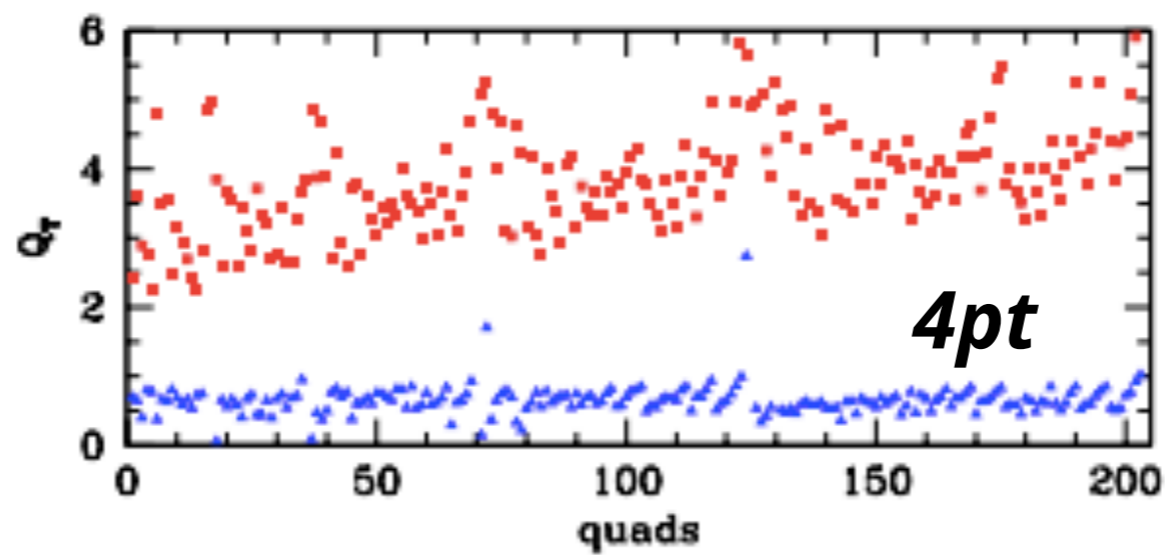
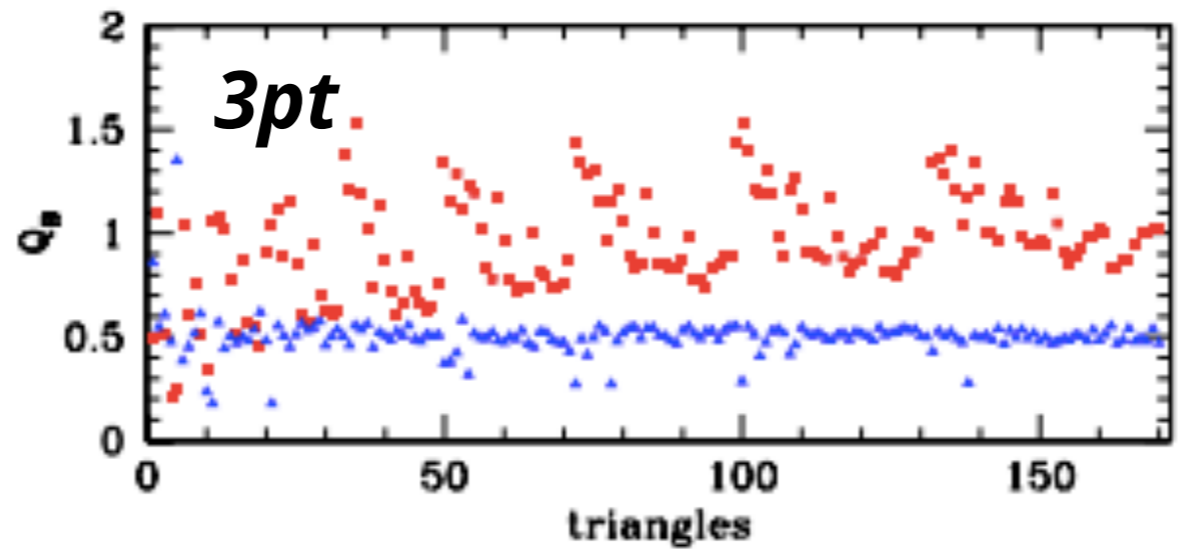
these two distributions have the *same power spectrum*



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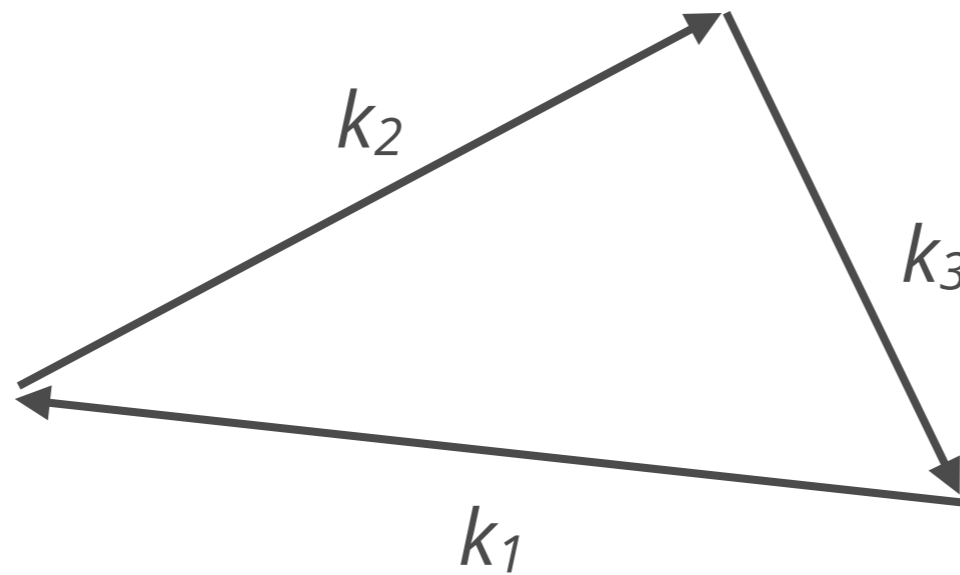


but very *different higher-order statistics*

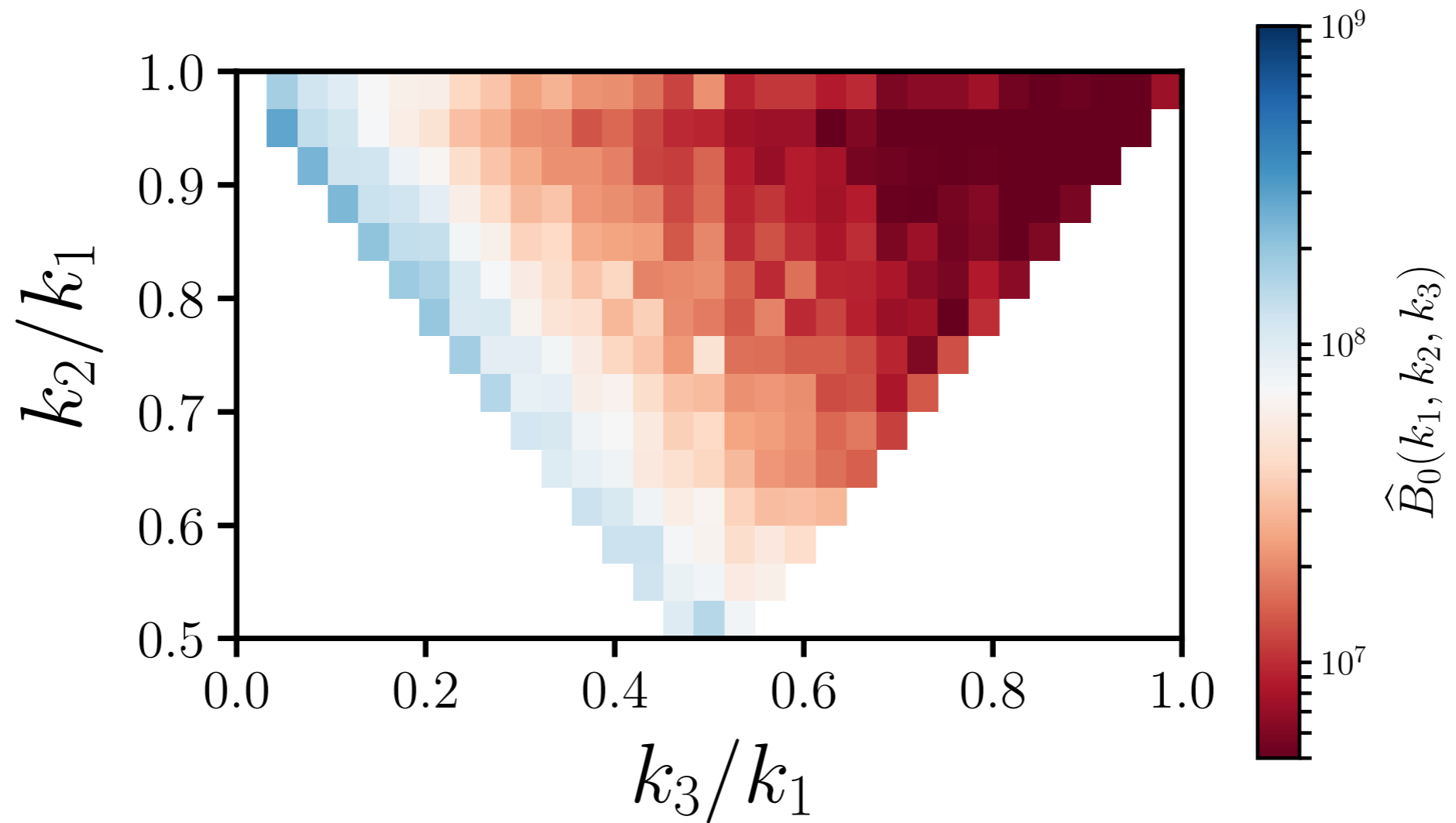


## introduction to *the bispectrum*

$$\langle \delta(\mathbf{k}_1), \delta(\mathbf{k}_2), \delta(\mathbf{k}_3) \rangle = \delta_D(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3) B(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3)$$

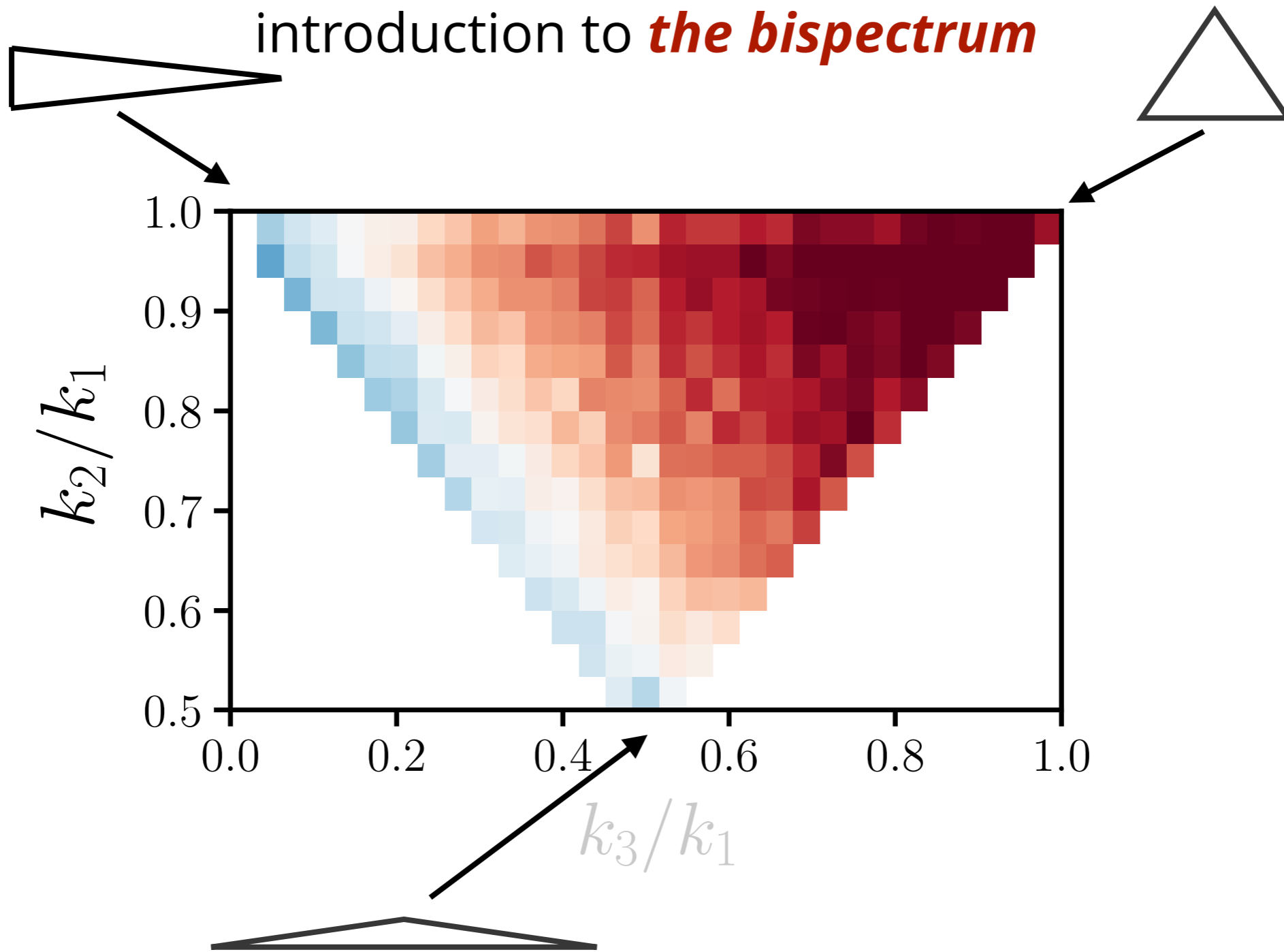


introduction to *the bispectrum*

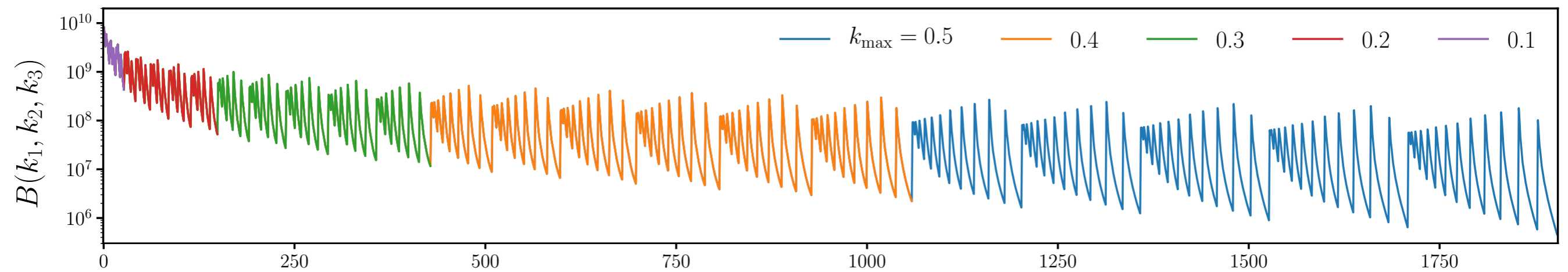




introduction to *the bispectrum*

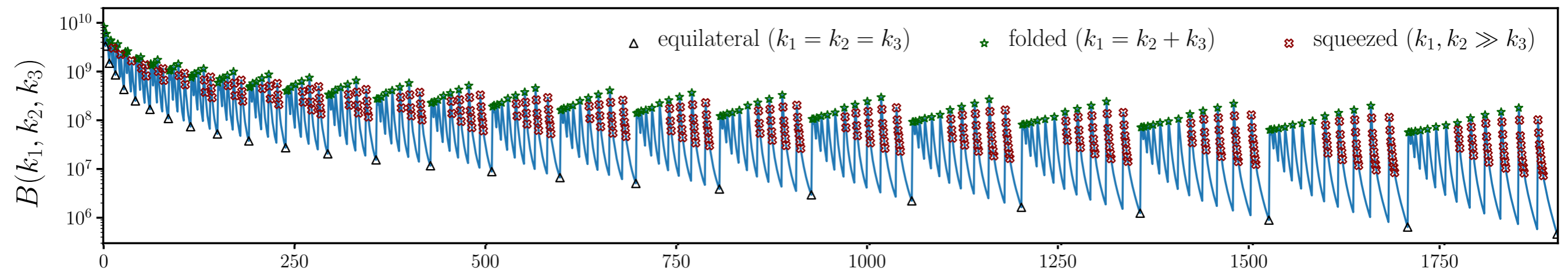


# introduction to *the bispectrum*



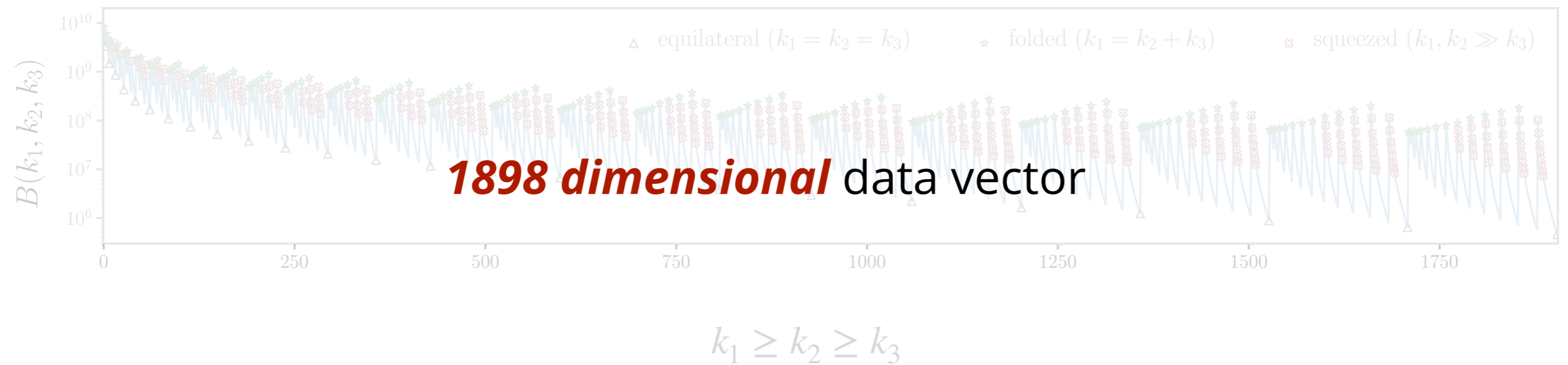
$$k_1 \geq k_2 \geq k_3$$

# introduction to *the bispectrum*



$$k_1 \geq k_2 \geq k_3$$

# introduction to *the bispectrum*



does *the bispectrum improve  $M_\nu$  constraints?*

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what's the **total information content** of *the bispectrum?*

# Quijote Simulations:

43,100 full N-body simulations designed to quantify the information content of cosmological observables

## Features

- Simulations run with the TreePM code Gadget-III
- More than 35 Million CPU hours
- Boxes of 1 Gpc/h. Combined total volume of 43100 (Gpc/h)<sup>3</sup>
- 17100 simulations for a fiducial Planck cosmology
- Between 500 and 1000 simulations/cosmology for 17 different cosmologies
- 11000 simulations in different latin-hypercubes
- More than 8.5 trillions of particles at a single redshift
- Billions of halos and voids identified
- Full snapshots at redshifts 0, 0.5, 1, 2, 3 and 127 (initial conditions)
- More than 200000 halo catalogues
- More than 200000 void catalogues
- More than 1 million power spectra
- More than 1 million bispectra
- More than 1 million correlation functions
- More than 1 million marked power spectra
- More than 1 million probability distribution functions
- More than 1 Petabyte of data publicly available

***all publicly available***

## ***Quijote Simulations:***

*43,100 full N-body simulations* designed to quantify the information content of cosmological observables

$$F_{ij} = \frac{1}{2} \text{Tr} \left[ \mathbf{C}^{-1} \left( \frac{\partial B^T}{\partial \theta_i} \frac{\partial B}{\partial \theta_j} + \frac{\partial B^T}{\partial \theta_j} \frac{\partial B}{\partial \theta_i} \right) \right]$$



## Quijote Simulations:

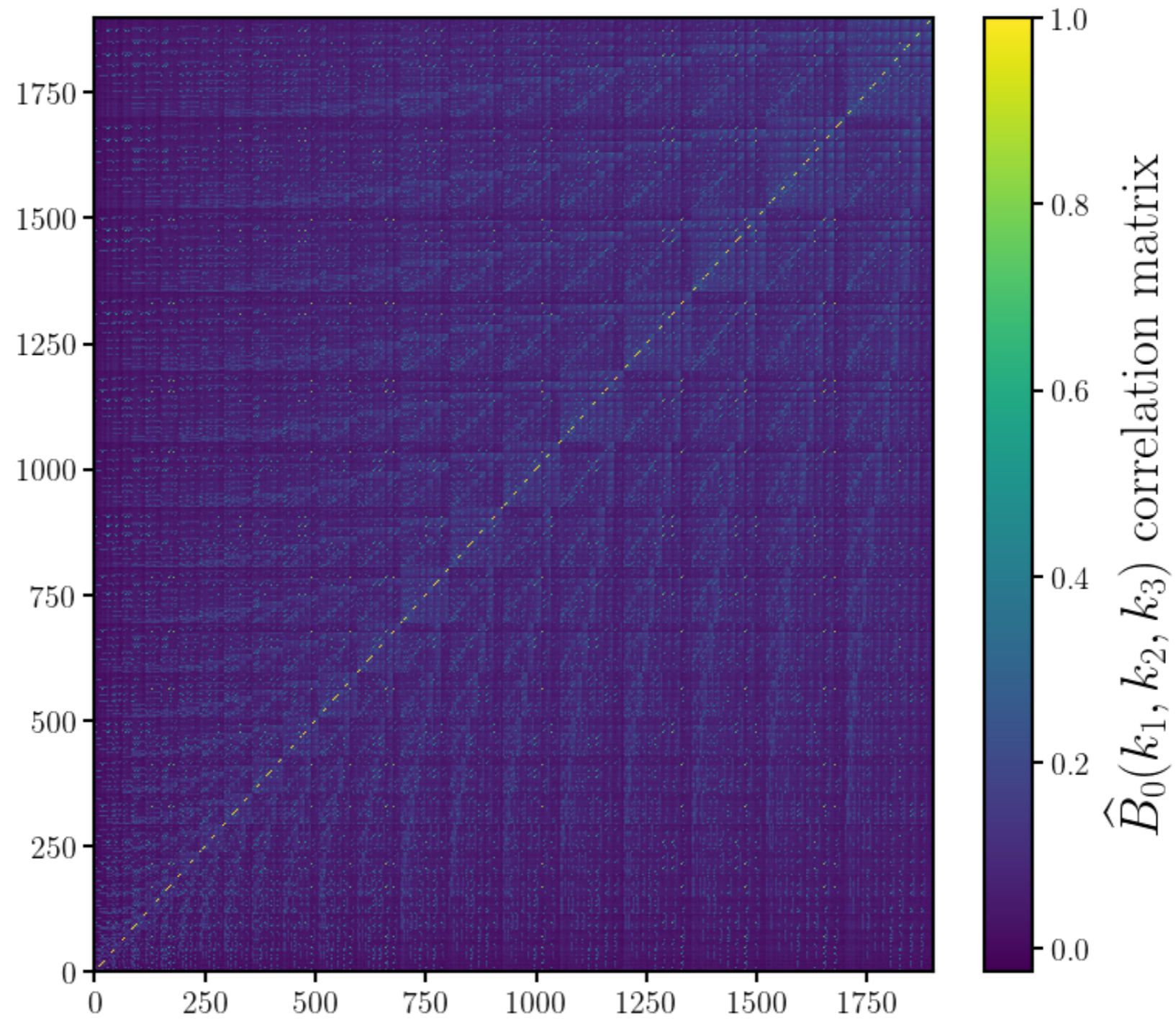
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difficult to accurately estimate the **1898x1898 covariance matrix**

*previous work used fewer triangles, PT estimates, ignored non-Gaussian term, etc*

C estimated using **15,000 N-body simulations!**



$$F_{ij} = \frac{1}{2} \text{Tr} \left[ \mathbf{C}^{-1} \left( \frac{\partial B^T}{\partial \theta_i} \frac{\partial B}{\partial \theta_j} + \frac{\partial B^T}{\partial \theta_i} \frac{\partial B}{\partial \theta_j} \right) \right]$$

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**fiducial**     $M_\nu = 0.0\text{eV}$      $\Omega_m = 0.3175$      $\Omega_b = 0.049$      $h = 0.6711$      $n_s = 0.9624$      $\sigma_8 = 0.834$

$$F_{ij} = \frac{1}{2} \text{Tr} \left[ \mathbf{C}^{-1} \left( \frac{\partial B^T}{\partial \theta_i} \frac{\partial B}{\partial \theta_j} + \frac{\partial B^T}{\partial \theta_i} \frac{\partial B}{\partial \theta_j} \right) \right]$$

$$\Omega_m^+ = 0.3275$$

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$$\Omega_m^- = 0.3075$$

$$F_{ij} = \frac{1}{2} \text{Tr} \left[ \mathbf{C}^{-1} \left( \frac{\partial B^T}{\partial \theta_i} \frac{\partial B}{\partial \theta_j} + \frac{\partial B^T}{\partial \theta_i} \frac{\partial B}{\partial \theta_j} \right) \right]$$

$$M_\nu^{+++} = 0.4$$

$$M_\nu^{++} = 0.2$$

$$M_\nu^+ = 0.1$$

$$\Omega_m^+ = 0.3275$$

$$\Omega_b^+ = 0.051$$

$$h^+ = 0.6911$$

$$n_s^+ = 0.9824$$

$$\sigma_8^+ = 0.849$$

**fiducial**

$$M_\nu = 0.0\text{eV}$$

$$\Omega_m = 0.3175$$

$$\Omega_b = 0.049$$

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$$n_s = 0.9624$$

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$$h^- = 0.6511$$

$$n_s^- = 0.9424$$

$$\sigma_8^- = 0.819$$

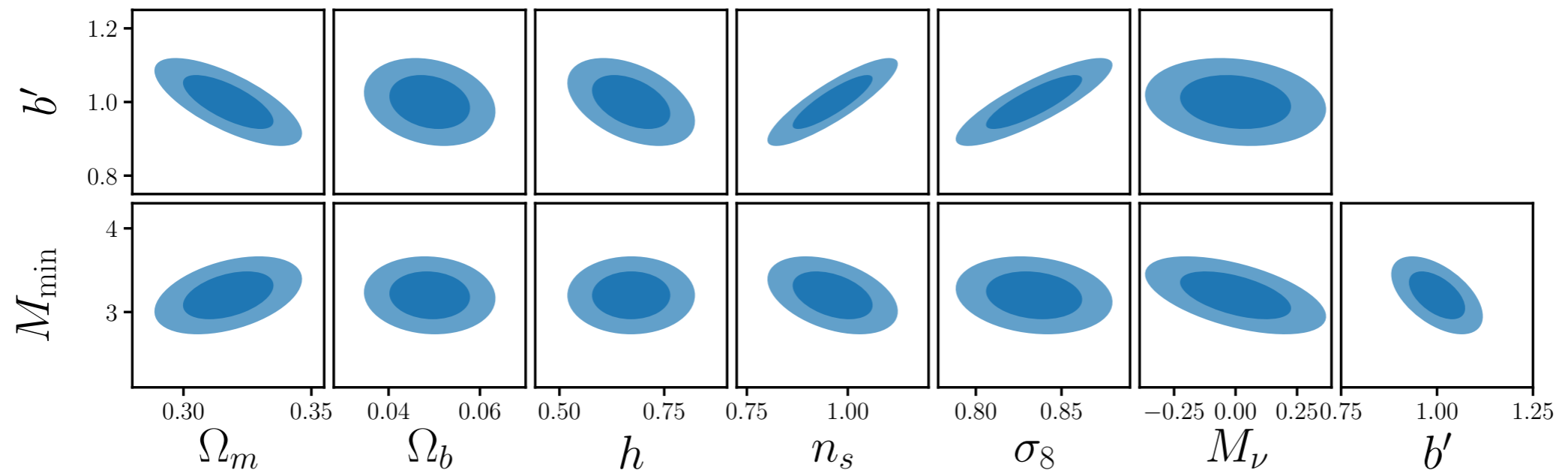
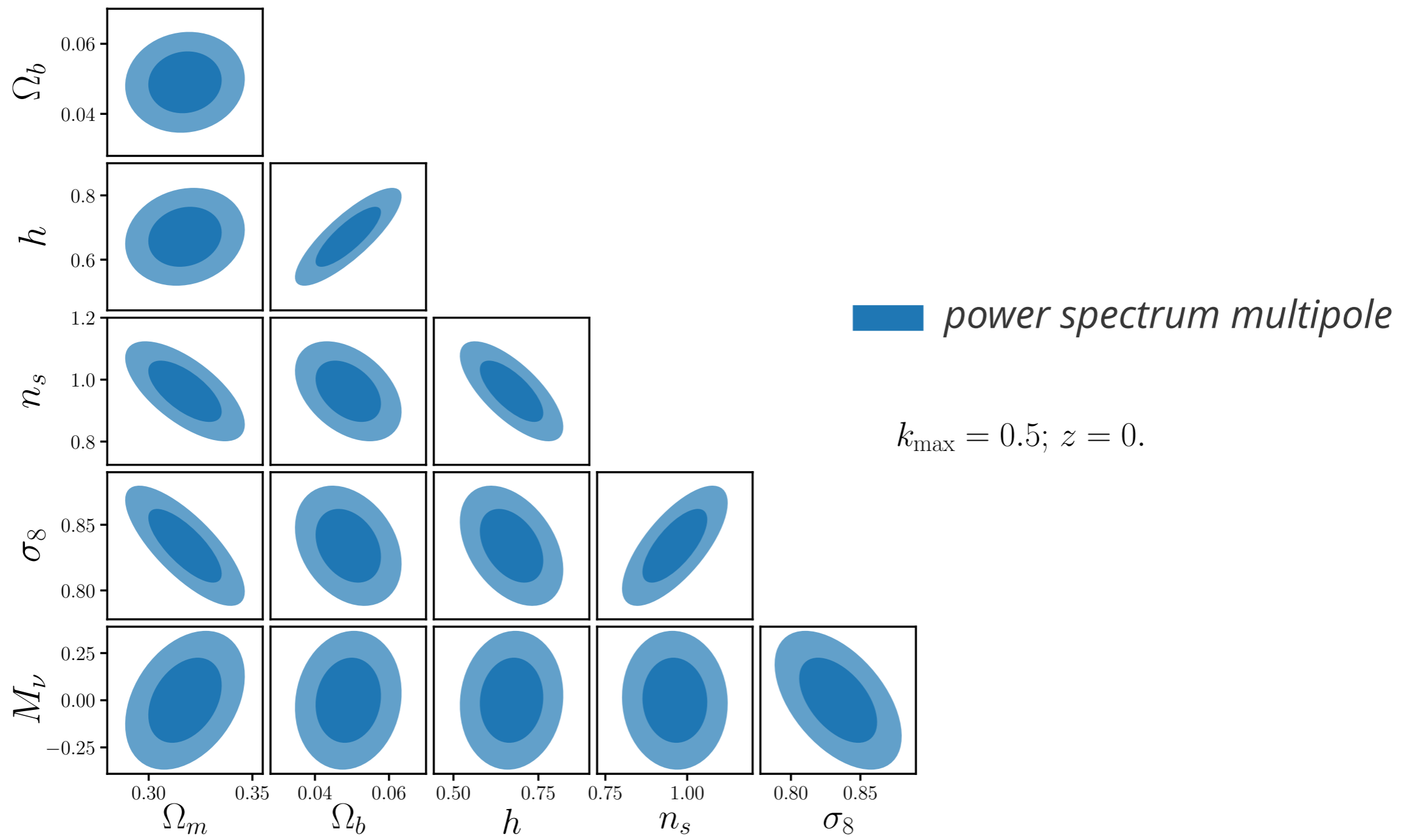
$$\frac{\partial B}{\partial \theta} \approx \frac{B(\theta^+) - B(\theta^-)}{\theta^+ - \theta^-}$$

$$F_{ij} = \frac{1}{2} \text{Tr} \left[ \mathbf{C}^{-1} \left( \frac{\partial B^T}{\partial \theta_i} \frac{\partial B}{\partial \theta_j} + \frac{\partial B^T}{\partial \theta_i} \frac{\partial B}{\partial \theta_j} \right) \right]$$

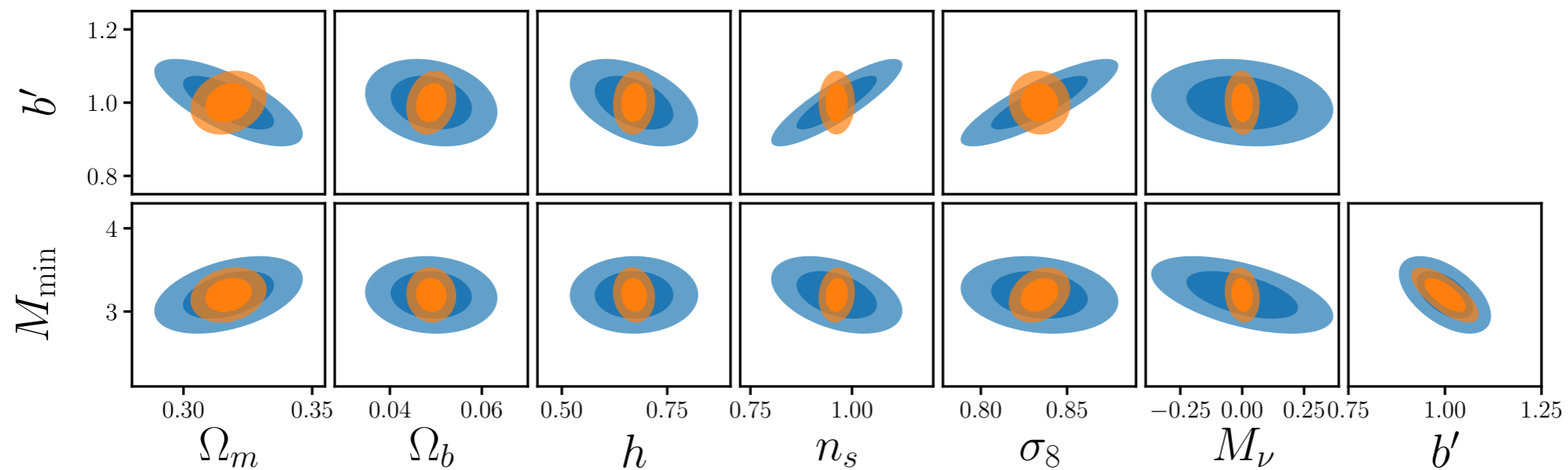
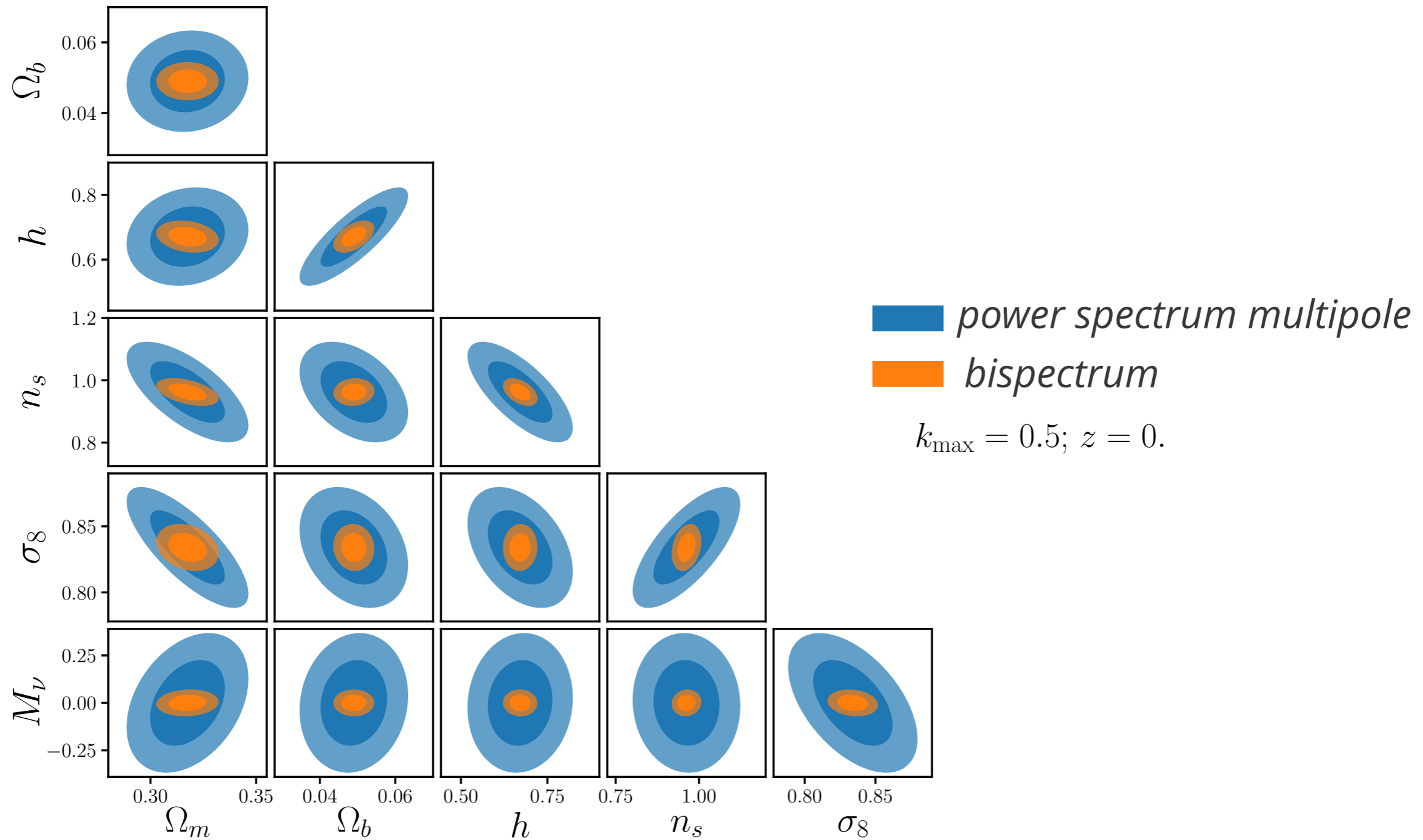
	$M_\nu^{+++} = 0.4$					
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	$M_\nu^+ = 0.1$	$\Omega_m^+ = 0.3275$	$\Omega_b^+ = 0.051$	$h^+ = 0.6911$	$n_s^+ = 0.9824$	$\sigma_8^+ = 0.849$
<b>fiducial</b>	$M_\nu = 0.0\text{eV}$	$\Omega_m = 0.3175$	$\Omega_b = 0.049$	$h = 0.6711$	$n_s = 0.9624$	$\sigma_8 = 0.834$
		$\Omega_m^- = 0.3075$	$\Omega_b^- = 0.047$	$h^- = 0.6511$	$n_s^- = 0.9424$	$\sigma_8^- = 0.819$

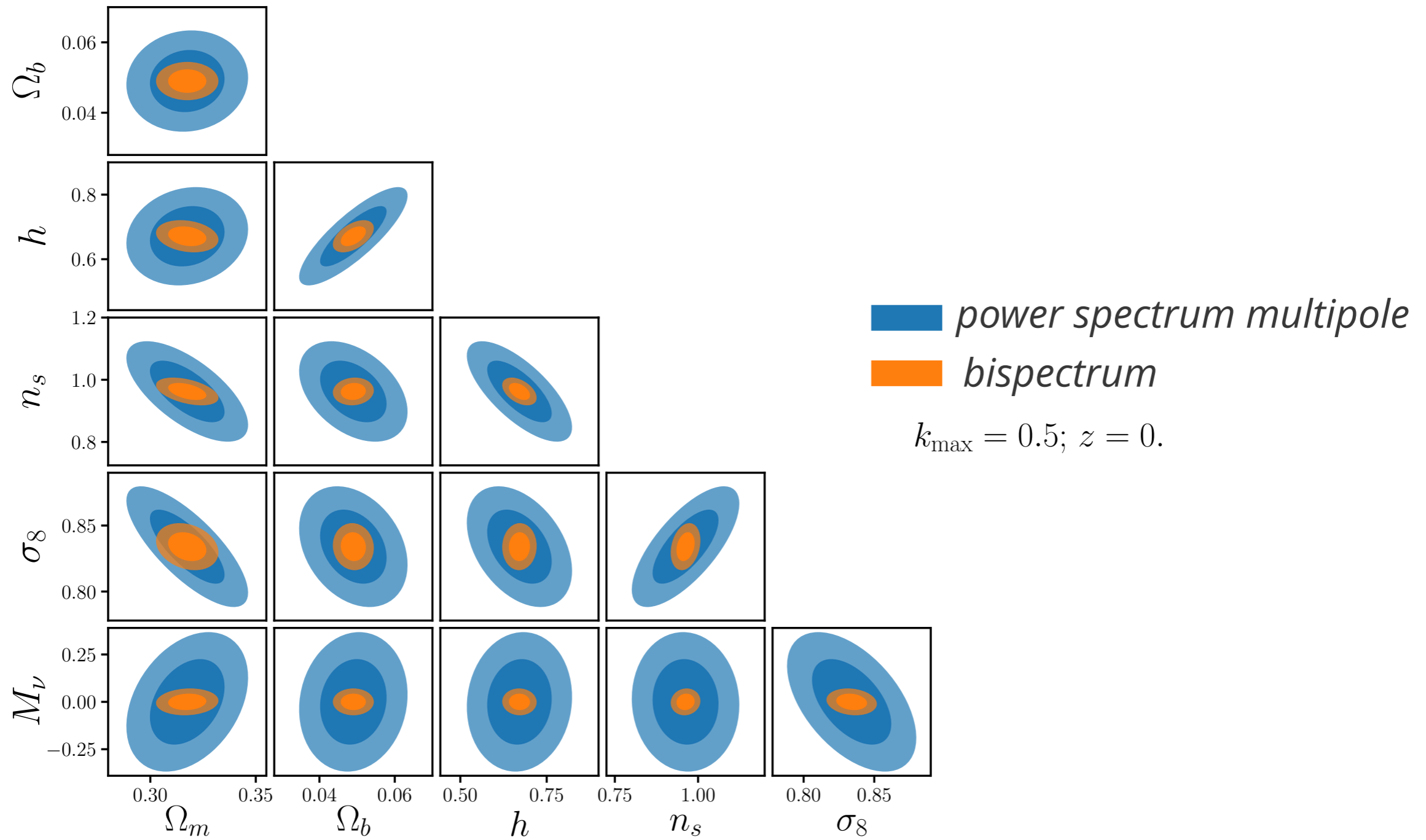
$$\frac{\partial B}{\partial \theta} \approx \frac{B(\theta^+) - B(\theta^-)}{\theta^+ - \theta^-}$$

each box is a different cosmology with **500 N-body** simulations

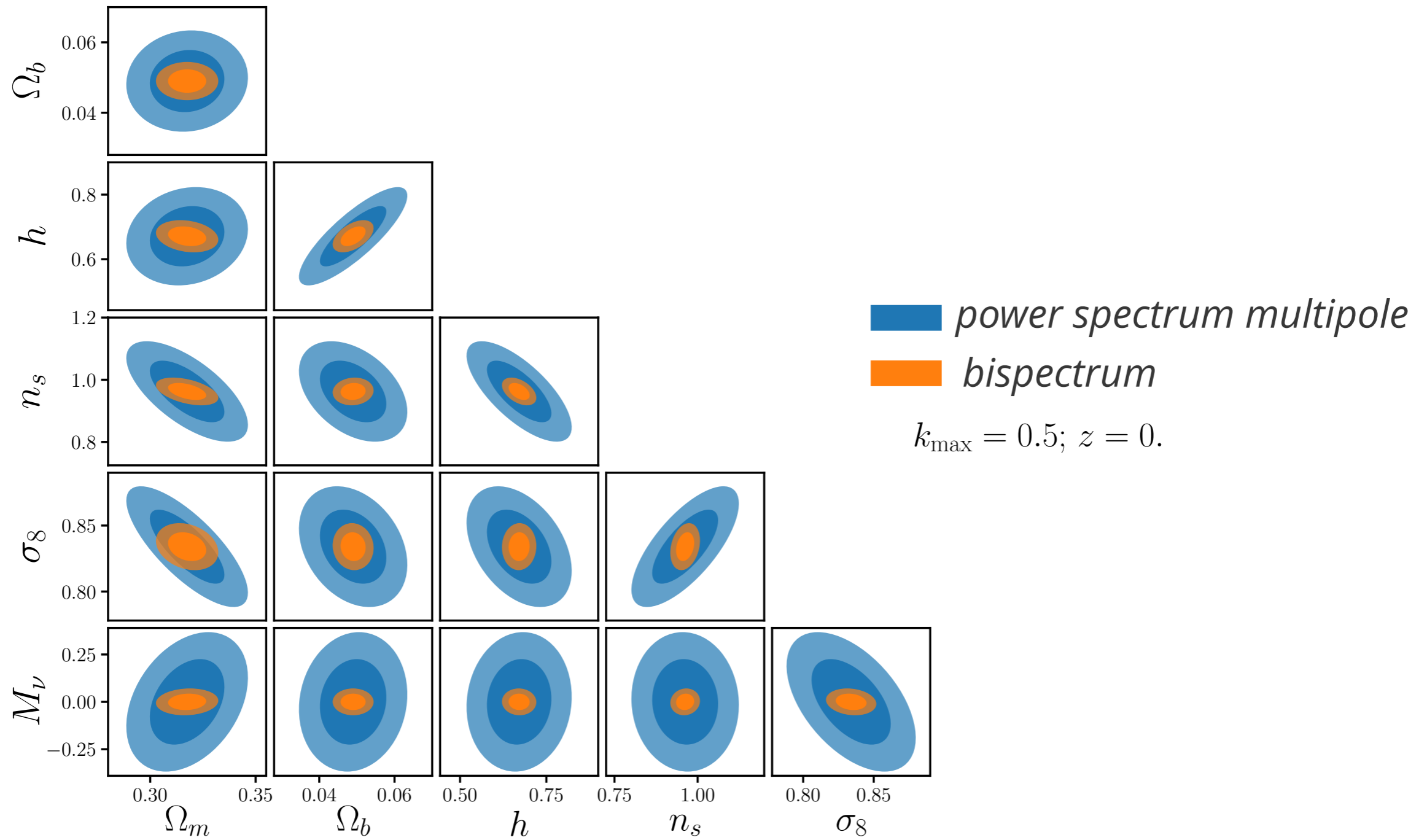








$\{\Omega_m, \Omega_b, h, n_s, \sigma_8\}$  constraints  $\{1.9, 2.6, 3.1, 3.6, 2.6\times\}$  **tighter**



*power spectrum*

$$\sigma_{M_\nu} = \pm 0.297$$

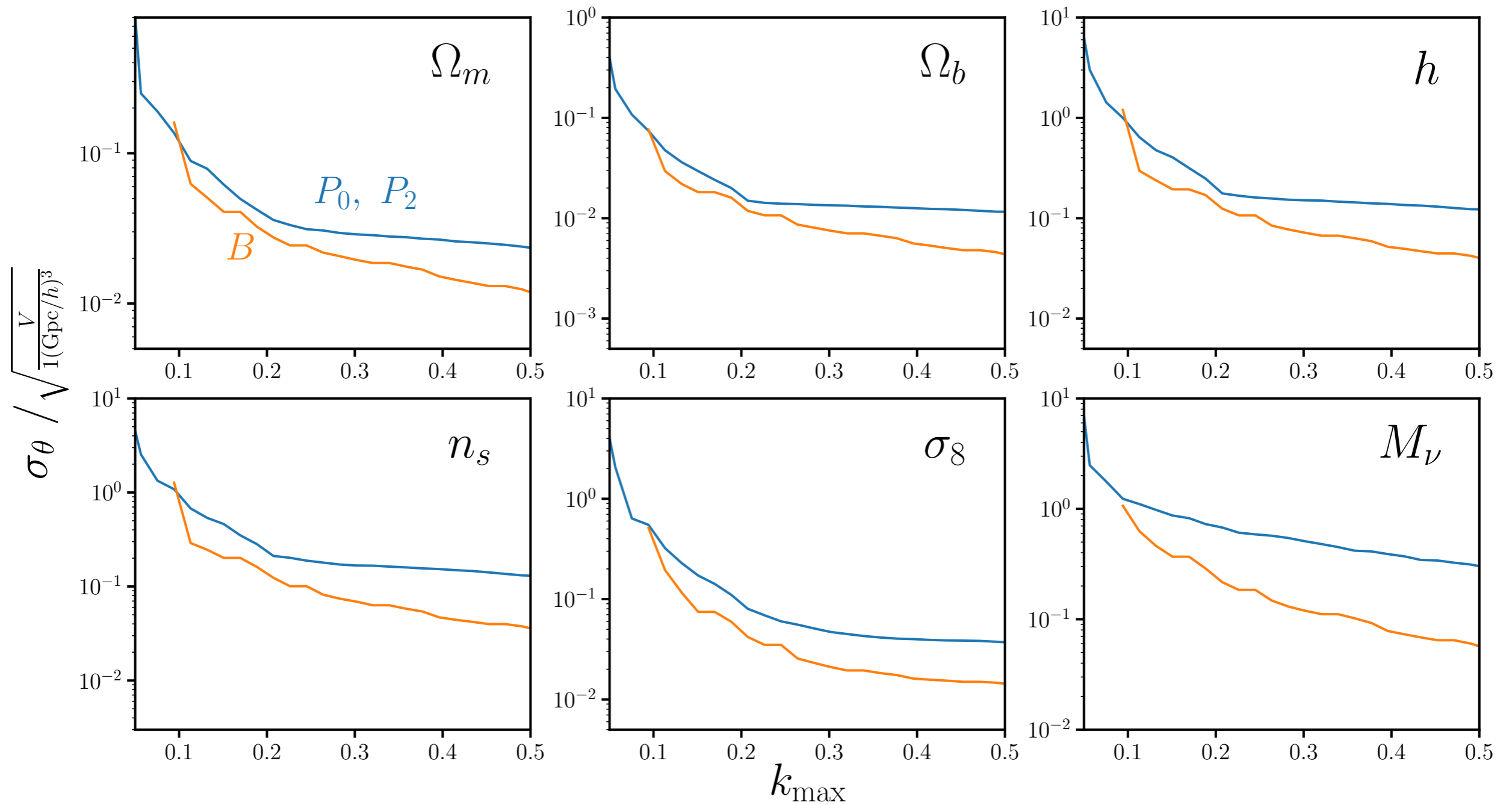


*bispectrum*

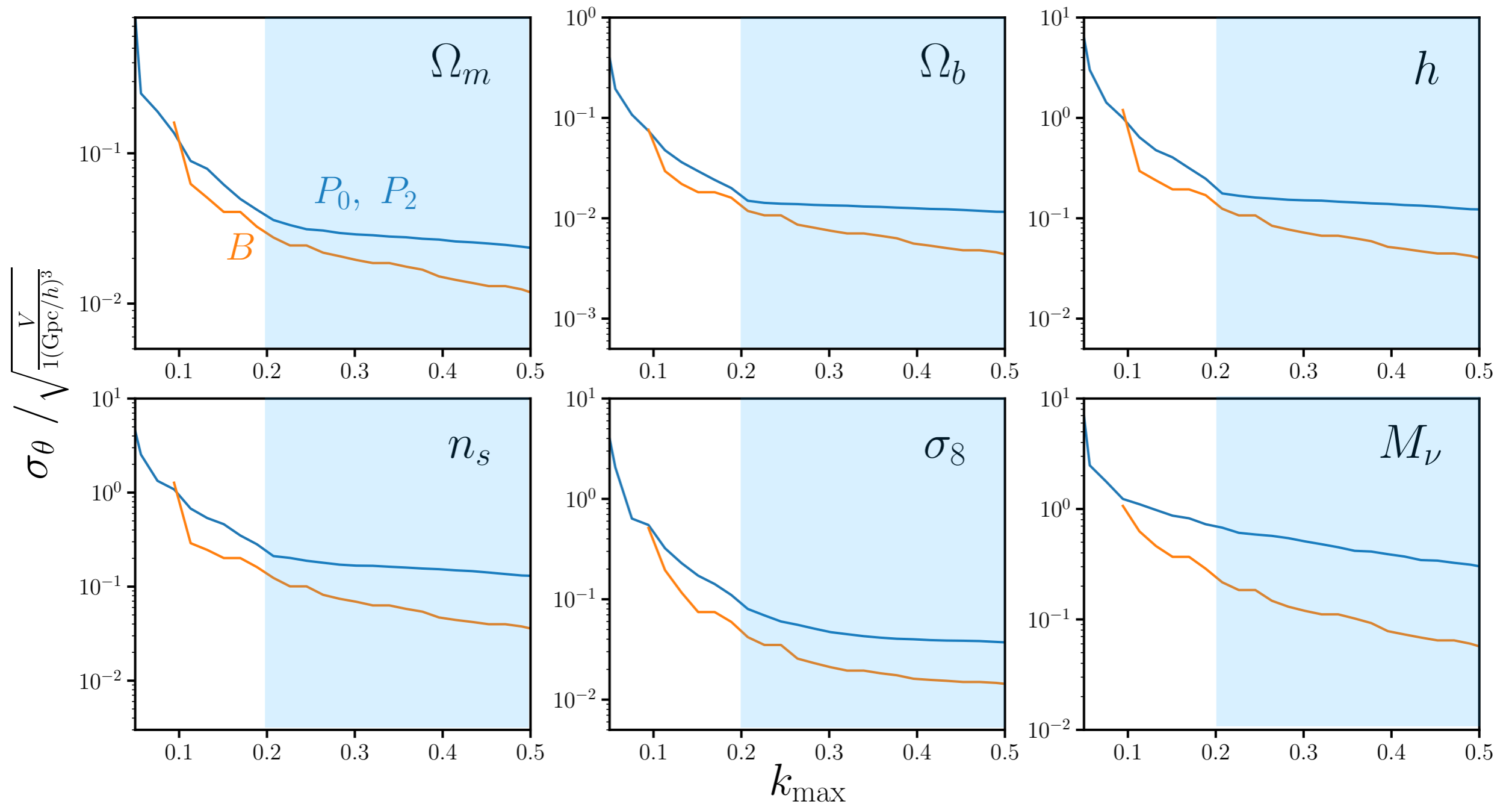
$$\sigma_{M_\nu} = \pm 0.057$$

**>5x tighter  $M_\nu$  constraints**

*the bispectrum* improves parameter constraints **even at low**  $k_{\max}$

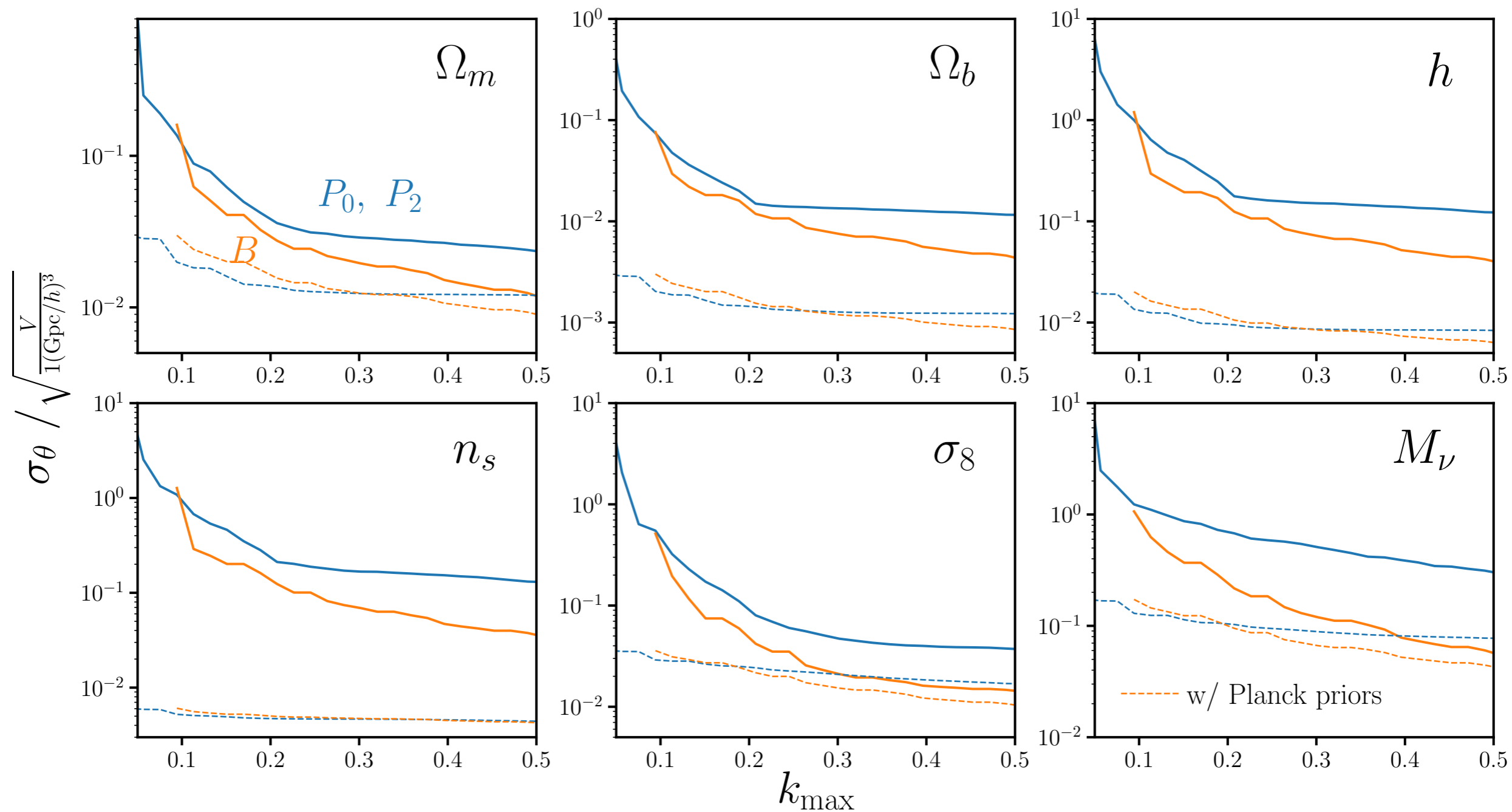


*the bispectrum* improves parameter constraints **even at low**  $k_{\max}$



information content of  $P_\ell$  saturates at  $k_{\max} > 0.2$  h/Mpc

*the bispectrum* improves parameter constraints **with Planck priors**



the redshift-space bispectrum breaks parameter degeneracies and ***significantly improves*** parameter constraints

1.9, 2.6, 3.1, 3.6,  $2.6 \times$  tighter than  $P_\ell$  for  $\Omega_m, \Omega_b, h, n_s, \sigma_8$   
 $5 \times$  tighter  $M_\nu$  constraints

the redshift-space bispectrum\* breaks parameter degeneracies  
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***halo bispectrum***



the redshift-space bispectrum\* breaks parameter degeneracies  
and ***significantly improves*** parameter constraints

what about the redshift-space ***galaxy*** bispectrum?

***halo bispectrum***

*marginalizing over  $b_1, b_2, \gamma_2, M_{\min}, A_{\text{SN}}, B_{\text{SN}}$*  **does not impact** the improvement

*marginalizing over  $b_1, b_2, \gamma_2, M_{\min}, A_{\text{SN}}, B_{\text{SN}}$  does not impact the improvement but **perturbation theory is not enough** to exploit the *substantial constraining power on nonlinear scales**

instead with **simulation-based methods** accuracy of *N-body simulations* can be exploited to analyze nonlinear scales

*e.g.*

*emulation (Zhai+2019, Wibking+2019)*

*approximate bayesian computation (Hahn+2017)*

*evidence modeling (Lange+2019)*

*density estimation (Alsing+2018, Hahn+2019a)*

...

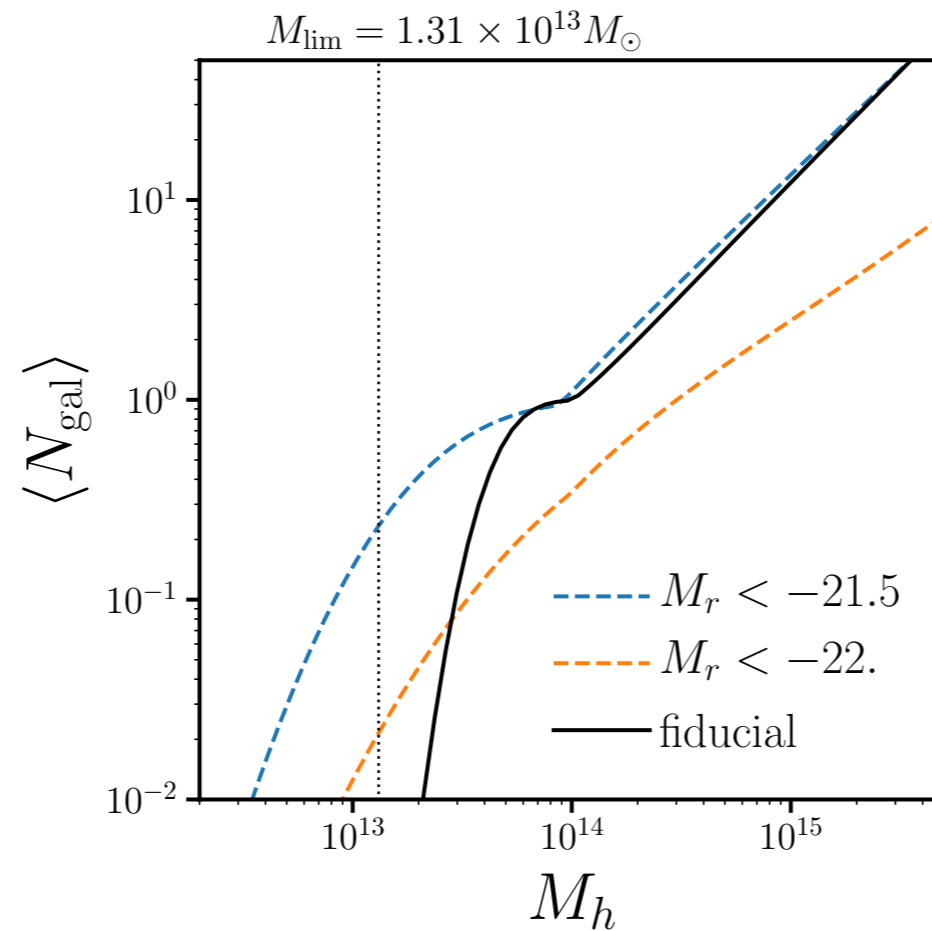
**Halo Occupation Distribution (HOD)** is the primary *galaxy bias framework* for simulation based methods

$$\langle N_{\text{cen}} \rangle = \frac{1}{2} \left[ 1 + \text{erf} \left( \frac{\log M_h - \log M_{\text{min}}}{\sigma_{\log M}} \right) \right] \quad \langle N_{\text{sat}} \rangle = \langle N_{\text{cen}} \rangle \left( \frac{M_h - M_0}{M_1} \right)^\alpha .$$

**Halo Occupation Distribution (HOD)** is the primary *galaxy bias framework* for simulation based methods

with *HOD + Quijote*, we can quantify the **total information content** of *the redshift-space galaxy bispectrum*

**fiducial HOD** based on best-fit HOD parameters for the *SDSS*  
 $M_r < -21.5$  and  $M_r < -22$  samples

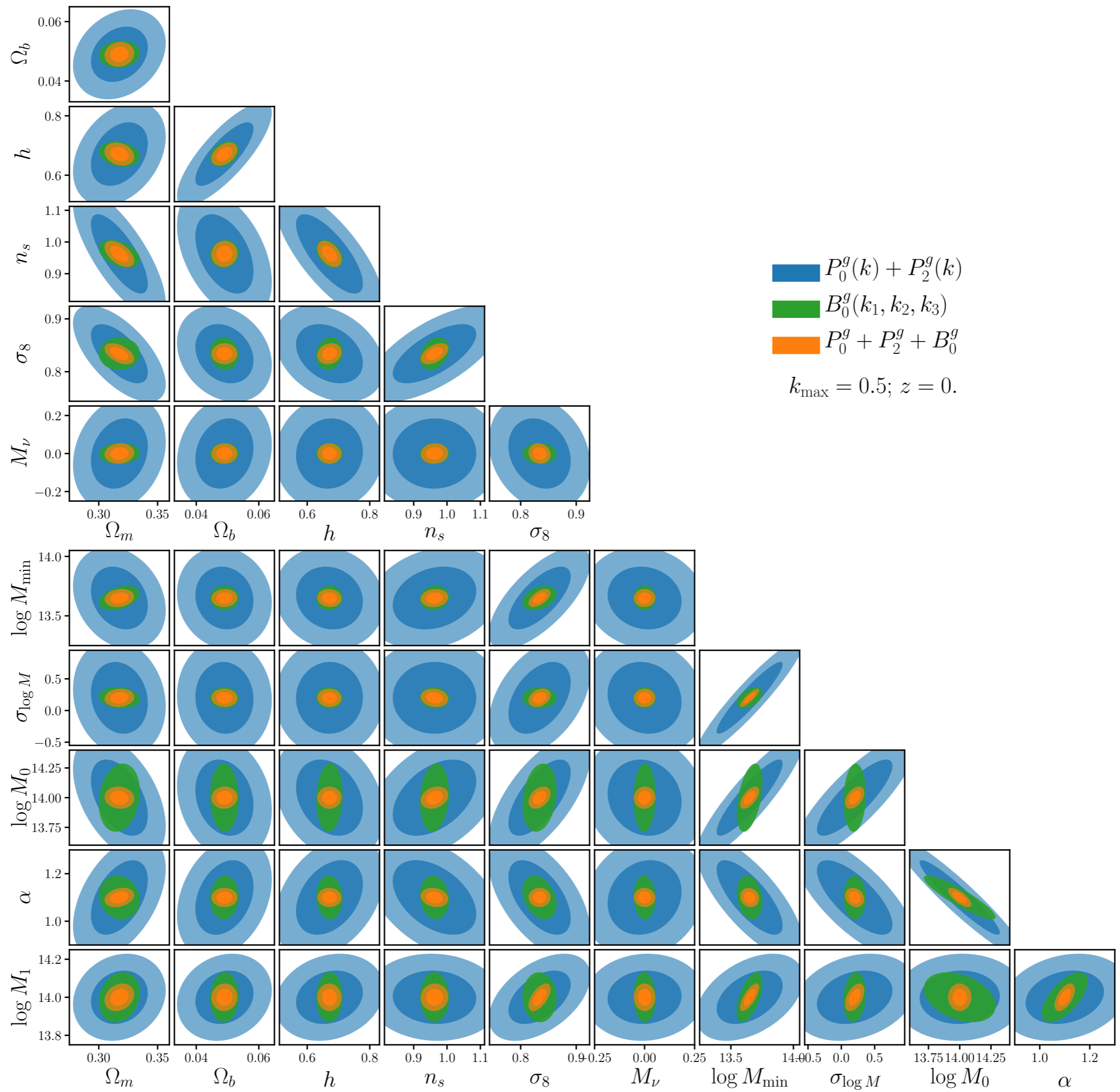


15,000 covariance matrix N-body mocks +  
 (500 N-body) x (5 HOD) x (3 RSD dir.) x (24 param)

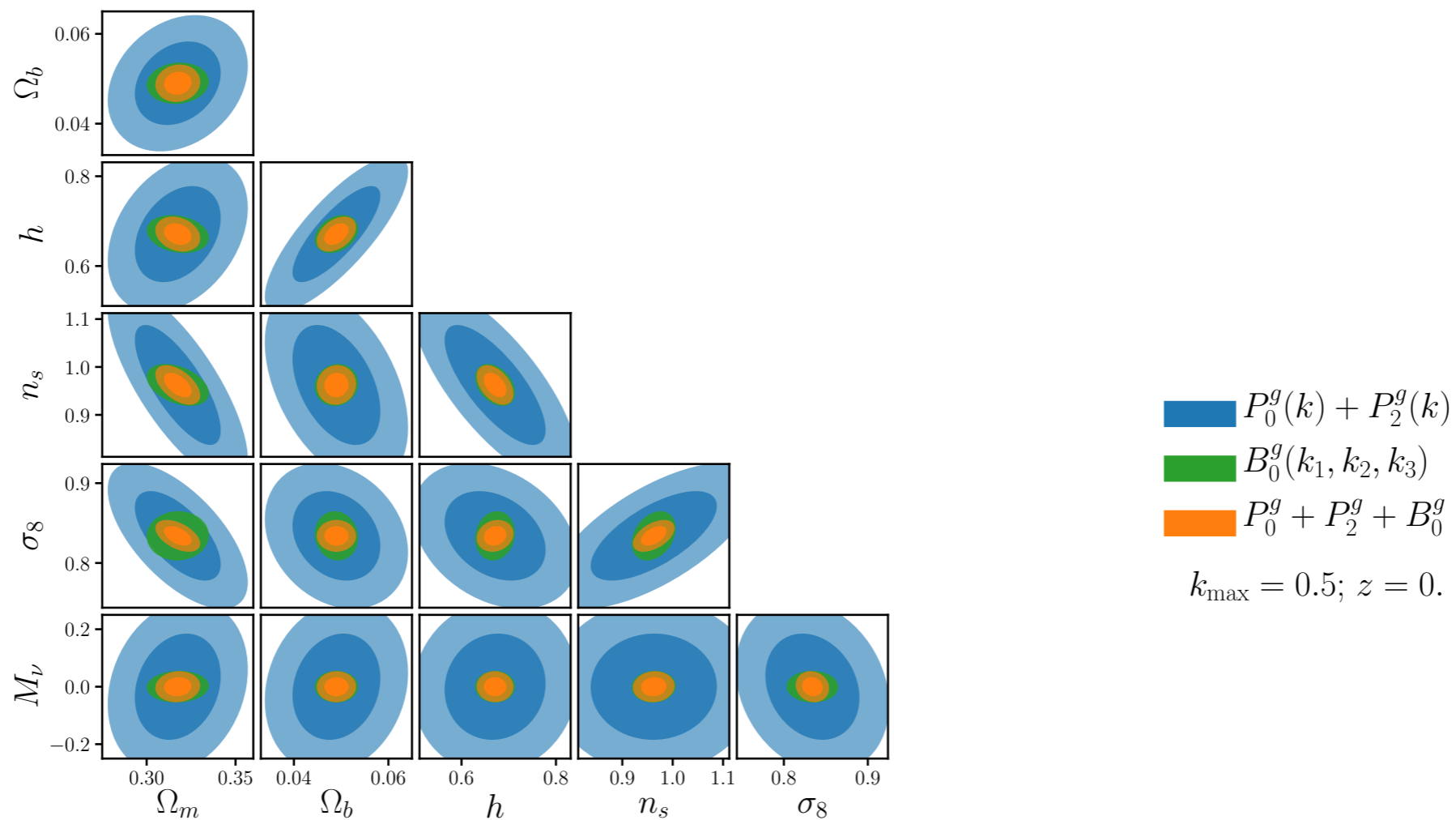
= **195,000 galaxy catalogs**

$M_\nu^{+++} = 0.4$											
$M_\nu^{++} = 0.2$											
$M_\nu^+ = 0.1$	$\Omega_m^+ = 0.3275$	$\Omega_b^+ = 0.051$	$h^+ = 0.6911$	$n_s^+ = 0.9824$	$\sigma_8^+ = 0.849$	$M_{\min}^+ = 13.7$	$\sigma_{\log M}^+ = 0.22$	$\log M_0^+ = 14.2$	$\alpha^+ = 1.3$	$\log M_1^+ = 14.2$	
$M_\nu = 0.0\text{eV}$	$\Omega_m = 0.3175$	$\Omega_b = 0.049$	$h = 0.6711$	$n_s = 0.9624$	$\sigma_8 = 0.834$	$M_{\min} = 13.65$	$\sigma_{\log M} = 0.2$	$\log M_0 = 14$	$\alpha = 1.1$	$\log M_1 = 14$	
	$\Omega_m^- = 0.3075$	$\Omega_b^- = 0.047$	$h^- = 0.6511$	$n_s^- = 0.9424$	$\sigma_8^- = 0.819$	$M_{\min}^- = 13.6$	$\sigma_{\log M}^- = 0.18$	$\log M_0^- = 13.8$	$\alpha^- = 0.9$	$\log M_1^- = 13.8$	

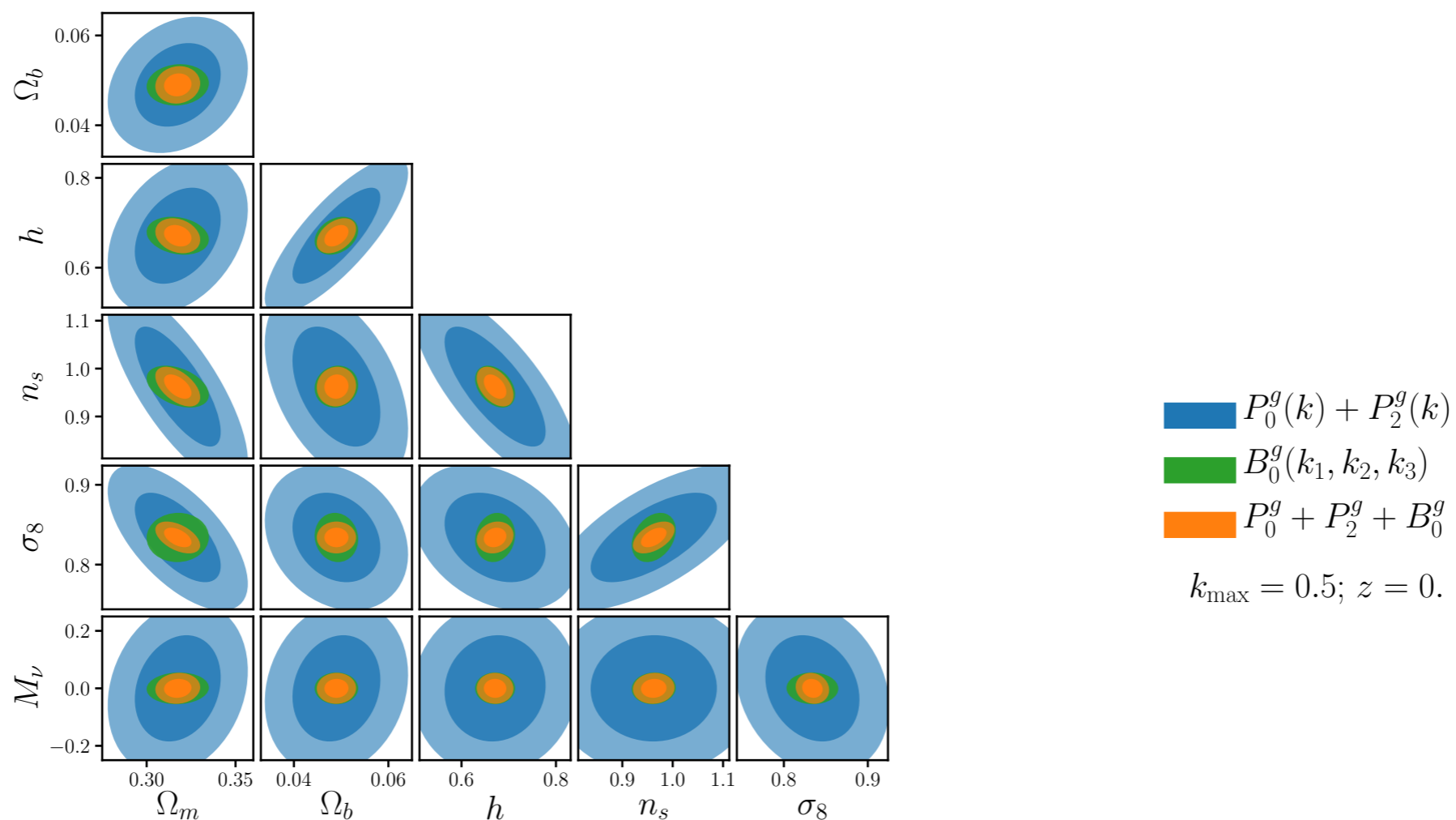




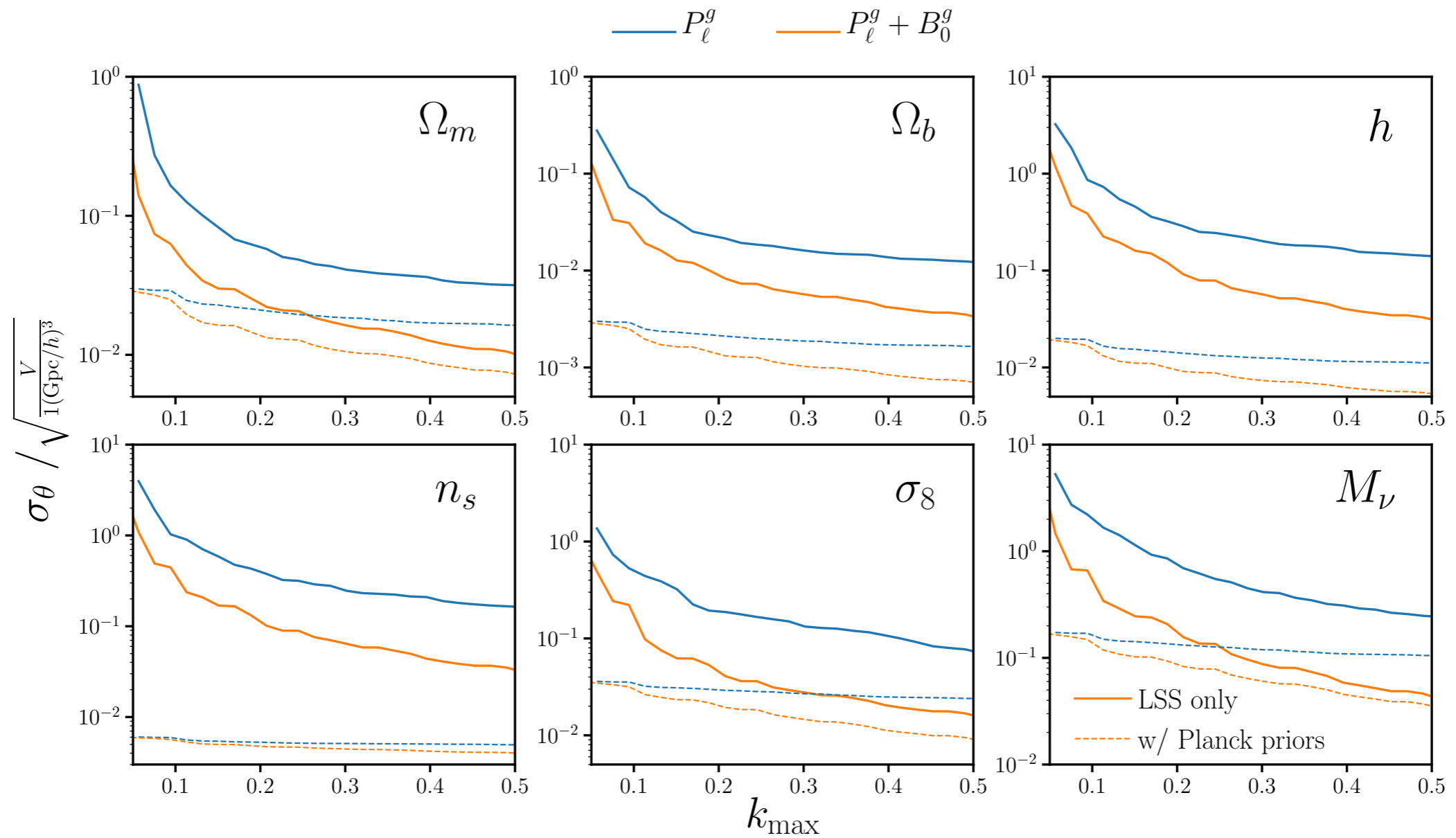
joint galaxy  $P_\ell + B$  constraints on  $\Omega_m$ ,  $\Omega_b$ ,  $h$ ,  $n_s$ ,  $\sigma_8$  are  $\gtrsim 3 \times$  **tighter** than the galaxy  $P_\ell$  alone



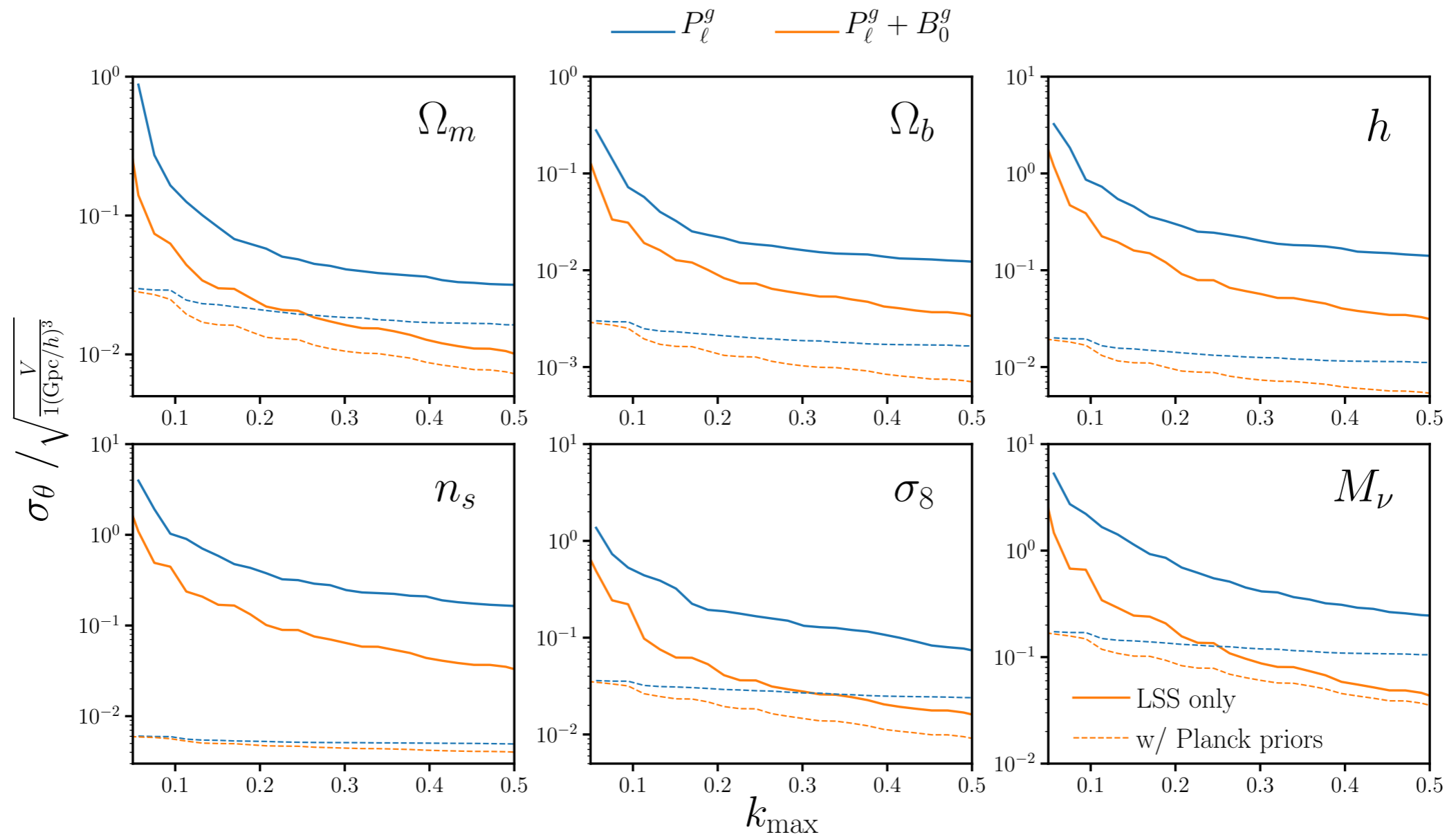
joint galaxy  $P_\ell + B$  constraints on  $\Omega_m$ ,  $\Omega_b$ ,  $h$ ,  $n_s$ ,  $\sigma_8$  are  $\gtrsim 3 \times$  **tighter** than the galaxy  $P_\ell$  alone;  $M_\nu$  constraint is  $\gtrsim 5 \times$  **tighter**



# significant **constraining power** on small *nonlinear scales*



$\gtrsim 2 \times$  **tighter** parameter constraints even *with Planck priors*



**some caveats:** assembly bias, velocity biases, baryonic effects, fisher forecast, survey geometry

*these caveats will also affect the power spectrum!*

**some caveats:** assembly bias, velocity biases, baryonic effects, fisher forecast, survey geometry

forecasts are *for*  $(1 \text{ Gpc}/h)^3$  box and  $\bar{n}_g \sim 1.64 \times 10^{-4}$  sample

*PFS 9 (Gpc/h)<sup>3</sup>*

*DESI 50 (Gpc/h)<sup>3</sup>*

*DESI BGS sample 20x higher  $\bar{n}$*

*DESI LRG sample 3x higher  $\bar{n}$*

the *galaxy bispectrum* significantly improves parameter constraints beyond the power spectrum — 5x tighter  $M_V$  constraints

the significant constraining power in the *nonlinear regime* can be exploited using **simulation-based methods**

next up: data compression, forward modeling systematics, fully simulation-based  $P_\ell$  and  $B$  SDSS-III BOSS reanalysis