# Small-Scale Structure from Interlopers

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based on *Quantifying the Line-of-Sight Halo Contribution to the Dark Matter Convergence Power Spectrum from Strong Gravitational Lenses* (arxiv:2006.07383)

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Cosmology from Home 2020

# Outline

1. Motivation: Small-scale structure
2. Previous work: Lensing
3. Previous work: Interlopers (line of sight)
4. Methods
5. Results
6. Conclusion

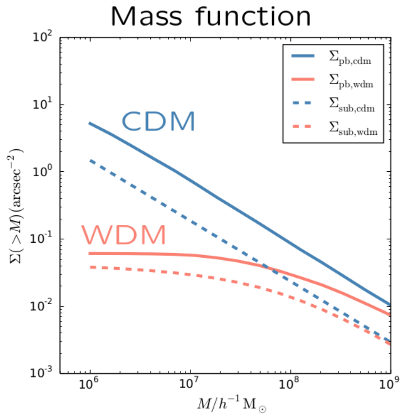
# 1. Motivation: Small-scale structure

# Motivation: Testing ΛCDM on galaxy scales

**Large-scale structure**: Observations match ΛCDM. ✓

**Small-scale structure** ($\lesssim 10^8\, M_\odot$): No clear constraints yet. ✗

This structure would shed light on dark matter!




(adapted from Li+ '17)

# Why no clear constraints yet?

- Small halos $\Rightarrow$ Low star formation $\Rightarrow$ Dark, hard to observe
- *Missing Satellites Problem*[1]: Invisible halos? Tidal stripping?

[1] Fewer local dwarf galaxies observed than expected under $\Lambda$CDM

# Why no clear constraints yet?

- Small halos $\Rightarrow$ Low star formation $\Rightarrow$ Dark, hard to observe
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Solution: Gravitational lensing (and interlopers).

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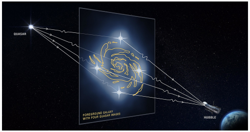
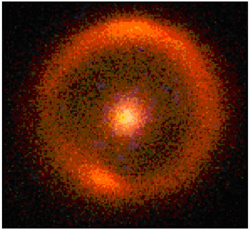
# 2. Previous work: Lensing

# Gravitational lensing




[NASA/ESA/Player (STScI)]

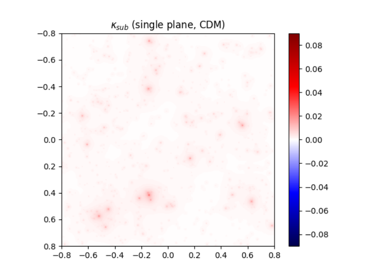


[Lagattuta/Keck]

- Perturbations to strong lensing systems
  - Possible to study quadruply-imaged quasars (Gilman+ 2018)
  - We focus on galaxy-galaxy systems
- Convergence field, $\kappa$
  - Proportional to mass in the case of single-plane lensing
  - Analyze small-scale structure in $\kappa$ as subhalos
  - Not directly observable, but larger individual subhalos (Vegetti+ 2012) and statistics of smaller subhalos (Hezaveh+ 2016) can be detected

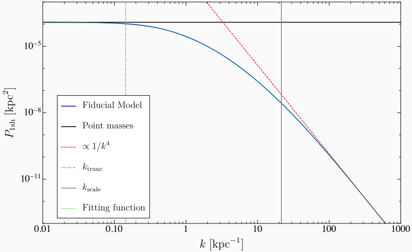
# Subhalo convergence power spectrum

Convergence ($\kappa_{\rm sub}$)




$\Rightarrow$

Power spectrum



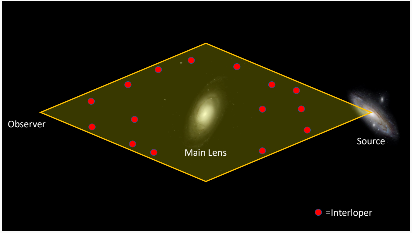

Power spectrum of convergence $\Rightarrow$ statistical properties

- Subhalo mass function
- Subhalo parameters (profile, characteristic radius)

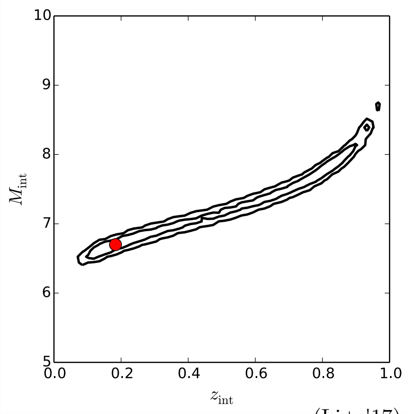
[A. Diaz Rivero, F.-Y. Cyr-Racine, C. Dvorkin 2018]

# 3. Previous work: Interlopers (line of sight)

# Interlopers intuition




Mass-Redshift Degeneracy




(Li+ '17)

Key idea: Interlopers produce deflections similar to subhalos.

# Interloper/subhalo relative abundance

Raw number: Interlopers $>$ Subhalos

- CDM $4\times$, WDM 1.5-$2\times$ for a fiducial system (Li$+$ 2017)
- CDM and WDM: $\sim$ 2-$10\times$ (Despali$+$ 2018)

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This suggests that interlopers contribute significantly to the "subhalo" power spectrum!

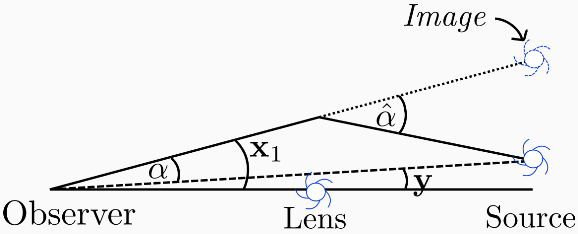
# 4. Methods

# Our probe

The convergence power spectrum of *interlopers*.

- Develop theory to quantify it analytically
- Confirm with numerical ray-tracing
- Compare with power spectrum of subhalos

# Single-plane lensing



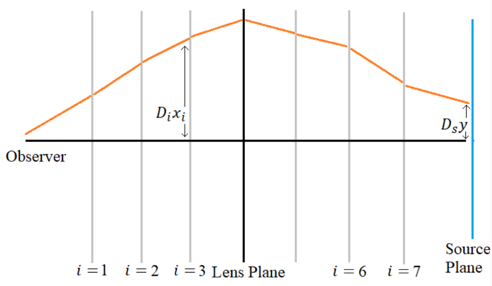

Thin-lens approximation: $\hat{\boldsymbol{\alpha}}$ is a function of lens mass distribution and $\perp$-distance from lens.

The *deflection angle* is $\boldsymbol{\alpha} = \frac{D_{ls}}{D_s}\hat{\boldsymbol{\alpha}}$.

The *lens equation* relates the image angular position, $\mathbf{x}_1$, the deflection angle, $\boldsymbol{\alpha}$, and the source angular position, $\mathbf{y}$:

$$\mathbf{y} = \mathbf{x}_1 - \boldsymbol{\alpha}$$

# Multi-plane lensing




The *multi-plane lens equation* traces rays from observer to source:

$$\mathbf{x}_j = \mathbf{x}_1 - \sum_{i=1}^{j-1} \beta_{ij} \boldsymbol{\alpha}_i(\mathbf{x}_i)$$

where $\beta_{ij} = \frac{D_{ij} D_s}{D_j D_{is}}$. We treat the source as a final plane $\mathbf{y} = \mathbf{x}_{N+1}$.

# Solving for effective convergence

Definition of $\kappa_{\text{eff}}$, analogous to $\kappa$ from single-plane lensing:

$$\frac{\partial \mathbf{y}}{\partial \mathbf{x}_1} = I - \begin{pmatrix} \kappa_{\text{eff}} + \gamma_{1,\text{eff}} & \gamma_{2,\text{eff}} \\ \gamma_{2,\text{eff}} & \kappa_{\text{eff}} - \gamma_{1,\text{eff}} \end{pmatrix} \mathbf{x}_1$$

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Derivative of multi-plane lens equation:

$$\frac{\partial \mathbf{y}}{\partial \mathbf{x}_1} = I - \underbrace{\sum_{i=1}^{l-1} \frac{\partial \boldsymbol{\alpha}_i(\mathbf{x}_i)}{\partial \mathbf{x}_1}}_{\text{foreground}} - \underbrace{\frac{\partial \boldsymbol{\alpha}_l(\mathbf{x}_l)}{\partial \mathbf{x}_1}}_{\text{main-lens coupling}} - \underbrace{\sum_{i=l+1}^{s-1} \frac{\partial \boldsymbol{\alpha}_i(\mathbf{x}_i)}{\partial \mathbf{x}_1}}_{\text{background}}$$

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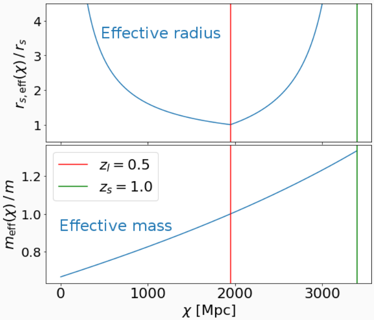
After CSB (Critical Sheet Born) approximation,

$$\kappa_{\text{eff}}(\mathbf{x}) = \underbrace{\sum_{i=1}^{l-1} (1-\beta_{il})\kappa_i(\mathbf{x})}_{\text{foreground + coupling}} + \underbrace{\sum_{i=l+1}^{N} (1-\beta_{li})\kappa_i\big((1-\beta_{li})\mathbf{x}\big)}_{\text{background}}$$

[A. Ç. Şengül, **A. Tsang**, A. Diaz Rivero, C. Dvorkin, H.-M. Zhu, U. Seljak, `arxiv:2006.07383`]

# Interlopers as effective subhalos

From $\kappa_{\text{eff}}$, we work out analytic expressions for the effective radius and mass of an interloper.




$$r_{\mathrm{s,eff}}(\chi) = \frac{D_l}{g(\chi) D_\chi} r_{\mathrm{s}}$$

$$m_{\mathrm{eff}}(\chi) = f(\chi) \frac{\Sigma_{\mathrm{cr},l}}{\Sigma_{\mathrm{cr},\chi}} \left( \frac{D_l}{g(\chi) D_\chi} \right)^2 m$$

[A. Ç. Şengül, **A. Tsang**, A. Diaz Rivero, C. Dvorkin, H.-M. Zhu, U. Seljak, `arxiv:2006.07383`]

# Interloper power spectrum

Two equivalent derivations:

- Limber approximation: 2D density as integral of 3D density with weighting (weak lensing formalism)
- Project interlopers onto main lens plane, then $\mathcal{F}$-transform the 2-pt correlation function

$$P_{\mathrm{2D}}(k) = \underbrace{\left(\frac{4\pi G}{c^2}\right)^2 D_l^2}_{\text{prefactor}} \int d\chi \underbrace{\left[\frac{f(\chi) D_{\chi s} \chi^2}{D_s D_\chi}\right]^2}_{\kappa_{\text{eff}}\text{ weighting}} \underbrace{\frac{1}{g^2(\chi)\chi^2}}_{\text{3D}\rightarrow\text{2D density}}$$

$$\times \int dm \underbrace{n(m,\chi)}_{\text{mass func.}} m^2$$

$$\times \underbrace{\int d^2\vec{q}\, \mathcal{P}(\vec{q}\,|\, m, \chi)}_{\text{integral over profile params}} \underbrace{\left|\phi\left(\frac{D_l r_{\mathrm{s}}}{g(\chi) D_\chi} k\,;\, \tau\right)\right|^2}_{\text{effective subhalo profile}}$$

[A. Ç. Şengül, **A. Tsang**, A. Diaz Rivero, C. Dvorkin, H.-M. Zhu, U. Seljak, `arxiv:2006.07383`]

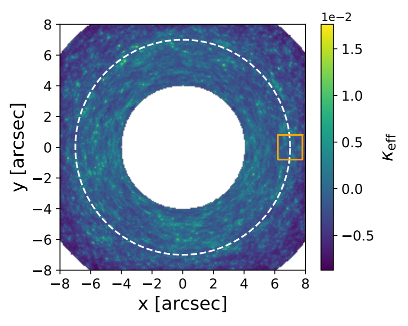
# Numerical calculation

To check validity of analytic approximations.

- `lenstronomy` multi-plane ray-tracing ⇒ deflection angles $\boldsymbol{\alpha}$

$$\kappa_{\text{eff}} = \frac{1}{2}\nabla \cdot \boldsymbol{\alpha}$$

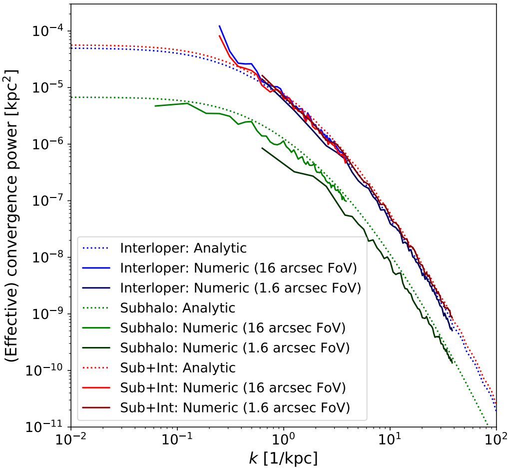
- Simulate at two resolution scales
- Annular mask at large scales




[A. Ç. Şengül, **A. Tsang**, A. Diaz Rivero, C. Dvorkin, H.-M. Zhu, U. Seljak, `arxiv:2006.07383`]

# 5. Results

# Analytic and numeric results match

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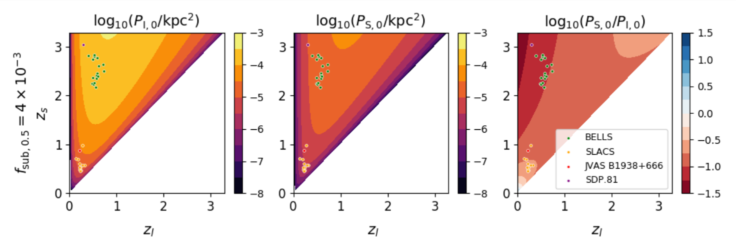
# Power spectrum depends strongly on redshift and mass function

Focus on *plateau* of the power spectrum curve ($k \to 0$ limit).

- Redshifts: $z_l$ and $z_s$
- Mass functions
  - Subhalo normalization: $f_{\rm sub} \equiv M_{\rm subhalo}/M_{\rm DM}$
  - Halo (interloper) mass function: Press-Schechter or Sheth-Tormen

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# $f_{\text{sub}}$ is poorly constrained

- Tidal stripping $\Rightarrow$ fewer/smaller subhalos
- Extent is not well quantified (with or without baryons)
- $f_{\text{sub}} \sim \mathcal{O}(10^{-3}$ or $10^{-2})$ for substructure in $[10^5\text{-}10^8]\, M_\odot$

Online widget with $f_{\text{sub}}$ slider at

`https://arthur-tsang.github.io/interloper_widget.html`

# 6. Conclusion

# Conclusion

Summary

- Want to test ΛCDM on small scales
- Focus on perturbations to galaxy-galaxy lensing systems
- Expect more interlopers (line of sight) than subhalos
- New probe: interloper effective convergence power spectrum
- As long as $f_{\text{sub}} \lesssim 2\%$, interlopers will dominate for SLACS-like lenses ($\lesssim 4\%$ for BELLS)
  - Simulations suggest this is likely

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Reflections & Future work

- Good news if interlopers dominate over subhalos
    - Stronger signal
    - Theoretical mass function is better understood
- Next step: extracting information from the curl, $\frac{1}{2}\nabla \times \boldsymbol{\alpha}$, to distinguish between interlopers and subhalos