Kinetic Field Theory: Velocity Statistics

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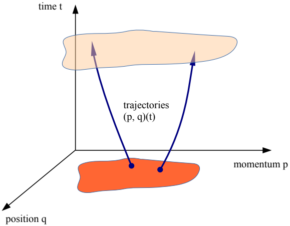
Basic Idea of KFT

- Generating Functional

$$Z[\mathbf{J}] = \int \mathrm{d}\mathbf{q}\mathrm{d}\mathbf{p} \quad \underbrace{P(\mathbf{q},\mathbf{p})}_{\text{initial conditions}} \quad \mathrm{e}^{\mathrm{i}<\mathbf{J},\bar{\mathbf{x}}>}$$

- Particle motion given by a retarded Green's function

$$\bar{\mathbf{x}} = \underbrace{\mathcal{G}(t,0)x^{(i)}}_{\text{inertial motion}} - \underbrace{\int \mathrm{d}t'\mathcal{G}(t,t')\mathcal{F}(t')}_{\text{interactions}}$$



Velocity Operator

- particle velocities contain no spatial information

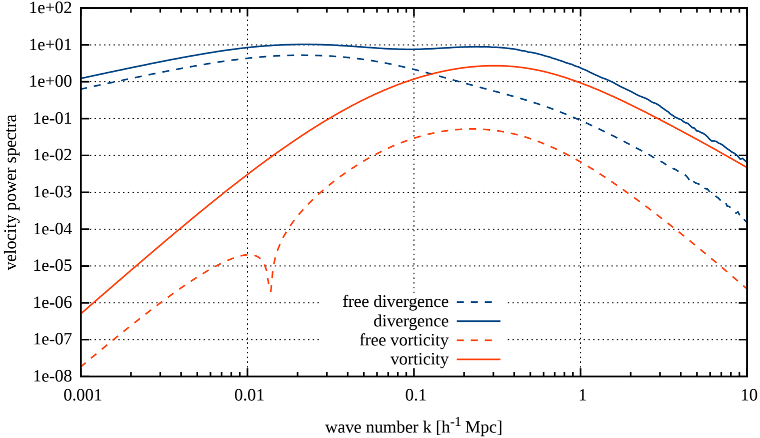
$$\vec{\pi} = \sum_{i=1}^{N} \vec{p}_i \delta_D(\vec{q} - \vec{q}_i) \rightarrow \sum_{i=1}^{N} \vec{p}_i \exp\left(-\mathrm{i}\vec{k} \cdot \vec{q}_i\right)$$

$$\hat{\vec{\pi}} = \sum_{i=1}^{N} \frac{\delta}{\delta J_{p_i}(t)} \exp\left(-\mathrm{i}\vec{k} \cdot \frac{\delta}{\delta J_{q_i}(t)}\right)$$

- 2-point correlation matrix:

$$M_\pi(k, a) = \hat{\vec{\pi}}(1) \otimes \hat{\vec{\pi}}(2) Z[\mathbf{J}] \,|_{J=0}$$

Velocity Field - Divergence and Vorticity



Sunyaev-Zeldovich Effect

- Comptonization of CMB photons
  - *thermal:* hot electron gas in clusters/galaxies
  - *kinetic:* structures moving relative to Hubble flow
- temperature distortion

$$\frac{\Delta T}{T} = -\frac{\sigma_T \bar{n}_e}{c} \int \frac{\mathrm{d}s}{a^2} \mathrm{e}^{-\tau} \vec{\pi} \cdot \hat{\gamma}$$

$$\vec{\pi} = (1 + \delta)\vec{v} \qquad \hat{\gamma} = \text{line-of-sight direction}$$

- angular power-spectrum

$$C_l = \left(\frac{\sigma_T \bar{n}_e}{c}\right)^2 \int \frac{\mathrm{d}s}{s^2 a^4} \mathrm{e}^{-2\tau} \mathbf{P}_{\pi_\perp}(k = l/s, a)$$

Linear Theory

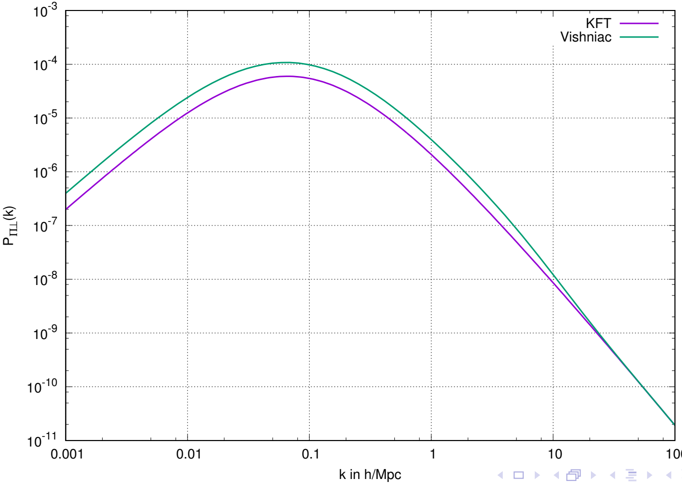
- classic result by Vishniac (1987)

$$P_{\pi,\perp}^{\mathrm{lin}}(k,t) \propto \int_{k'} P_\delta(k',t) P_\delta(\Delta,t) \frac{k^2(k-2k'\mu)^2(1-\mu^2)}{k'^2\Delta^4}$$

- KFT result

$$P_{\pi,\perp}^{\mathrm{KFT}}(k,t) \propto \int_{k'} P_\delta(k',t) P_\delta(\Delta,t) \frac{k^2(k-k'\mu)(k-2k'\mu)(1-\mu^2)}{k'^2\Delta^4}$$

Vishniac (1987) vs. KFT



Non-linear Theory

- Park et al. (2016) schematically:

$$P_{\pi,\perp}(k,t) = \langle\delta\delta\rangle\langle vv\rangle + 2\langle v\delta\rangle^2 + \langle v\delta v\delta\rangle_c$$

  - need to evaluate connected term at 1-loop order
- KFT with mean-field interactions

$$\begin{aligned} P_{\pi,\perp}(k,t) &= -2\mathrm{e}^{i\langle S_I\rangle - Q_D}\int \mathrm{d}^3q\, a_\perp(\xi'_{12},\xi''_{12})\mathrm{e}^{Q-i\vec{k}\cdot\vec{q}} \\ Q &= -g_{qp}^2(t)k^2 a_\parallel(\xi'_{12},\xi''_{12}) \end{aligned}$$

Park et al. (2016) vs. KFT

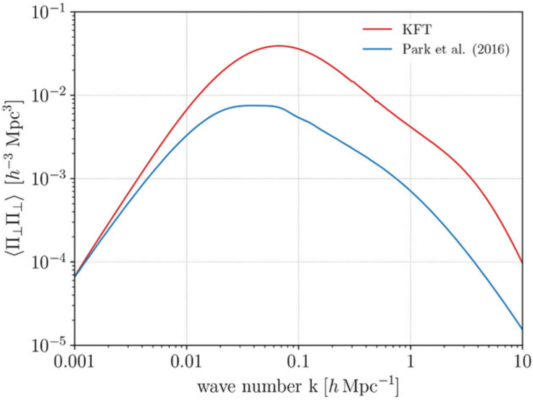


Figure: from: Bartelmann et al., Cosmic Structure Formation with Kinetic Field Theory. ANNALEN DER PHYSIK 2019, 531, 1800446

Summary

- KFT is an analytical, particle-based approach for cosmic large-scale structure formation
- momentum-density correlations can be calculated with less effort compared to standard perturbation theory
- we recover basic properties of the transverse momentum-density field